Structural Mechanics

Structural Mechanics is the study of behavior of solid / Rigid bodies subjected to various types of loading.

→ These behaviors are further used to Predict the failure of structure.

1. Review of Basic concepts

1.1: Basic Principle of Mechanics:

Force :- A force represents the action of one body on another.

→ force can be generated either by the direct contact of bodies or by their effect at a distance.

→ Force is a vector quantity & represented by the symbol F.

→ Unit → N, KN.

Moment :- It is also known as Bending Moment.

→ It is the measure of the bending effect due to forces acting on a beam. It is measured in terms of force & distance.

→ Moment = force x distance.

Support Conditions

Support in a structure is a member which helps other members to resist loads.

→ It transfers the load to the ground and provides stability to the structure supported on it.

Types

There are mainly 3 types of supports which join a built structure to it's foundation are :-

1. Roller
2. Pinned or hinged
3. Fixed.

Roller support / simple support

Roller supports are free to rotate and translate along the surface upon which the roller rests. ex:- 

→ It moves in x direction, allows rotation
→ It doesn't allow movement in y dirn.

 → It doesnot allow movement in x dirn.

 ex:→ Rubber bearings, set of gears.

Pinned or Hinged support

Pinned support attaches the only web of a beam to a girder called shear connection.

→  Allows rotation, but doesnot allow movement in x & y dirn.

ex:→ Door Hinge.
→ Park (swinging belt)

Fixed Support / Rigid Support.

Rigid or fixed supports maintain the relationship between the joined elements and provide both force & moment resistance.

→ It exerts forces acting in any direction and prevents all translational movements (horizontal and vertical) as well as all rotational movement of a member. ex-  → It does not allow movement in any dirn.

Name	~~Schematic diagram~~	figure	Movement V	Movement H	Movement R (moment)	Reaction Dirn	Reaction Number
Roller or simple.			NO	yes	yes.		1
Pinned or Hinged			NO	NO	yes		2
Fixed or Rigid			NO	~~yes~~	NO		3

Conditions of equilibrium

Equilibrium :— A structure is in equilibrium, when all forces or moments acting upon it are balanced.

→ This means that each and every force acting upon a body, or part of the body, is resisted by either another equal and opposite force or set of forces whose net result is zero.

Conditions

1st condition (The resultant force acting on the object is zero.)

If a resultant force acting on a particle is zero, then the particle will not be having any acceleration.

→ This means that both the net force and the net torque on the object must be zero.

$$F_{net} = 0$$

$$F_x = 0 \quad \text{(summation of forces in x-dir}^n\text{)}$$

$$F_y = 0 \quad \text{(" " " " y-dir}^n\text{)}$$

Second condition (The sum of the moments acting on an object must be zero)

The net torque acting on the object must be zero.

→ Torque means a rotational or twisting effect of a force.

$$\Sigma M = 0.$$

C.G. :- Centre of Gravity (for 3-D)

The centre of gravity of the body may be defined as the point through which the whole weight of the body may be assumed to act, so that if supported at this point the body would remain in equilibrium.

→ It is denoted by C.g. or simply by G.

→ It depends upon the shape of the body.

→ It's position is determined by unit m or mm or cm.

Centroid (for 2-D)

The centroid or centre of area is defined as the point where the whole area of the figure (like triangle, circle etc.) is assumed to be ~~center~~ concentrated.

Moment of Inertia (M.I)

The moment of a force (also called the first moment of force) about any point is the product of the force & the perpendicular distance between them.

→ If this first moment is again multiplied by the perpendicular distance between them, the product so obtained is called the second moment of force or moment of moment of the force.

→ If instead of force, the area of the figure or mass of the body is considered, it is called the second moment of area or second moment of mass.

→ They are also termed as Moment of Inertia (M.I)

→ It is generally denoted by 'I'.

→ It's unit is m^4 or mm^4 or cm^4

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Defn

A quantity expressing a body's tendency to resist angular or area acceleration, which is the sum of product of mass of each particle in a body with the square of distance from the axis of rotation.

$$I = \sum_{i=1}^{i=n} M_i r_i^2$$

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots M_n r_n^2$$

M = mass of body

r = distance of body from the axis of rotation.

It is of 2 types.

1. Mass M.I. → $I = Mr^2$
2. Area M.I. → $I = Ar^2$

Free Body Diagram

A diagram, which shows a body separately, indicating all the external forces acting on it, is called as a free Body Diagram.

→ It is a graphical Illustration which is used to visualize the applied forces, moments & resulting reactions on a body in a given condition.

Review of CG & MI of different Sections.

Steps for finding out C.G.

1st step : Reference Axis

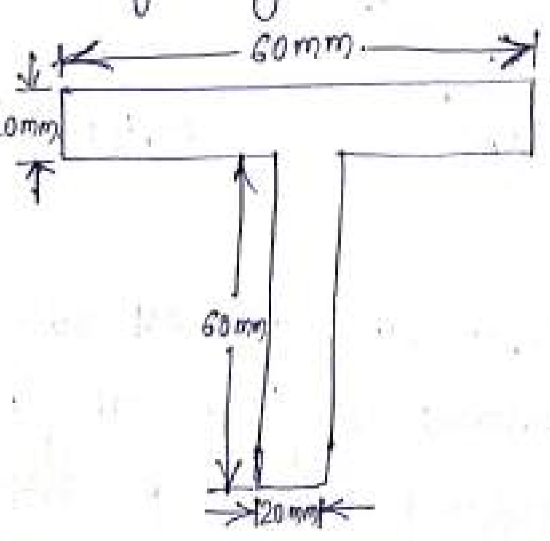
2nd step : Check the given section is symmetrical about any axis or not?

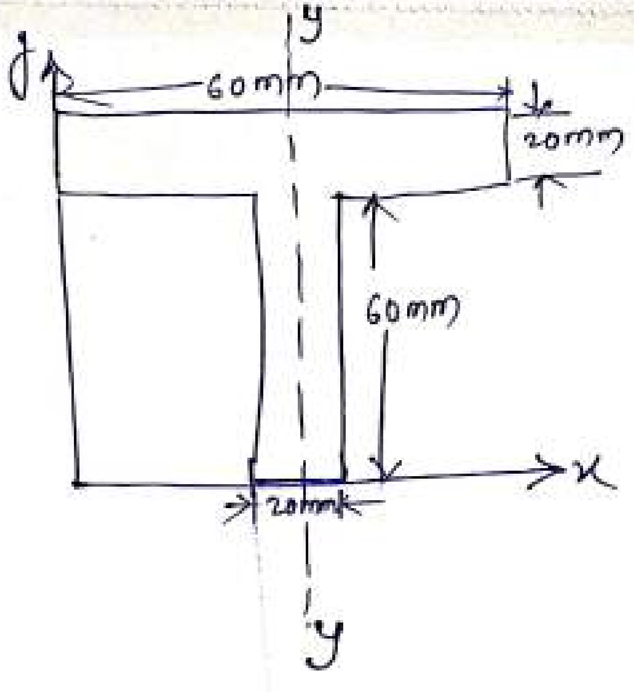
3rd step :- Divide the section in ____ parts.

4th step :- find Area, $\bar{x}$ & $\bar{y}$ from the reference axis. whichever is required.

5th step : finding which ever is required.

Q.1. find center of gravity of the given T-section.





This section is symmetrical about y axis.

So, dividing the section in 2 parts i.e.

Part 1 → 60 × 20 ( ) , Part 2 → 60×20 or 20×60

Part - 1

Area $= a_1 = 60 \times 20 = 1200\ mm^2$

$y_1 = 60 + (20/2) = 60 + 10 = 70mm$

Part - 2

Area $= a_2 = 20 \times 60 = 1200\ mm^2$

$y_2 = 60/2 = 30mm.$

$$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(1200 \times 70) + (1200 \times 30)}{2400} = \frac{84000 + 36000}{2400} = 50mm$$

$\bar{x} = 60/2 = 30mm.$ (already known as it is symmetric about y-axis.)

$\therefore \bar{x} = 30mm,\ \bar{y} = 50mm.$ (Ans).

Q. Steps for finding out M.I.

Step-1 First, split up the given section into plane areas (i.e. rectangular, triangular, circular etc. and find the centre of gravity of the section).

Step-2

find the moment of Inertia of these areas about their respective centre of gravity.

Step-3

Now transfer these moment of Inertia about the required axis by the Theorem of Parallel axis, i.e.

$$I = I_g + Ar^2 \text{ or } I_g + ah^2$$

I_g = Moment of Inertia of a section about its center of gravity & Parallel to the axis.

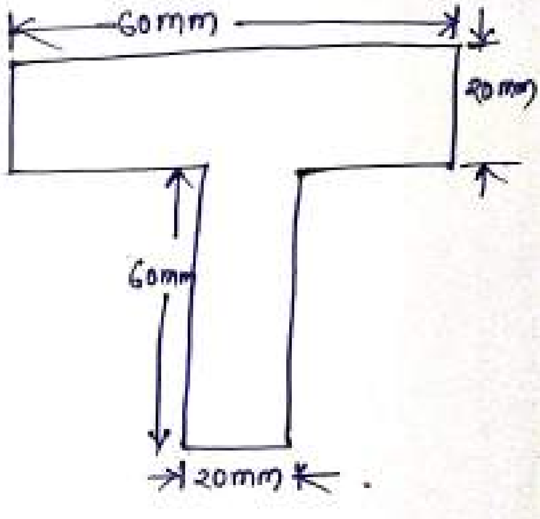
A or a = Area of the section

r or h = Distance between the required axis and center of gravity of the section.

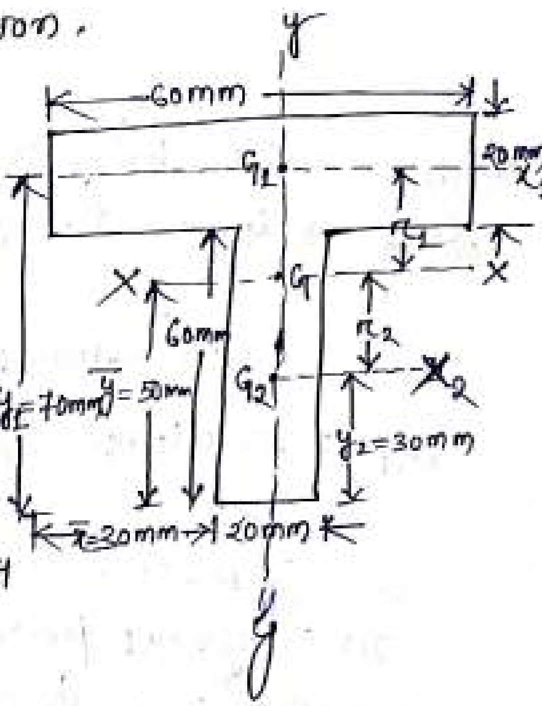
Step-4

The moment of Inertia of the given section may now be obtained by the algebraic sum of the moment of Inertia about the required axis.

Q. find the moment of Inertia of a T-section with flange as 60mm × 20mm & web as 60mm × 20mm about x-x and y-y axes through the center of gravity of the section.



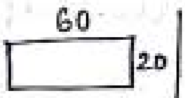
Q. Find the moment of Inertia of a T-section with flange as 60 mm × 20 mm & web as 60 mm × 20 mm about X-X and Y-Y axes through the center of gravity of the section.

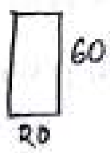


Solution:- From previous problem we have calculated $\bar{x} = 30mm$ & $\bar{y} = 50mm$.

M.I. of Plane areas about Horizontal axis

Part-1	Part-2
$I_{G_1} = \frac{bd^3}{12}$	$I_{G_2} = \frac{bd^3}{12}$
$= \frac{60 \times 20^3}{12} = 40{,}000 mm^4$	$= \frac{20 \times 60^3}{12}$
$= 0.04 \times 10^6 mm^4$	$= 36{,}000 mm^4 = 0.036 \times 10^6 mm^4$





I_{x_1} = moment of inertia of Part-1 about centre of gravity of T-section

$= I_{G_1} + a_1 r_1^2 = (0.04 \times 10^6) + 1200 \times (50-70)^2 = (0.04 \times 10^6) + 4{,}80{,}000$
$= 5{,}20{,}000 mm^4 = 0.52 \times 10^6 mm^4$

I_{x_2} = (Moment of Inertia of Part-2 about C.G. of T-section)

$= I_{G_2} + a_2 r_2^2 = (0.36 \times 10^6) + 1200 \times (50-30)^2 = 0.36 \times 10^6 + 4{,}80{,}000$
$= 0.84 \times 10^6 mm^4$

∴ Moment of Inertia about X-X axis through the C.G. of the section

$= I_{x-x} = I_{x_1} + I_{x_2} = (0.52 \times 10^6) + (0.84 \times 10^6) = 1.36 \times 10^6 mm^4$ (Ans.)

M.I. of plane areas about vertical axis.

For Part-1, $I_{G_1} = \frac{db^3}{12} = \frac{20 \times 60^3}{12} = 0.36 \times 10^6 mm^4 = I_{y_1}$

For part-2, $I_{G_2} = \frac{db^3}{12} = \frac{60 \times 20^3}{12} = 0.04 \times 10^6 mm^4 = I_{y_2}$.

∴ Moment of Inertia about y-y axis through the C.G. of the section

$= I_{y-y} = I_{y_1} + I_{y_2} = (0.36 \times 10^6) + (0.04 \times 10^6) = 0.4 \times 10^6 mm^4$ (Ans.)

∴ $I_{xx} = 1.36 \times 10^6 mm^4$, $I_{yy} = 0.4 \times 10^6 mm^4$.

Chapter-2

Simple & Complex stress, strain.

Simple stresses & strains

Mechanical Properties of materials

Rigidity :- It refers to a material's resistance to bending.

> The more resistant to bending it is, the more rigid it is.

Elasticity :- The property of material by which, it retains it's original shape after removal of external load, is called Elasticity.

Plasticity :- The property of material by which material take permanent deformation after removal of load, is called plasticity.

Compressibility :- The property of material by which material reduces it's thickness under increased pressure or compressive loading, is called compressibility.

Hardness :- The property of material which resist the penetration or scratch is called hardness.

Toughness :- The property of material by which it can bend, twist and stretch under a high stress without any rupture is called toughness.

Stiffness :- The property of material by which material resist the deformation under any stress or applied force is called stiffness.

→ It is also similar to rigidity.

Brittleness :- The property of material in which no deformation take place by the application of external load and it fails by rupture, is called brittleness.

→ Lack of ductility property in the material describes it's brittleness.

Ductility :- The property of material by which it can drawn into thin wire without any rupture, is called ductility.

Malleability :- The property of material by which it can convert into a thin shape when ~~by~~ beaten by hammer without any rupture, is called malleability.

Creep :- The plastic deformation of material due to load applied for long time is called creep.

Fatigue :- When the material is subjected to fluctuation or repeating load, ~~the~~ ~~qual~~ it tends to develop a ~~chara~~ characteristic behaviour which different from that under steady load, is called fatigue.

→ By this behaviour, the material fails due to a cyclic load, by showing cracks in the body.

Tenacity :- The property of a material, by which it resists to any type of stress such as crushing, bending, breaking or tearing etc. is called Tenacity.

Durability :- The property of material, by which it resists or withstands a stress or load for a long time.

→ It is also defined as the ability of a material to remain serviceable in the surrounding.

Stress :- It is the internal resisting force offered by the body to resist the deformation per unit c/s. area of the body.

→ It is denoted by 'σ'.

$$\sigma = P/A$$

where, P = Load or force acting on the body, &

A = cross-sectional area of the body.

Unit :- In M.K.S → kg/cm^2

S.I. → N/mm^2 or N/m^2

N/m^2 is also called Pascal (Pa)

Types of stress :-

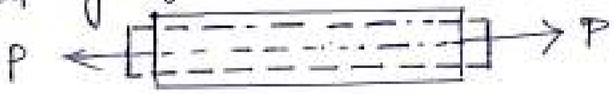
There are mainly 3 types of stresses.

1. Tensile stress
2. Compressive stress.
3. Shear stress.

1. Tensile Stress (σ_t)

When a section is subjected to two equal and opposite pulls and the body tends to increase it's length, the stress induced is called tensile stress.

→ It is denoted by 'σ_t'



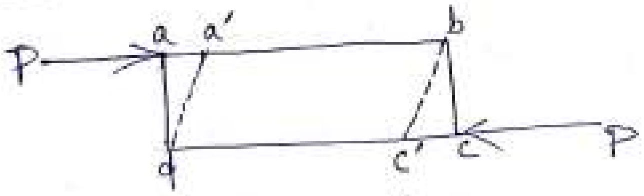
2. Compressive Stress (σ_c)

When a section is subjected to two equal and opposite pushes and the body tends to shorten it's length, the stress induced is called compressive stress.

→ It is denoted by 'σ_c'.

3. Sheare Stress (τ) or (σ_s)

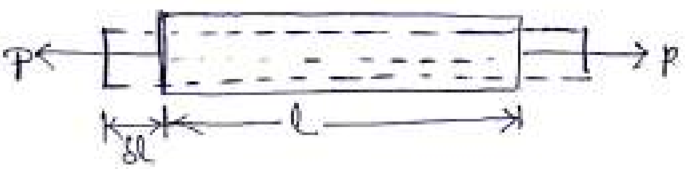
When a section is subjected to two equal and opposite parallel forces having different line of action and the body tends over slide, the stress induced is called sheare stress.



→ It is denoted by 'τ' or 'σ_s'

Strain :- whenever a single force acts on a body, it undergoes some deformation. This deformation per unit length is known as strain.

→ It is the ratio of change in dimension of the member to the original dimension.



→ It is denoted by 'ε' or 'e'.

$$\varepsilon = \frac{\delta l}{l} \text{ or } \delta l = \varepsilon \times l$$

where, δl = change of length of the body / change of dimension of the body.

l = Original length of the body.

→ It is unitless.

Types of strain :-

There are mainly 3 types of strains

1. Tensile strain
2. Compressive strain
3. Shear strain.

1. Tensile strain (ε_t or e_t)

It is the measure of the deformation of an object under tensile stress and defined as the ratio of increase in length to the original length of the body due to tensile force.

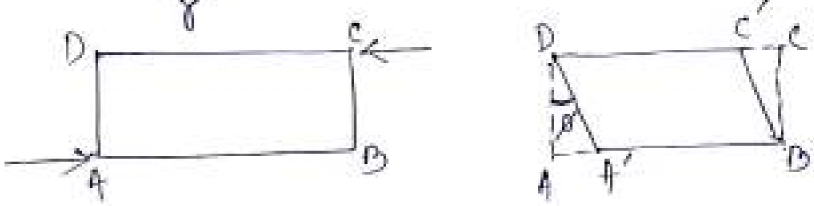
$$\varepsilon_t \text{ or } e_t = \frac{\text{Increase in length}}{\text{Original length}} = \frac{\delta l}{l}$$

2. Compressive Strain (ε_c or e_c)

It is the measure of the deformation of an object under compressive stress and defined as the ratio of decrease in length to the original length of the body due to compressive force.

$$\varepsilon_c \text{ or } e_c = \frac{\text{Decrease in length}}{\text{Original length}} = \frac{\delta l}{l}$$

3. Shear strain (ε_s or e_s or γ)



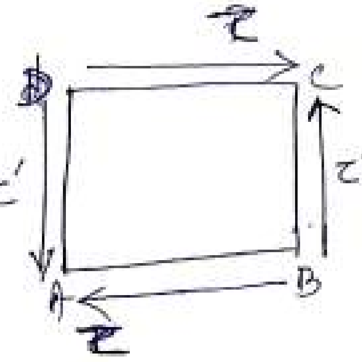
$$\varepsilon_s \text{ or } e_s = \frac{AA'}{AD} = \tan\phi$$

→ It is the measure of angle through which a body is distorted when shear forces are acting on it.

→ It is defined as the ratio of the change in deformation to it's original length perpendicular to the axes of the member due to shear stress.

Complementary stress :

Whenever a shear stress 'τ' is applied on parallel surface of body then to keep the body in equilibrium τ' a shear stress 'τ'' is induced on remaining surface of body.



These stresses form a couple. This resisting shear stress τ' is known as complementary shear stress.

Longitudinal & Lateral strain :

Longitudinal or Linear strain

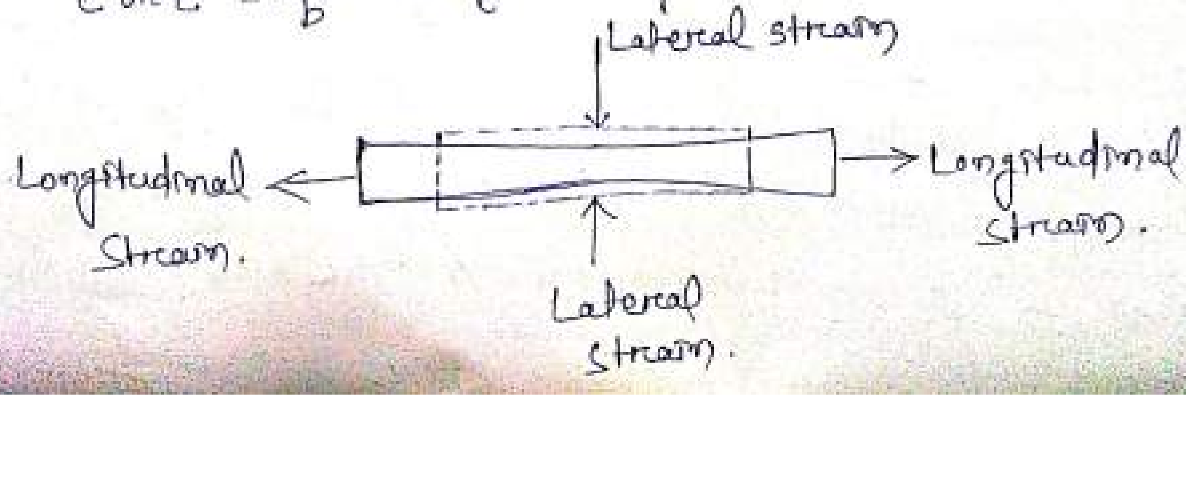
The change in dimensions occurs in the direction of applied load is called as linear or longitudinal strain.

$$e \text{ or } \varepsilon = \frac{\delta l}{l}$$

Lateral strain

The change in dimensions occurs in the direction perpendicular to the line of action of applied load is called as Lateral strain.

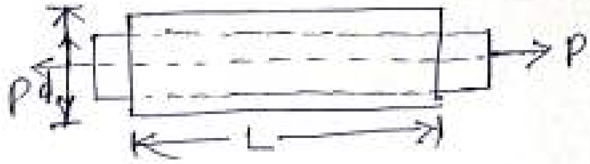
$$e \text{ or } \varepsilon = \frac{\delta b}{b} \text{ or } \frac{\delta t}{t} \text{ or } \frac{\delta d}{d} \ldots$$



Poisson's ratio : (It is named after French Mathematician Poisson)

It is defined as the ratio between lateral strain & longitudinal strain.

→ It is denoted by 'μ' or $\frac{1}{m}$



Mathematically, $\mu = \frac{(\delta d/d)}{(\delta l/l)}$

$= \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

→ It is a constant.

→ It is always < 1 ($\because$ Lateral strain is always < Longitudinal strain)

Volumetric Strain

→ When a body is subjected to a system of forces, it undergoes some changes in its dimensions. The volume of the body is changed.

→ The ratio of the change in volume to the original volume is known as volumetric strain.

→ volumetric strain ε_v or $e_v = \frac{\delta V}{V}$

where, δV = Change in volume

V = Original volume.

Hooke's Law : (It is named after Robert Hooke, in 1678)

It states, "When a material is loaded, within its elastic limit, the stress is directly proportional to the strain." Mathematically,

$\frac{\text{Stress}}{\text{strain}} = E = \text{Constant}$.

→ 'E' is also called as Modulus of Elasticity or Young's Modulus.

Modulus of Rigidity (G)

→ It is the ratio of shear stress to shear strain.

→ It is denoted by 'G'

$$G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\varepsilon_s \text{ or } \phi}$$

→ Unit of Modulus of rigidity is N/mm2

Young's Modulus or Modulus of Elasticity (E)

→ It is the ratio of stress to the strain.

→ It is denoted by 'E'

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon}$$

→ unit of young's modulus is N/mm2

Bulk Modulus (K)

→ It is the ratio of Normal stress to volumetric strain.

→ It is denoted by 'K'.

$$K = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{\sigma_n}{\varepsilon_v \text{ or } e_v}$$

→ Unit of Bulk Modulus is N/mm2

Relationship Between Elastic Constants (μ, E, G & K)

Relation betn E, G & μ

1. $$\boxed{E = 2G(1+\mu)}$$

Relation betn E, K & μ

2. $$\boxed{E = 3K(1-2\mu)}$$

Relation betn E, K & G

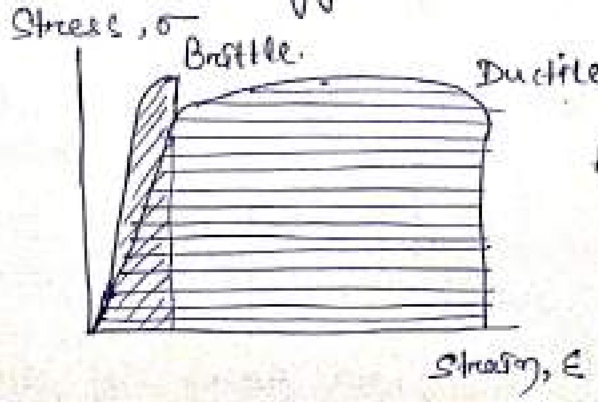
3. $$\boxed{E = \frac{9KG}{3K+G}}$$

2.2 Application of simple stress & strain

Behaviour of ductile & brittle materials under direct load

→ Brittle materials fracture at low strains and absorb little energy.

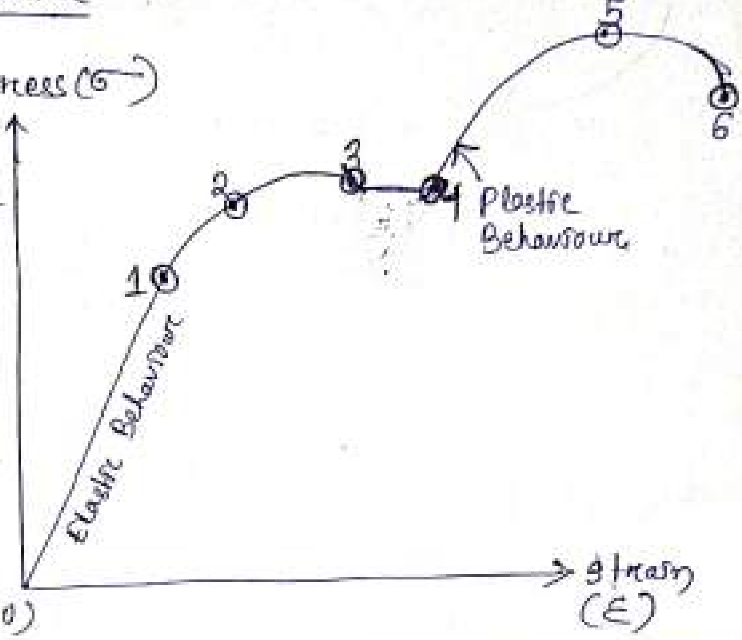
→ Conversely, ductile materials fail after significant deformation and absorb more energy.



Area under curve = Absorbed energy.

Stress-strain curve for ductile material

Here, we have taken an example of mild steel and shown the stress-strain curve during application of load.



0-1 →	Proportional Limit
1-2 →	Elastic limit
2-3 →	Upper yield limit (plastic limit)
3-4 →	Lower yield limit
4-5 →	Ultimate stress limit (strain hardening)
5-6 →	fracture limit. (Necking)

0-1 → (Proportional limit)

In this limit, when the material is under application of load,

As stress increases, strain also increases.

→ up to point '1',

stress ∝ strain

$\sigma \propto \varepsilon$

It means, material obeys Hooke's law.

1-2 → (Elastic limit)

In this limit, relation betⁿ stress & strain is not linear.

→ stress is not proportional to strain, so it is not a straightline and shown as a curved line.

2-3 & 3-4 → (Plastic limit)

Beyond elastic limit, when stress increases, strain also increases rapidly and the amount of strain becomes larger.

→ This phenomena is called as yielding of material.

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'3' point is called as <u>upper yield point</u> &
'4' point is called as <u>lower yield point</u>.

→ The stress-strain curve in this part (3-4) of the graph is almost horizontal, which implies that there is an appreciable increase in strain for a negligible increase in stress.

→ yielding starts at '3' & ends at '4', the deformation is of nearly permanent nature.

4-5 (Ultimate stress limit)

This is the limit which shows the maximum stress that a material can take before it fails.

→ The specimen however doesn't fail at this point.

→ After this point/limit the curve starts dropping.

→ The point '5' is called as ultimate stress point.

5-6 (Fracture limit)

→ After point '5', the specimen can not take more stress and goes for failure & finally fails at point '6'.

→ Point '6' is called breaking point.

→ Here in this limit, necking of the specimen takes place, which causes a loss in the load carrying capacity of the specimen & ultimately causes it to fail.

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Complex Stress & Strain

Complex stress & strain are also known as principal stress & strain.

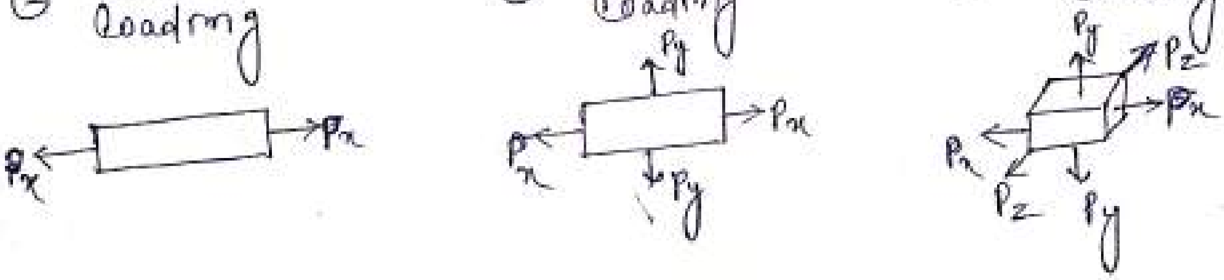
Normally loading about axis or axial loading is of 3 types.

Axial loading.

① Uniaxial Loading

② Biaxial Loading

③ Triaxial loading.



1. <u>Uniaxial loading</u>:- When a specimen is subjected to a loading in one direction only (Basically Longitudinal axis), then the load is called uniaxial loading.

→ Corresponding stress also acts in one direction only.

2. <u>Biaxial Loading</u>.

When a specimen is subjected to loading in two directions, then the loading is called Biaxial loading.

→ Corresponding stresses also acts in two directions.

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3. Triaxial loading:-

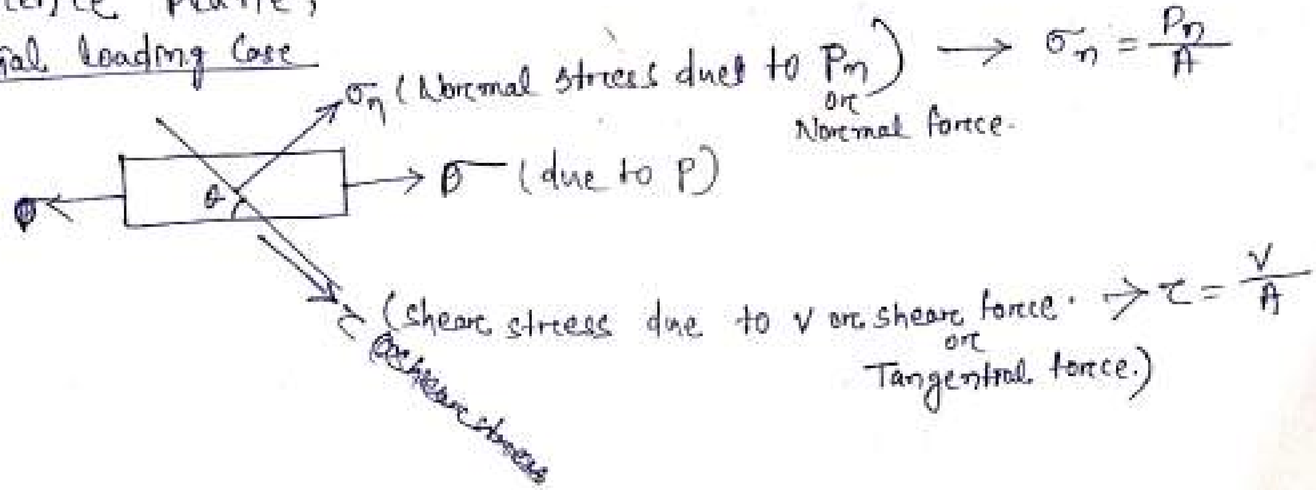
When a specimen is subjected to loading in three directions, then the loading is called triaxial loading.

→ Corresponding stresses also act in three directions.

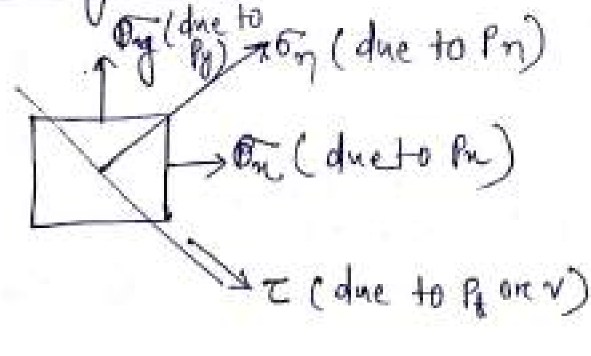
Normal stress & shear stress on oblique plane.

So, if we take an oblique plane at an angle θ to the reference plane,

Uniaxial loading case



Biaxial loading case.



Like this, in triaxial loading case also there is a 'σ_n' in normal direction, & 'τ' in tangential direction, in any oblique plane in the specimen.

Principal Plane

An oblique plane, at which shear stress ~~[illegible]~~ is zero, is called as principal plane.

→ In this plane $\tau = 0$,

→ It is of 2 types (i) Major principal plane. (ii) Minor principal plane. } These are mutually ⊥ar.

Principal stress

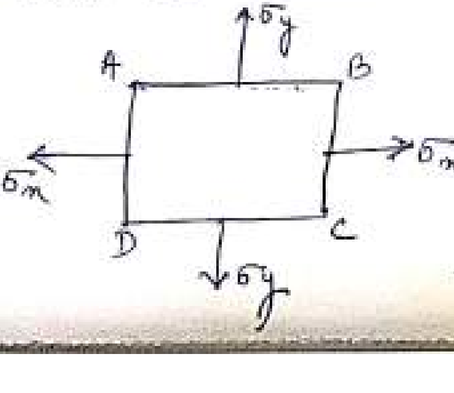
The normal stress acting on the principal plane is called as principal stress.

Principal stress is of two types
(i) Major principal stress
(ii) Minor principal stress.

(i) Major principal stress :- The normal stress (σ_n) which carries greatest value ~~along~~ on the major principal plane, is called as Major principal stress.

(ii) Minor principal stress :- The normal stress (σ_n) which carries lowest value on the minor principal plane, is called as Minor principal stress.

Ex-



→ Here σ_x & σ_y are normal stresses.
→ AB & CD are principal plane, at which shear stress = 0. (Also, AD & BC)
→ If $\sigma_x > \sigma_y$, AD & BC → Major principal plane
AB & CD → Minor principal plane
→ σ_x & σ_y are principal stresses.
σ_x → Major principal stress. σ_y → Minor principal stress

Principal Strain

The shear strain ~~acting~~ on the principal plane is called as principal strain.

Mohr's Circle

Mohr's circle (named after Otto Mohr (German Scientist)) is a graphical technique to transform stress/strain from one co-ordinate system to another, and to find maximum normal and shear stresses.

→ The construction of Mohr's circle of stresses as well as determination of normal, shear and resultant stress is very easier than the analytical method.

Mohr's circle of stresses can be drawn for the following cases:

1. A body subjected to a direct stress in one plane.
2. A body subjected to direct stresses in two mutually ⊥ar dir's.
3. A body subjected to a simple shear stress.
4. A body subjected to a direct stress in one plane accompanied by a simple shear stress.
5. A body subjected to direct stresses in two mutually ⊥ar directions accompanied by a simple shear stress.

Sign Conventions

For normal stress

Tensile stress → +ve

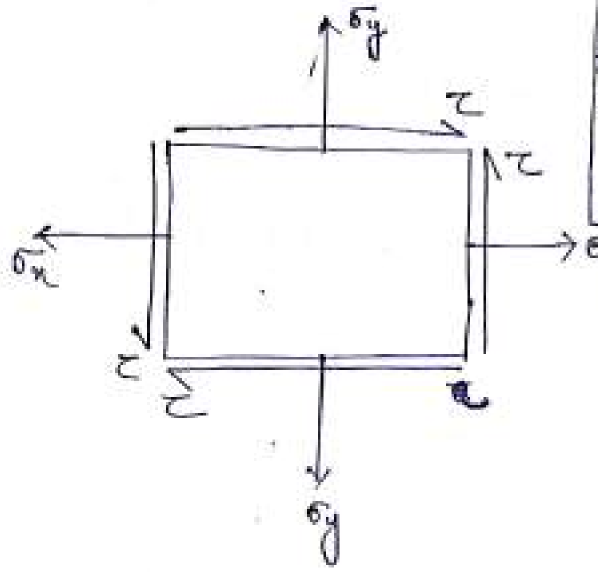
Compressive stress → −ve.

For shear stress

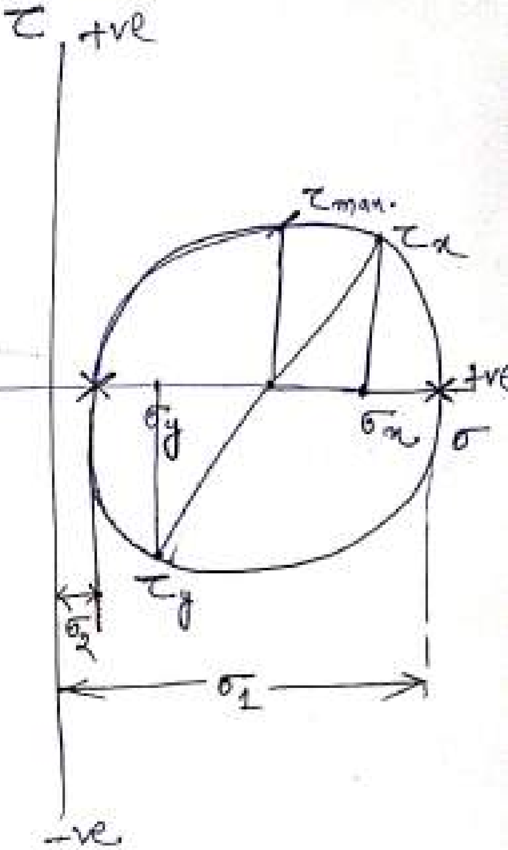
For clockwise → −ve

For Anticlockwise → +ve

Taking the example of difficult case i.e. case-5, we will draw the Mohr's circle.



$$\boxed{\begin{aligned} R \text{ or } \tau_{max} &= \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2} \\ \sigma_1 &= \sigma_{Avg}+R \\ \sigma_2 &= \sigma_{Avg}-R \\ \sigma_{Avg} &= \frac{\sigma_x+\sigma_y}{2} \end{aligned}}$$



If $\sigma_x > \sigma_y$, the diagram will be like this.

. Here σ_1 – Maximum Principal stress

σ_2 – Minimum Principal stress.

τ_{max} – (Radius of Mohr's circle) – Maximum Shear stress.

From Analytical Method

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

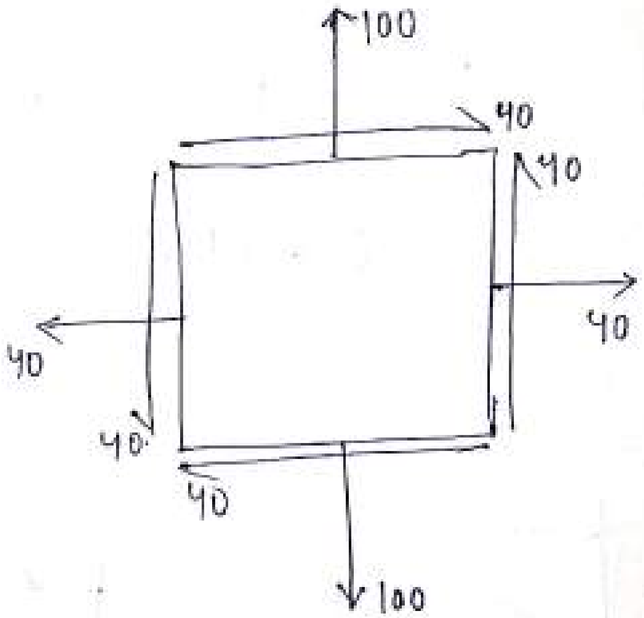
Problem-1

The state of stress at a point under plane stress condition is $\sigma_x = 40$ mpa and $\sigma_y = 100$ mpa & $\tau = 40$ mpa.

(i) find the value of major principal stress, minor principal stress & maximum shear stress. by analytical & Graphical method.

(ii) find the value of radius of mohr's circle.

solution:-



Analytical Method

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

$$= \left(\frac{40+100}{2}\right) + \sqrt{\left(\frac{40-100}{2}\right)^2+40^2}$$

$$= 70 + 50$$

$$= 120 \text{ mpa.}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

$$= 70 - 50 = 20 \text{ mpa.}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau^2}$$

$$= 50 \text{ mpa.}$$

Graphical Method.

Scale

Taking, 1 cm = 10 mpa.

$\sigma_x = 4$ cm.

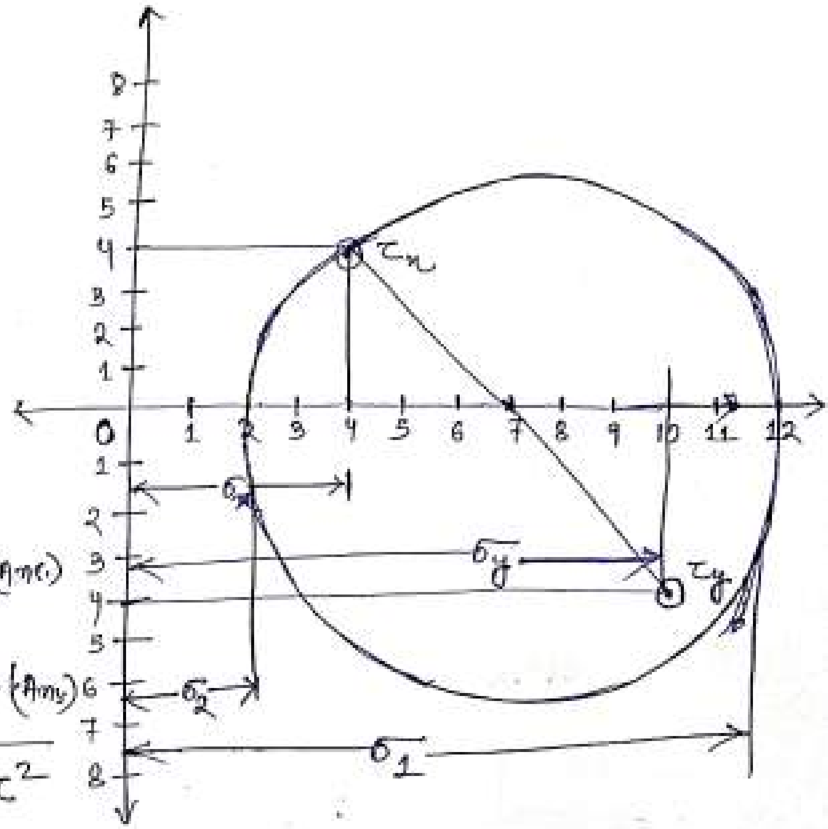
$\sigma_y = 10$ cm.

$\tau = 4$ cm.

Co-ordinate

$x \rightarrow +40, +40$

$y \rightarrow +100, -40$



$\sigma_1 = 12$ cm $= 12 \times 10 = 120$ mpa (Ans.)

$\sigma_2 = 2$ cm $= 2 \times 10 = 20$ mpa (Ans.)

τ_{max} or Radius $= \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau^2}$

$= \sqrt{\left(\frac{10-4}{2}\right)^2 + 4^2}$

$= \sqrt{3^2 + 4^2} = 5$ cm $= 50$ mpa (Ans.)

∴

(i) σ_1 = Maximum or Major Principal Stress = 120 mpa.

(ii) σ_2 = Minimum or Minor Principal stress = 20 mpa.

(iii) τ_{max} = Maximum Shear Stress = 50 mpa.

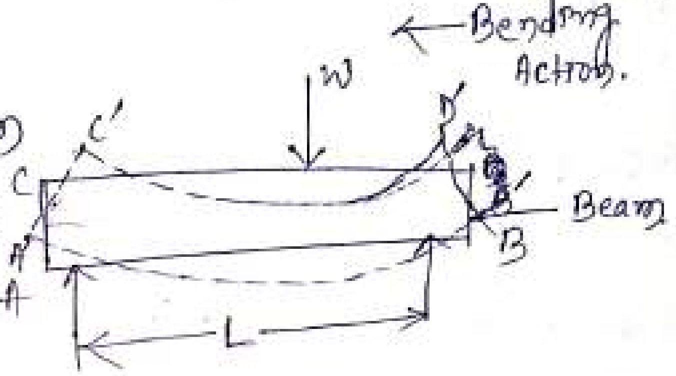
(iv) Radius of Mohr's Circle = τ_{max} = 50 mpa.

Chapter-3

Stresses in Beams & Shafts

Stresses in Beams due to bending:

Bending stress in beams:



The bending moment at a section tends to bend or deflect the beam and the internal stresses resist the bending.

→ The Process of bending stops, when every cross-section sets up full resistance to the bending moment.

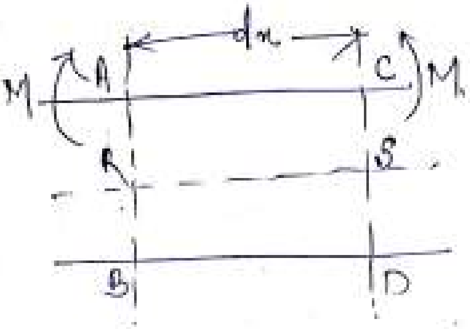
→ The resistance, offered by the internal stresses, to the bending, is called bending stress, and the relevant theory is called the theory of simple bending or theory of pure bending.

Assumptions in the Theory of Simple Bending.

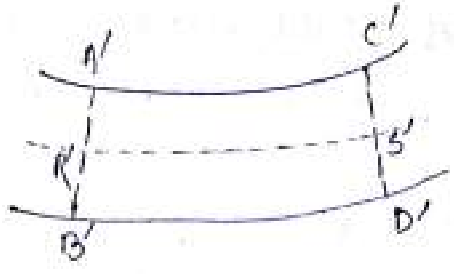
The following assumptions are made in the theory of simple bending:

1. Beam is initially straight & it is straight before loading & remains straight even after load is removed.
2. The material of the beam is perfectly homogeneous (i.e. of the same kind throughout) and isotropic (i.e. of equal elastic properties in all direction).
3. The material of the beam obeys Hooke's Law. (i.e. $\sigma \propto \varepsilon$)
4. The transverse sections, which were plane before bending remains plane after bending also.
5. The beam is in equilibrium & value of Young's modulus (E) is same in tension and compression.

Theory of simple Bending:-



(a) Before bending



(b) After Bending

Consider a small length of a simply supported beam subjected to a bending moment as shown in fig (a). Now consider two sections AB and CD, which are normal to the axis of the beam RS.

→ Due to action of the bending moment, the beam as a whole will bend as shown in figure.

→ The top layer of the beam AC has suffered compression and reduced to A'C'.

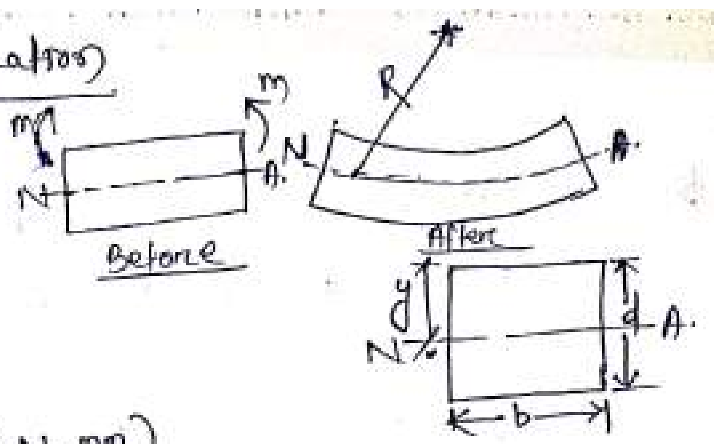
→ The middle layer of the beam RS has suffered no change in its length, though bent into R'S'.

— It is known as neutral plane or neutral layer or neutral axis.

→ The lower layer of the beam BD has suffered tension and stretched to B'D'.

This theory of bending is called theory of simple bending.

Flexural (Bending) stress Equation



$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

Where,

M = Bending Moment (N-mm)

I = Moment of Inertia for Beam c/s (mm^4)

σ_b = Flexural stress or Bending stress (N/mm^2)

y = Distance of layer subjected to bending from N.A. (mm)

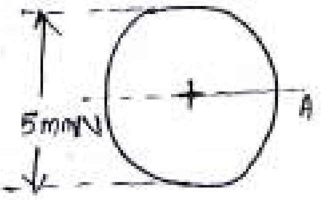
E = Young's Modulus or Modulus of Elasticity (N/mm^2) for beam-material (Different value for different material)

R = Radius of curvature of beam (mm)

Q.1. A steel wire of 5 mm diameter is bent into a circular shape of 5m radius. Determine the maximum stress induced in the wire. Take E = 200 GPa.

Sol:

Given data.



Diameter of steel wire (d) = 5 mm;

Radius of circular shape (R) = 5 m = 5×10^3 mm,

Modulus of elasticity (E) = 200 GPa = $200 \times 10^3 \; N/mm^2$

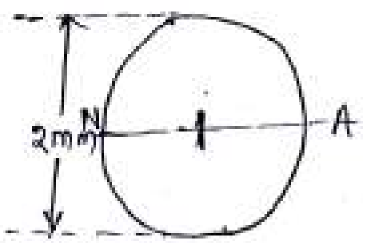
The distance between the neutral axis of the wire & its extreme fibre,

$$y = \frac{d}{2} = \frac{5}{2} = 2.5 \text{ mm}.$$

and maximum bending stress induced in the wire,

$$\sigma_{b(max)} = \frac{E}{R} \times y = \frac{200 \times 10^3}{5 \times 10^3} \times 2.5 = 100 \; N/mm^2 = 100 \text{ MPa (Ans.)}$$

Q.2: A copper wire of 2 mm diameter is required to be wound around a drum. Find the minimum radius of the drum, if the stress in the wire is not to exceed 80 MPa. Take modulus of elasticity for the copper as 100 GPa.



Solⁿ:-

Given Data

Diameter of wire (d) = 2mm,

Maximum bending stress $(\sigma_b)_{max}$ = 80 MPa = 80 N/mm²

and modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm².

The distance between the neutral axis of the wire & its extreme fibre.

$$y = \frac{2}{2} = 1\,mm.$$

∴ Minimum radius of the drum

$$R = \frac{y}{\sigma_{b\,max}} \times E = \frac{1}{80} \times 100 \times 10^3 = 1.25 \times 10^3\,mm$$

$$= 1.25\,m \text{ (Ans.)}$$

Q.3. A metallic rod of 10 mm diameter is bent into a circular form of radius 6m. If the maximum bending stress developed in the rod is 125 MPa, find the value of Young's modulus for the rod material.

Solⁿ: Given data

Diameter of rod (d) = 10mm, Radius (R) = 6m = 6×10^3 mm

Maximum bending stress $\sigma_{b\,max}$ = 125 MPa = 125 N/mm²

Distance of neutral axis from extreme fibre of wire

$$y = \frac{10}{2} = 5\,mm$$

∴ Value of Young's modulus for the rod material,

$$E = \frac{\sigma_b}{y} \times R = \frac{125}{5} \times 6 \times 10^3 = 150 \times 10^3\,N/mm^2 = 150\,GPa \text{ (Ans.)}$$

Moment of Resistance

When a beam subjected to loading, after bending, on one side of the neutral axis there are compressive stresses and on the other there are tensile stresses. These stresses form a couple, whose moment must be equal to the external moment (M).

→ The moment of this couple, which resists the external bending moment, is known as moment of resistance.

Position of Neutral Axis

→ The line of intersection of the neutral layer, with any normal cross-section of a beam, is known as neutral axis of that section.

→ To locate the neutral axis of a section, first we have to find out the centroid of the section and then to draw a line passing through this centroid and normal to the plane of bending.

→ This line will be the neutral axis of the section.

→ Here, AB is neutral layer & BC is neutral axis.

Flexural rigidity :-

Bending moment required for unit radius of curvature is known as flexural rigidity.

→ It means the radius of curve which is created by bending of beam is ~~called as~~ of unit length i.e. in SI system it is 1m.

→ from Bending equation, $M = \frac{EI}{R}$

For unit radius, Bending moment $= \boxed{M = EI}$

or

Flexural Rigidity

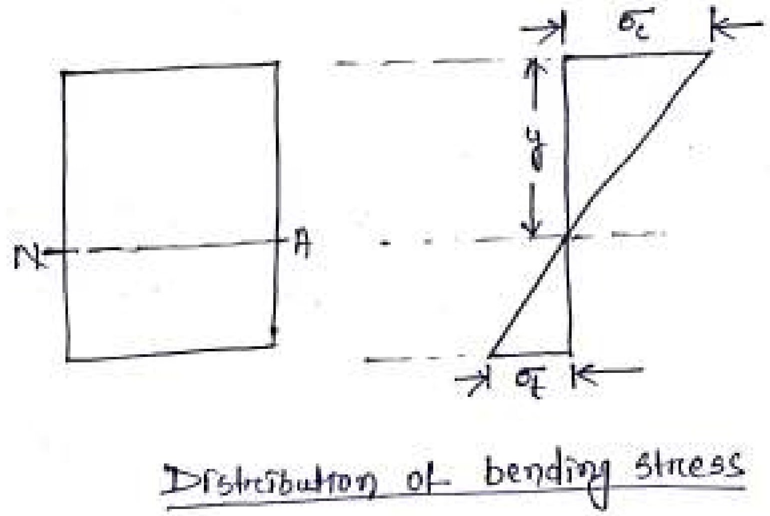
Flexural or Bending stress Distribution:

We know that, there is no stress at the neutral axis. In a simply supported beam, there is a compressive stress above the neutral axis and a tensile stress below it.

→ $\sigma_b = \frac{Ey}{R}$

From this equation, we know that the bending stress at a point is directly proportional to its distance from the neutral axis.

→ If we plot the stresses in a simply supported beam section, we shall get a figure as shown in figure.



Distribution of bending stress

→ The maximum stress (either compressive or tensile) takes place at the outermost layer.

Or, in other words, while obtaining maximum bending stress at a section, the value of y is taken as maximum.

Section Modulus:

It is defined as the ratio of moment of inertia of a section about it's centroidal axis & to the distance of extreme layer from neutral axis.

Mathematically,

$$Z = \frac{I}{y}$$

where, I = Moment of Inertia of the section about any axis

y = Distance of the extreme layer from N.A.

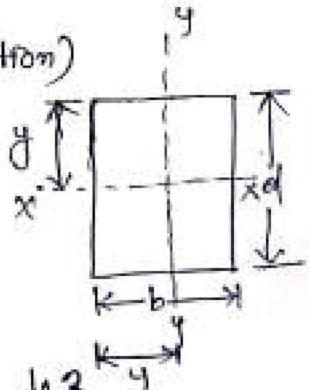
Unit

It's unit will be mm^3 or cm^3 or m^3

Example

1. Rectangular Section

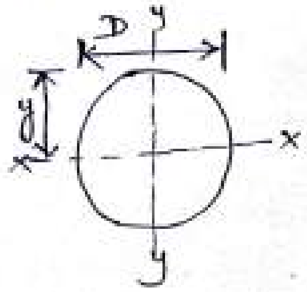
$I_{xx} = \frac{bd^3}{12}$, $y = \frac{d}{2}$ (due to symmetric section)



$$Z_{xx} = \frac{I_{xx}}{y} = \frac{bd^3}{12} \Big/ \frac{d}{2} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$$

$I_{yy} = \frac{db^3}{12}$, $Z_{yy} = \frac{I_{yy}}{y} = \frac{db^3}{12} \times \frac{2}{b} = \frac{db^2}{6}$

2. Circular Section



$I_{xx} = I_{yy} = \frac{\pi D^4}{64}$, $y = \frac{D}{2}$

$\therefore$ $Z_{xx} = Z_{yy} = \frac{\pi D^4}{64} \times \frac{2}{D} = \frac{\pi D^3}{32}$

$\left(\frac{I_{xx}/I_{yy}}{y}\right)$

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Shear stresses in Beams

Shear stress :- It is defined as the ratio of shear force to the cross-sectional area.

→ It is denoted by 'τ' or 'σ_s'.

$$\text{shear stress } (\tau) = \frac{\text{Shear force } (F)}{\text{Cross-sectional Area } (A)}$$

$$\Rightarrow \boxed{\tau = \frac{F}{A}}$$

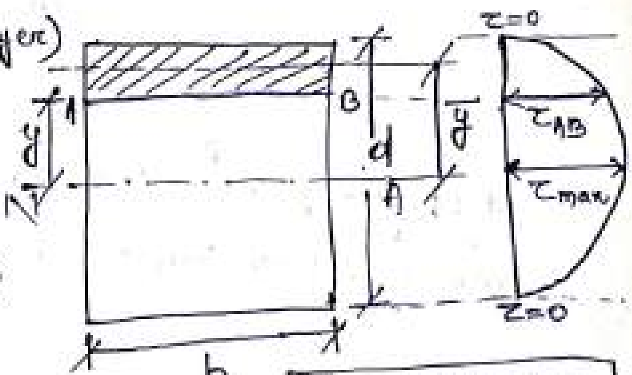
→ Unit is N/mm^2

→ In shear stresses, Load is tangential to the cross-sectional area.

Shear stress distribution diagram for Rectangular section:-

Shear stress is given by (At any layer)

$$\boxed{\tau = \frac{FA\bar{y}}{Ib}}$$

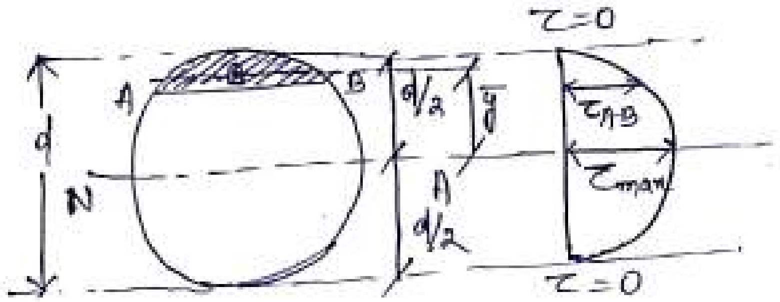


$$\boxed{\tau_{max} = 1.5\,\tau_{Av.}}$$

$$\tau_{Avg} = \frac{F}{A}$$

where,
τ = shear stress (N/mm^2)
F = shear force. (N)
A = Area of section where shear stress is to be calculated (mm^2)
$\bar{y}$ = Distance from N.A. to the centroid of Area
I = Moment of Inertia of given figure about N.A.
b = width of given section where shear stress is to be calculated

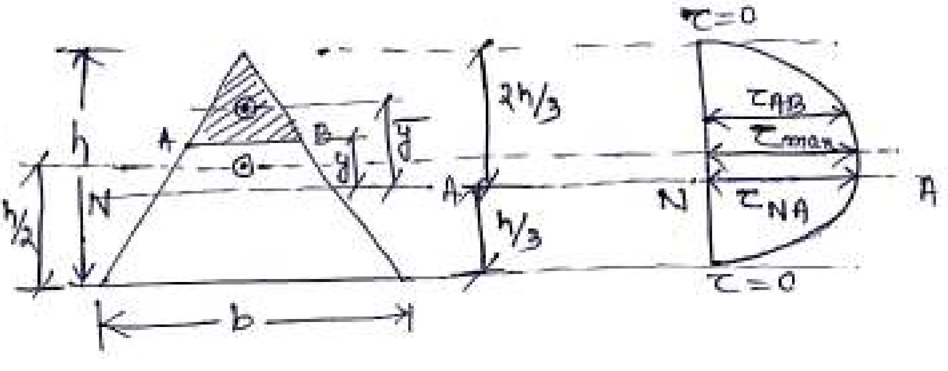
Shear stress distribution diagram for circular section



$$\tau_{Avg} = \frac{F}{A}$$

$$\boxed{\tau_{max} = \frac{4}{3} \times \tau_{Avg}.}$$

Shear stress distribution diagram for Triangular section



$$\tau_{Avg} = \frac{F}{A}$$

$$\boxed{\tau_{max} = \frac{3}{2}\tau_{avg} = 1.5\,\tau_{Avg}}$$

$$\boxed{\tau_{NA} = \frac{4}{3}\tau_{Avg}.}$$

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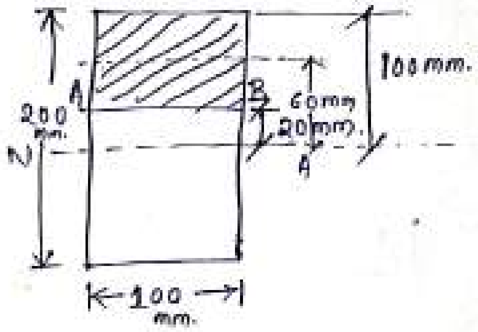
Problem-1

A beam of rectangular cross-section 100 mm × 200 mm is subjected to a shear force = 30 KN. Calculate the shear stress induced across the section at a layer 20 mm. away from N.A.. Also, draw the shear stress distribution diagram, for the cross-section.

Given data: b = 100 mm, d = 200 mm.

F = 30 KN = 30×10^3 N, τ = ?



Solⁿ: Shear stress at any given layer is written as :—

$$\boxed{\tau = \frac{F A \bar{y}}{I b}} \quad \text{---- (1)}$$

Area of section = $A = b \times d = 100 \times (100 - 20) = 8 \times 10^3 \, mm^2$

$$\bar{y} = 20 + (80/2) = 60 \, mm.$$

I = Moment of Inertia for complete rectangle is given by

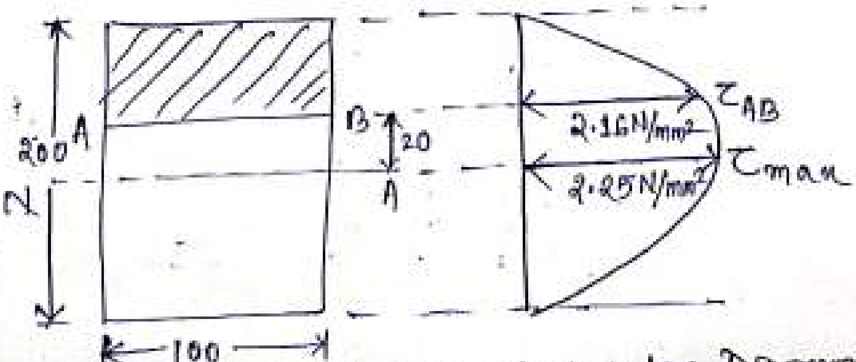
$$\Rightarrow I = \frac{bd^3}{12} = \frac{100 \times 200^3}{12} = \frac{8 \times 10^8}{12} = 6.67 \times 10^7 \, mm^4$$

Putting all values in eqⁿ (1)

$$\tau_{AB} = \frac{(30 \times 10^3) \times (8 \times 10^3) \times 60}{6.67 \times 10^7 \times 100} = 2.16 \, N/mm^2 \quad \text{(Ans.)}$$

Average shear stress = $\tau_{Avg.} = \frac{F}{A} = \frac{30 \times 10^3}{100 \times 200} = 1.5 \, N/mm^2$

$$\tau_{max} = 1.5 \, \tau_{Avg} = 1.5 \times 1.5 = 2.25 \, N/mm^2$$



Shear-Stress Distribution Diagram

3.3 Stresses in shafts due to torsion

Torsion :- The action of twisting, specially of one end of a body while the other is fixed is called torsion.

→ In case of a shaft torsion can be seen.

→ It is also called as the body is subjected to turning moment or twisting moment or torsional moment or torque.

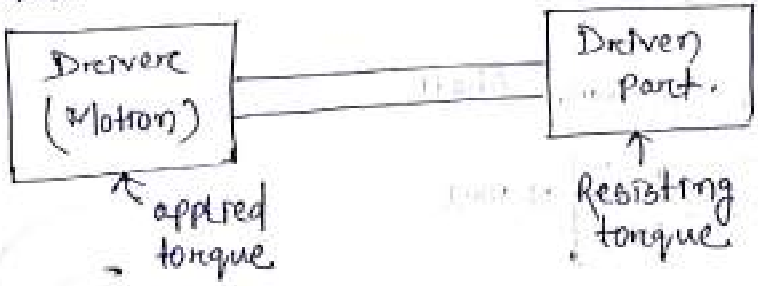
Shaft

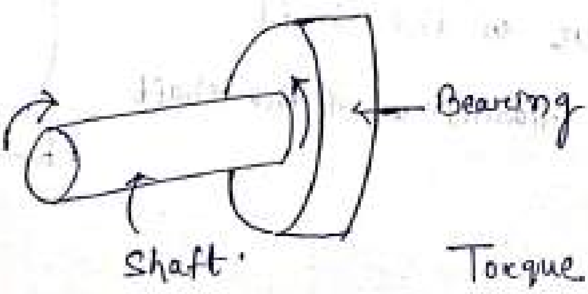
It is a member, which rotates & supported by a bearing.

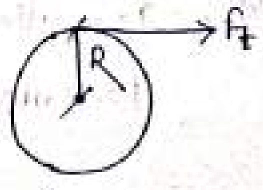
→ when it rotates, it produces a rotation & the fixed bearing produces a opposite rotation to oppose it's motion.

→ This rotation gives torque, which is the product of turning force and the distance between the point of application of the force and the axis of the shaft.

Shaft is usually a member/machine element used to transfer motion and power from one place to another place. in machines.







Torque $(T) = F_t \times R$

= Tangential or Turning force × Radius.

→ Unit will be N-mm.

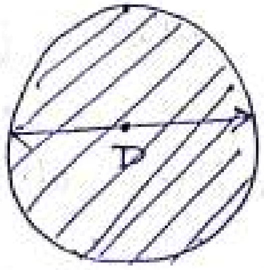
Basic assumptions of pure torsion :-

Following assumptions are made, when a circular shaft is subjected to torsion:

1. The material of the shaft is uniform throughout the length.
2. The twist along the shaft is uniform.
3. The shaft is of uniform circular section throughout the length.
4. Cross section of the shaft, which are plane before twist remain plane after twist.
5. All diameters of the normal cross-section, which were straight before the twist, remain straight with their magnitude unchanged, after the twist.

Torsion of a solid circular shaft :-

$$\boxed{T = \frac{\pi}{16} \times \tau \times D^3} \text{ N-mm}$$



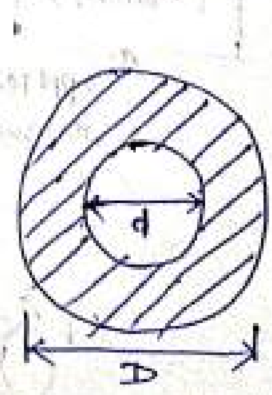
Where, T = Torque.

τ = Shear stress developed in the outermost layer of the shaft

D = Diameter of the shaft which is equal to $2R$.

Torsion of a Hollow circular shaft :-

$$\boxed{T = \frac{\pi}{16} \times \tau \times \frac{D^4 - d^4}{D}} \text{ N-mm.}$$



where, D = External diameter of the shaft

d = Internal diameter of the shaft

Problem-1

A circular shaft of 50mm diameter is required to transmit torque from one shaft to another. find the safe torque, which the shaft can transmit, if the shear stress is not to exceed 40 MPa.

Soln: Given data: Diameter of shaft (D) = 50 mm.

Maximum shear stress (τ) = 40 mpa = 40 N/mm^2

The safe torque, which the shaft can transmit

$$T = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 40 \times (50)^3 = 0.982 \times 10^6 \ N \cdot mm$$

$$= 0.982 \ KN \cdot m \ \text{(Ans.)}$$

Problem-2

A solid steel shaft is to transmit a torque of 10 KN·m. If the shearing stress is not to exceed 45 mpa. find the minimum diameter of the shaft.

Soln:

Given data: Torque (T) = 10 KN-m = 10×10^6 N.mm

Maximum shearing stress (τ) = 45 mpa = 45 N/mm^2

If D = ~~Diam~~ Minimum diameter of the shaft,

Torque transmitted by the shaft (T)

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$\Rightarrow 10 \times 10^6 = \frac{\pi}{16} \times 45 \times D^3$$

$$\Rightarrow D^3 = 1.132 \times 10^6$$

$$\Rightarrow D = 1.04 \times 10^2 = 104 \ mm \ \text{(Ans.)}$$

Problem - 3

A hollow shaft of external and internal diameter of 80 mm & 50 mm is required to transmit torque from one end to the other. What is the safe torque it can transmit, if the allowable shear stress is 45 mpa.?

Soln: Given data.

External diameter (D) = 80 mm, Internal diameter(d) = 50 mm

and allowable shear stress (τ) = 45 mpa = 45 N/mm²

Torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right)$$

$$= \frac{\pi}{16} \times 45 \times \left\{\frac{(80)^4 - (50)^4}{80}\right\}$$

$$= 3.83 \times 10^6 \text{ N-mm} = 3.83 \text{ KN.m (Ans.)}$$

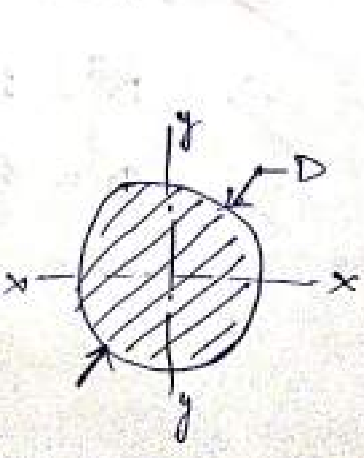
Polar Moment of Inertia

> Polar moment of Inertia is the moment of inertia about the third axis i.e., Z-axis & it is denoted by 'J' or 'I_p'.

> Mathematically, $\boxed{J = I_{xx} + I_{yy}}$

> Unit is mm⁴.

1.) Polar M.I. of solid Circular shaft



$$J = I_{xx} + I_{yy}$$

$$= \frac{\pi D^4}{64} + \frac{\pi D^4}{64}$$

$$= 2 \times \frac{\pi D^4}{64} = \frac{\pi D^4}{32}$$

$$\therefore \boxed{J = \frac{\pi D^4}{32}} \text{ mm}^4$$

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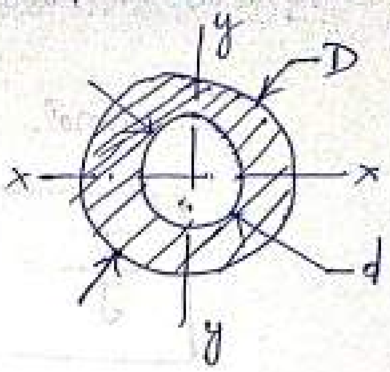
2. Polar M.I. of Hollow circular shaft.



$J = I_{xx} + I_{yy}$

$= \frac{\pi}{64}(D^4 - d^4) + \frac{\pi}{64}(D^4 - d^4)$

$= 2 \times \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{32}(D^4 - d^4)$

$\therefore \boxed{J = \frac{\pi}{32}(D^4 - d^4)}$ mm^4

Torsional Rigidity

→ The amount of resistance, a cross section has against torsional deformation is called as Torsional rigidity.

The higher the rigidity, the more resistance the c/s has.

→ It is also defined as the product of modulus of rigidity and polar moment of Inertia.

$K = G \times J$

where, K = Torsional rigidity

G = ~~Polar mo~~ modulus of rigidity

J = Polar moment of Inertia.

→ S.I. unit of torsional rigidity is Nm^2.

Torsional stiffness

It is defined as the torque required to produce unit angle of twist.

→ $T = \frac{GJ\theta}{l} \Rightarrow \boxed{\frac{T}{\theta} = \frac{GJ}{l}}$

For unit angle, $T = \frac{GJ}{l}$

→ It's unit is N.m/rad.

Equation of torsion:

Torsional eqn is given by :-

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$

where,

T = Torque or Twisting moment (N-mm)

J = Polar Moment of Inertia (mm^4)

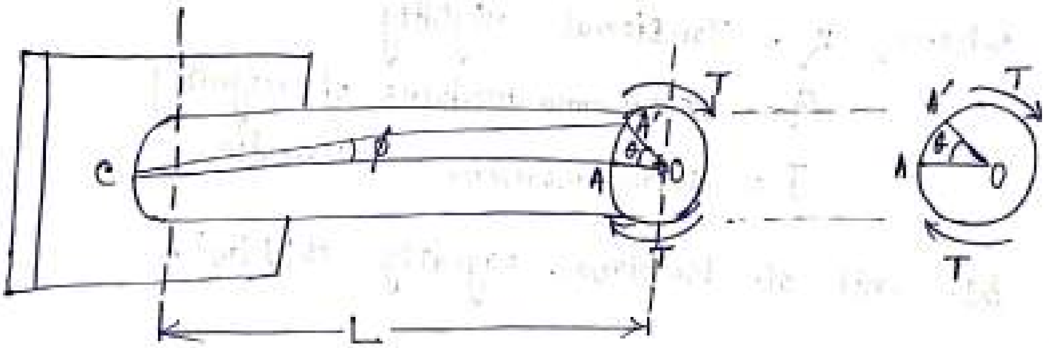
τ = Shear stress (N/mm^2)

R = Radius of shaft (mm)

G = Modulus of rigidity (N/mm^2)

θ = Angle of twist (radian)

L = Length of shaft. (mm)



Here, considering a circular shaft which is fixed at one end and subjected to a torque at other end as shown in fig, as a result of this torque, every c/s of the shaft will be subjected to shear stresses.

Let the line CA on the surface of the shaft be deformed to CA' and OA to OA' as shown in figure. Here, AOA' = θ in radians

(i) Based on strength criteria.

$$\boxed{\frac{T}{J} = \frac{\tau}{R}}$$

(ii) Based on rigidity criteria

$$\boxed{\frac{T}{J} = \frac{G\theta}{L}}$$

3.4 Combined bending and direct stresses:

Here, in this figure when a member is subjected to a downward load 'p' at the central axis, it will be subjected to compressive stress and a reaction 'p' is shown which will act at the bottom.

→ In this case, when the load will act at the central axis of the member; then the member will be subjected to direct stresses.

here, $\sigma_d = \frac{Load}{Area} = \frac{P}{A}$

→ And here, in this figure when a member is subjected to a loading at a distance 'e' from the central axis. (eccentricity)

→ In this case, the member is subjected to a eccentric loading and is under direct stress & bending stress.

So, here the Resultant stress or total stress

$$= \sigma_{total} = \sigma_R = \sigma_d + \sigma_b = \text{direct stress} + \text{bending stress}$$

(in N/mm^2) $\left(\frac{P}{A}\right) + \left(\frac{M}{I} \cdot y\right)$

<u>Direct stress</u> :- when a body is subjected to an axial tension or compression, it will be under direct stress.

<u>Bending stress</u> :- when a body is subjected to a loading at an eccentricity from axis, it creates a bending moment, and it will be under bending stress as well as direct stress. for the weight of the member.

Eccentric Loading:

A load, whose line of action does not coincide with the axis of a member, is known as an eccentric load.

Ex:- A bucket full of water, carried by a person in his hand

The simple reason for the case is that if he carries the bucket in his hands, then in addition to his carrying bucket, he has also to lean or bend on the other side of the bucket, so as to counteract any possibility of his falling towards the bucket.

Thus we can say that he is subjected to:

(Direct Stress)1. Direct load, due to the weight of bucket (including water)

and

(Bending stress)2. Moment due to eccentricity of the load.

→ So, under application of eccentric load, direct & bending stresses develop

Eccentricity:

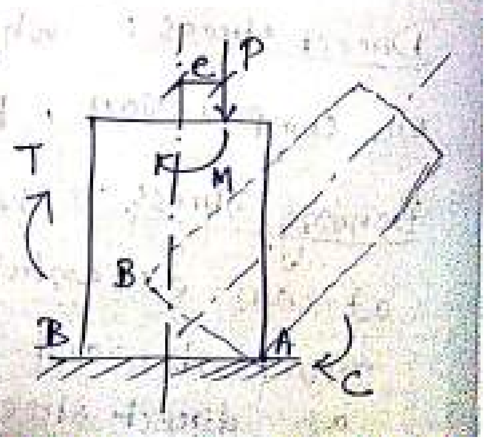
The distance between the actual line of action of compressive or tensile loads and the line of action that would produce a uniform stress over the cross-section of the specimen is called as eccentricity.

→ It is denoted by 'e'.

→ Or, The horizontal distance between the longitudinal axis of the member and line of action of load is called as eccentricity.

Maximum & Minimum stresses in Sections

When, P Load is applied at a distance 'e' which is nearer to face 'A', & far from 'B', the 'A' point will be in compression & 'B' point will be lifted out due to tension.



So, Max^m stress will be at face 'A' & min^m stress will be at 'B'.

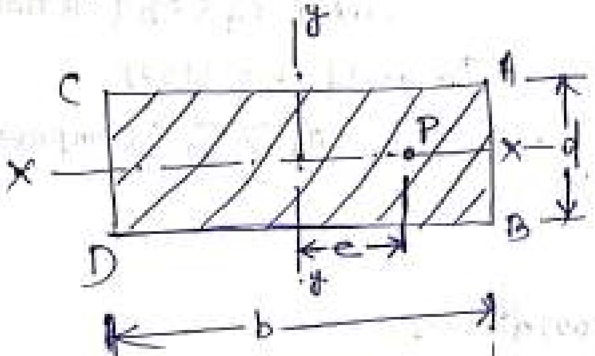
→ But at point 'B', Tension is not allowed because the member will be lifted out from it's support.

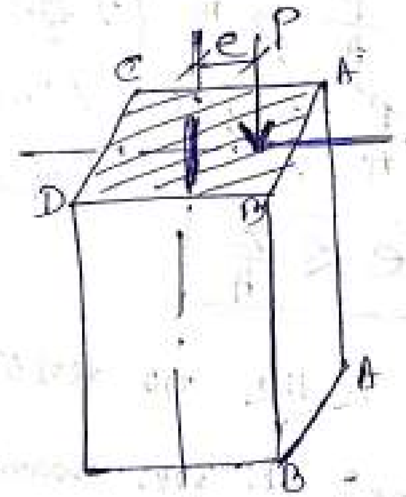
So,

1. $\boxed{\sigma_{max} = \sigma_d + \sigma_b}$ or, $\sigma_{max} = \frac{P}{A} + \frac{M}{Z} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$

2. $\boxed{\sigma_{min} = \sigma_d - \sigma_b}$ or, $\sigma_{min} = \frac{P}{A} - \frac{M}{Z} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$

Stress distribution diagrams for direct & bending stresses.





Case-1

If $\sigma_d > \sigma_b$

σ_{max} is +ve

σ_{min} is +ve.

Case-2

If $\sigma_d = \sigma_b$

σ_{max} is +ve

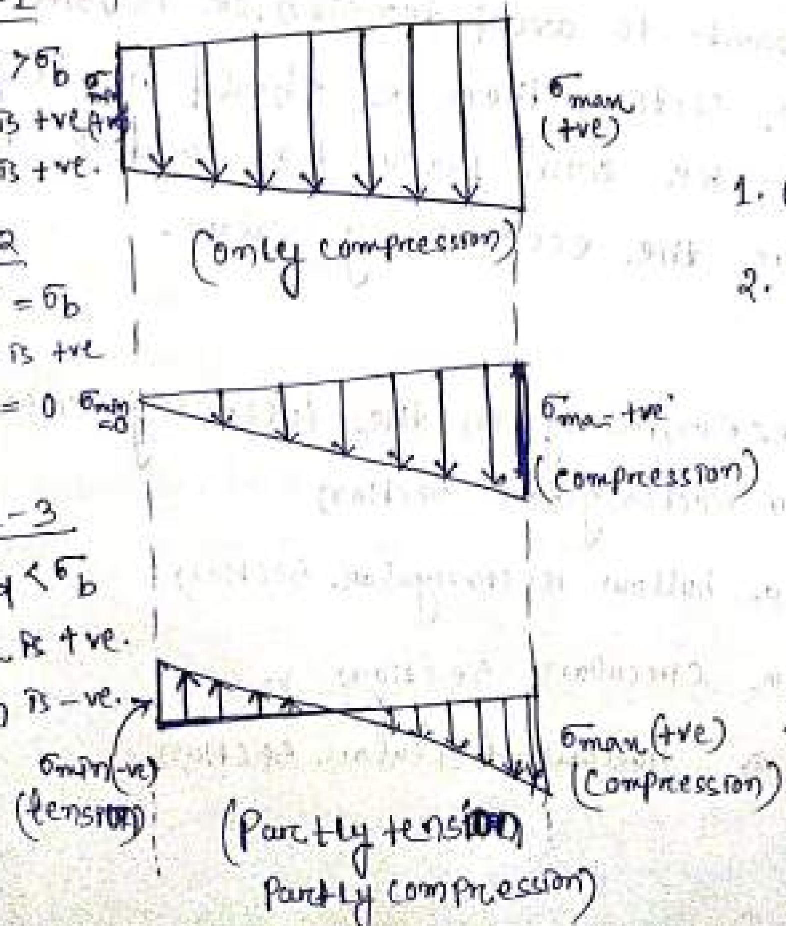
$\sigma_{min} = 0$

Case-3

If $\sigma_d < \sigma_b$

σ_{max} is +ve.

σ_{min} is -ve.



1. $\sigma_{max} = \sigma_d + \sigma_b$

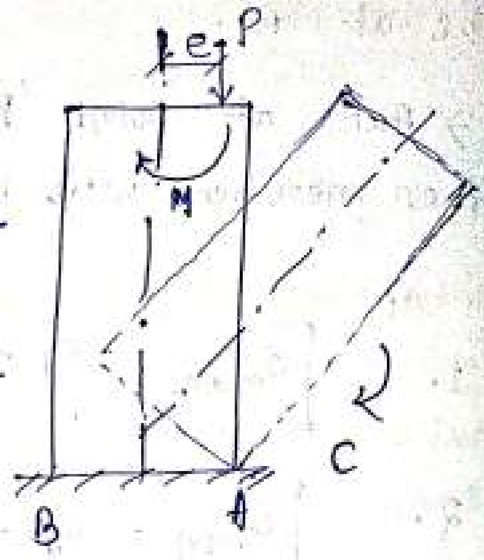
2. $\sigma_{min} = \sigma_d - \sigma_b$

As we know, we have to avoid the case of tension because the member will be lifted out from it's support.

→ So, case-1 & case-2 will be accepted, but case-3 will be avoided.

Conditions for no tension in the section :-

If the tensile stress develops at the base of the member, the member will be lifted from it's support and such conditions are not allowed. T to protect the structure.



→ so, we have to avoid the tension at the base.

$\sigma_d \geq \sigma_b$

$\sigma_{max} = \sigma_d + \sigma_b$

$\sigma_{min} = \sigma_d - \sigma_b$

when $\sigma_d < \sigma_b$ (Tensile)

∴ To avoid tension,

∴ $\sigma_d \geq \sigma_b$ (compressive)

$\therefore \frac{P}{A} \geq \frac{M}{Z}$ $\left(\because \frac{M}{I} = \frac{\sigma_b}{y}\right.$

$\Rightarrow \cancel{\frac{P}{A}} \geq \frac{\cancel{P}.e}{Z}$ $\Rightarrow \sigma_b = \frac{M.y}{I}$

$\Rightarrow \sigma_b = \frac{M}{\left(\frac{I}{y}\right) \to Z}\Big)$

$\Rightarrow \boxed{e \leq \frac{Z}{A}}$

This is the no tension condition.

→ It means, if we want to avoid tension or tensile stress being developed in the section then 'e' should be less or equal to 'Z/A'. so, that we will place the load 'P' to avoid tension as per the eccentricity value.

Limit of eccentricity

The limit of eccentricity in the following cases:

1. for a rectangular section
2. for a hollow rectangular section
3. for a circular section
4. for a hollow circular section..

1. Limit of eccentricity for a rectangular section

No tension condition,

$$\boxed{e \leq \frac{d}{6}}$$

2. Limit of eccentricity for a hollow rectangular section

No tension condition,

$$\boxed{e \leq \frac{BD^3 - bd^3}{6D(BD - bd)}}$$

3. Limit of eccentricity for a circular section

No tension condition,

$$\boxed{e \leq \frac{d}{8}}$$

4. Limit of eccentricity for a hollow circular section

No tension condition,

$$\boxed{e \leq \frac{(D^2 - d^2)}{8d}}$$

The maximum distance of load from the centre of column, such that if load acts within this distance, there is no tension in the column, is called limit of eccentricity (e limit).

When load is acting within e limit,
σ_{min} will be compressive (+ve)
When load is acting at the point of e limit,
σ_{min} will be zero.
When load is acting beyond e limit,
σ_{min} will be tensile (−ve)

Core of a section / Kernel of a section

→ It is a region within which if the load is placed then, there will be 'no tension' in the section.

→ we know that for 'No tension' condition,

$$\text{eccentricity } (e) \leqslant \frac{Z}{A}$$

where, Z = section modulus.

A = Cross-sectional area.

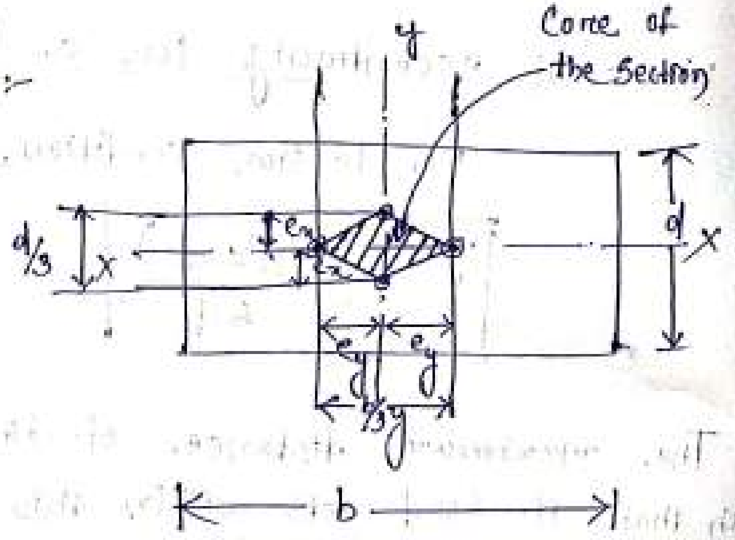
→ Middle one-third rule.

Middle one-third Rule states that no tension is developed in a wall or foundation of rectangular section, if the resultant force lies within the middle third of the structure.

For rectangular section

For No Tension condition:-

$$\boxed{e \leqslant \frac{Z}{A}}$$



$$\therefore e_x \leqslant \frac{Z_{xx}}{A}$$

$$\Rightarrow e_x \leqslant \frac{I_{xx}}{y} \Big/ A$$

$$\Rightarrow e_x \leqslant \frac{\frac{bd^3}{12}}{\frac{d}{2}} \Big/ bd$$

$$\Rightarrow e_x \leqslant \left(\frac{\cancel{b}d^{\cancel{3}}}{\cancel{12}_6} \times \frac{\cancel{2}}{\cancel{d}}\right) \times \frac{1}{\cancel{bd}}$$

$$\Rightarrow \boxed{e_x \leqslant \frac{d}{6}}$$

Total $e_x = 2 \times \frac{d}{6} = \frac{d}{3}$

$$e_y \leq \frac{Z_{yy}}{A}$$

$$\Rightarrow e_y \leq \frac{I_{yy}}{y} \Big/ A$$

$$\Rightarrow e_y \leq \frac{\frac{db^3}{12}}{\frac{b}{2}} \Big/ bd$$

$$\Rightarrow e_y \leq \left(\frac{db^{\cancel{3}}}{\cancel{12}_6} \times \frac{\cancel{2}}{\cancel{b}}\right) \times \frac{1}{\cancel{bd}}$$

$$\Rightarrow \boxed{e_y \leq \frac{b}{6}}$$, Total $e_y = \frac{b}{6} \times 2 = \frac{b}{3}$

→ By joining the eccentricity limits, we will get a area/region. This area is called core of the section.

→ It means, whenever inside this area ~~we~~ or on to the boundary of this area we are placing a load, there will be no tension in the member.

But when we will place the load outside this area, the member will be in tension.

→ So, to avoid tension in the member, we have to apply load in this region.

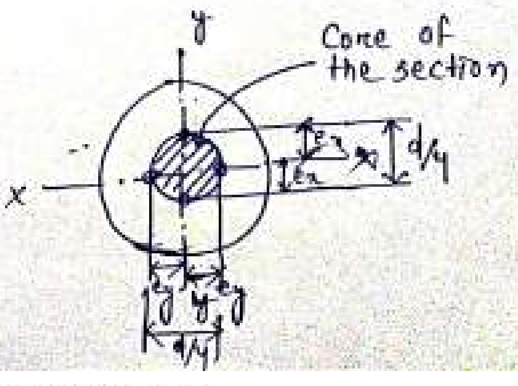
→ <u>Middle one-fourth Rule./middle quarter Rule.</u>

Middle one-fourth Rule states that no tension is developed in a member of circular section, if the resultant force lies within the middle-fourth of the structure.

<u>For circular section</u>

For No Tension condition:—

$$\boxed{e \leq \frac{Z}{A}}$$



$\therefore e_x \leq \frac{Z_{xx}}{A}$

$\Rightarrow e_x \leq \frac{I_{xx}}{y} / A$

$\Rightarrow e_x \leq \left(\frac{\frac{\pi d^4}{64}}{\frac{d}{2}}\right) \times \frac{1}{\frac{\pi}{4} d^2}$

$\Rightarrow e_x \leq \frac{\pi d^4}{64} \times \frac{2}{d} \times \frac{4}{\pi d^2}$

$\Rightarrow \boxed{e_x \leq \frac{d}{8}}$ $\quad\therefore$ Total $e_x = \frac{d}{8} \times 2 = \frac{d}{4}$

For circular section $I_{xx} = I_{yy}$

so, $e_x = e_y$

$\therefore \boxed{e_y \leq \frac{d}{8}}$ $\quad$ Total $e_y = \frac{d}{8} \times 2 = \frac{d}{4}$

→ By joining the eccentricity limits, we will get the core of the section.

→ So, to avoid tension, we have to place the load inside this core/region of the section or on the boundary of the core.

→ If we apply load out-side this area, the member will be in tension.

Maximum & Minimum stress in case of Dams, Retaining walls, Chimneys

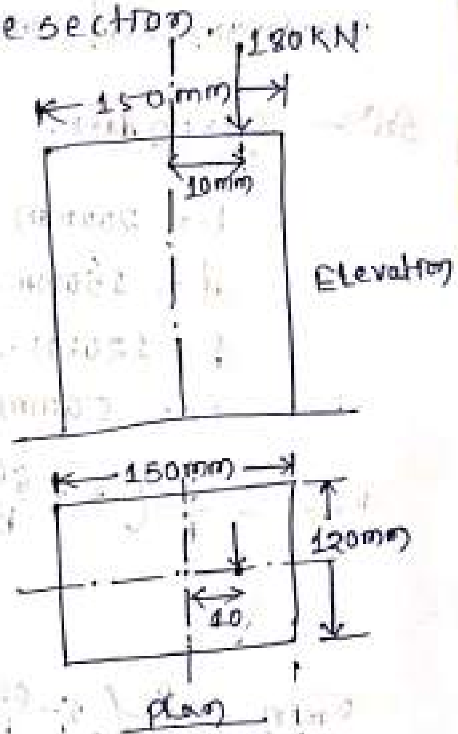
$$\boxed{\begin{aligned} \sigma_{max} &= \frac{W}{b}\left(1 + \frac{6e}{b}\right) \\ \sigma_{min} &= \frac{W}{b}\left(1 - \frac{6e}{b}\right) \end{aligned}}$$

where, W = weight of structure per unit length

b = width of structure.

e = eccentricity

1. A rectangular member is 150mm wide & 120 mm thick. It carries a load of 180 kN at an eccentricity of 10mm in a plane bisecting the thickness. Find the maximum & minimum intensities of stress in the section.



Soln:- Given data. $b = 150mm$

$d = 120mm.$

$P = 180KN = 180\times10^3 N$

$e = 10mm.$

$\sigma_{max} = \sigma_d + \sigma_b$

$\sigma_{min} = \sigma_d - \sigma_b$

$$\sigma_{max} = \sigma_d + \sigma_b$$

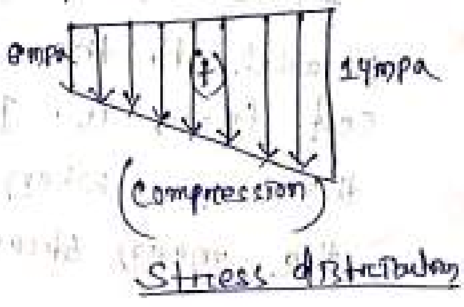
$$= \frac{P}{A} + \frac{M}{Z}$$

$$= \frac{180\times10^3}{(150\times120)} + \frac{180\times10^3\times10}{\left(\frac{120\times150^3}{12} / \frac{150}{2}\right)}$$

(here, I_{yy} will be used because e value is given from yy axis)

$$= 10 + 4$$

$$= 14 \text{ mpa. (Ans.)}$$



$$\sigma_{min} = \sigma_d - \sigma_b$$

$$= \frac{P}{A} - \frac{M}{Z}$$

$$= \frac{180\times10^3}{150\times120} - \left(\frac{180\times10^3\times10}{\frac{120\times150^3}{12} / \frac{150}{2}}\right)$$

$$= 10 - 4$$

$$= 6 \text{ mpa. (Ans.)}$$

or,

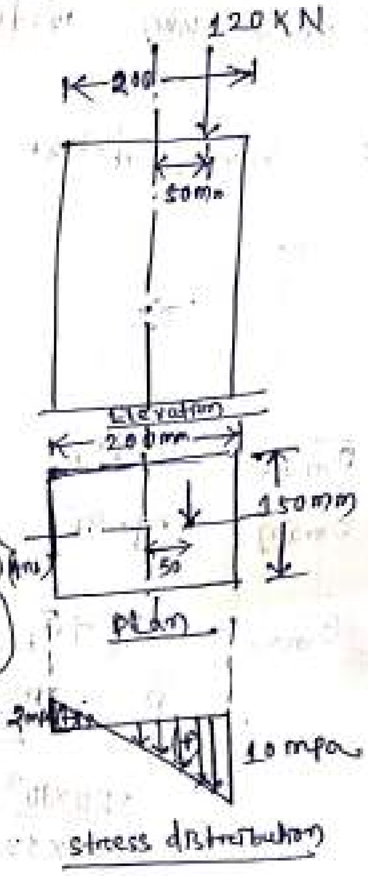
$$\sigma_{max} = \frac{P}{A}\left(1+\frac{6e}{b}\right) = \left(\frac{180\times10^3}{150\times120}\right)\times\left(1+\frac{6\times10}{150}\right)$$

$$= 10\times(1+0.4) = 10\times1.4 = 14 \text{ mpa}$$

$$\sigma_{min} = \frac{P}{A}\left(1-\frac{6e}{b}\right) = \left(\frac{180\times10^3}{150\times120}\right)\times\left(1-\frac{6\times10}{150}\right)$$

$$= 10\times(1-0.4) = 10\times0.6 = 6 \text{ mpa.}$$

2. A rectangular section 200 mm wide and 150 mm thick is carrying a vertical load of 120 KN at an eccentricity of 50 mm in a plane bisecting the thickness. Determine the maximum and minimum intensities of stress in the section.



Solⁿ:— Given data:

$b = 200 mm$

$d = 150 mm.$

$P = 120 KN = 120 \times 10^3 N$

$e = 50 mm.$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right) = \left(\frac{120 \times 10^3}{200 \times 150}\right)\left(1 + \frac{6 \times 50}{200}\right)$$

$$= 4 \times (1 + 1.5) = 10 \text{ mpa (comp.) (Ans)}$$

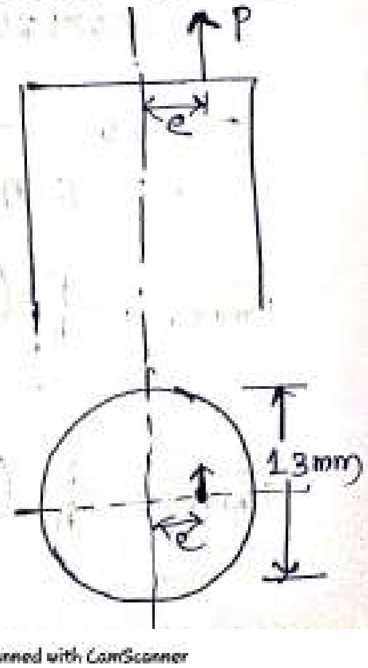
$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right) = \left(\frac{120 \times 10^3}{200 \times 150}\right)\left(1 - \frac{6 \times 50}{200}\right)$$

$$= 4 \times (1 - 1.5) = 4 \times (-0.5)$$

$$= -2 \text{ mpa}$$

$$= 2 \text{ mpa (Tension) (Ans)}$$

3. In a specimen of 13 mm diameter, the line of pull is parallel to the axis of the specimen, but it is displaced from it. Determine the distance of the pull from the axis, when the maximum stress is 15% greater than the mean stress on the specimen normal to the axis.



Solⁿ:— $d = 13 mm.$ (diameter)

$\sigma_{max} = 15\%$ greater than σ_{mean}

$$\sigma \text{ or } \sigma_{mean} = \frac{P}{A}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{Z} \text{ or } \sigma_d + \sigma_b$$

$$= \frac{P}{A} + \frac{P \times e}{I/y}$$

$$\Rightarrow \sigma_{max} = \frac{P}{A} + \frac{P.e}{\frac{\pi d^4}{64}/\frac{d}{2}} = \frac{P}{A} + \frac{P.e}{\frac{\pi d^4}{64} \times \frac{2}{d}} = \frac{P}{A} + \frac{P.e}{\frac{\pi d^3}{32}}$$

According to given stress condition

$\cancel{\frac{115}{100} \times \frac{P}{A}}$ σ_{max} = 15% greater than σ_{mean}

$$= \frac{115}{100} \times \frac{P}{A}$$

We can write

$$\frac{115}{100} \times \frac{P}{A} = \frac{P}{A} + \frac{P.e}{\frac{\pi d^3}{32}}$$

$$\Rightarrow \frac{115}{100} \times \frac{\cancel{P}}{A} = \cancel{P}\left(\frac{1}{A} + \frac{e}{\frac{\pi d^3}{32}}\right)$$

$$\Rightarrow \frac{115}{100} \times \frac{1}{A} = \frac{1}{A} + \frac{e}{\left(\frac{\pi d^2}{4}\right) \times \frac{d}{8}}$$

$$\Rightarrow \frac{115}{100} \times \frac{1}{A} = \frac{1}{A} + \frac{e}{A \times \frac{d}{8}} \quad \left(\because A = \frac{\pi d^2}{4}\right)$$

$$\Rightarrow \frac{115}{100} \times \frac{1}{\cancel{A}} = \frac{1}{\cancel{A}}\left(1 + \frac{8e}{d}\right)$$

$$\Rightarrow \frac{115}{100} = 1 + \frac{8e}{13}$$

$$\Rightarrow \frac{115}{100} = \frac{13 + 8e}{13}$$

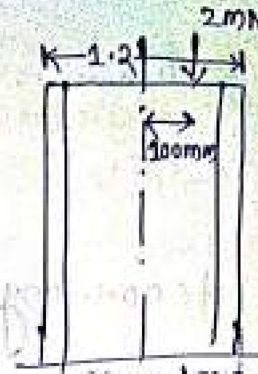
$$\Rightarrow 8e = \left(\frac{13 \times 115}{100}\right) - 13$$

$$\Rightarrow 8e = 14.95 - 13$$

$$\Rightarrow e = 1.95/8 = 0.24 \text{ mm. (Ans.)}$$

4. A hollow rectangular masonry pier is 1.2m x 0.8m wide and 150mm thick. A vertical load of 2MN is transmitted in the vertical plane bisecting 1.2m side and at an eccentricity of 100 mm from the geometric axis of the section.
Calculate the maximum and minimum stresse intensities in the section.

Sol^n:— Given data:



Elevation

Outer width $(B) = 1.2m = 1.2\times10^3$ mm

Outer thickness $(D) = 0.8m = 0.8\times10^3$ mm

Load $(P) = 2$ MN $= 2\times10^6$ N

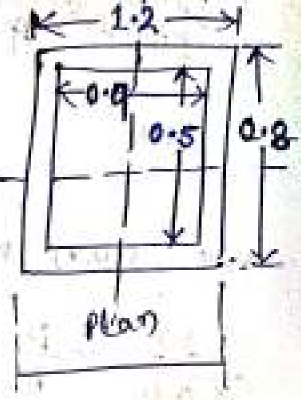
Thickness $(t) = 150$ mm. $\therefore b = 1.2 - (2\times0.15)$

eccentricity $(e) = 100$ mm. $= 1.2 - 0.3$

$= 0.9m = 0.9\times10^3$ mm

$\therefore d = 0.8 - (2\times0.15) = 0.8 - 0.3$

$= 0.5m = 0.5\times10^3$ mm

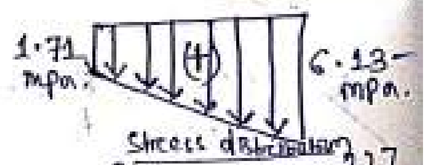


The area of the pier $= A = BD - bd$

$= \{(1.2\times10^3)\times(0.8\times10^3)\} - \{(0.9\times10^3)\times(0.5\times10^3)\}$

$= (0.96\times10^6) - (0.45\times10^6)$

$= 0.51\times10^6$ mm^2



Stress distribution

Section Modulus $= Z$

$Z = \frac{1}{6}(BD^2 - bd^2) = \frac{1}{6}\left[\{(1.2\times10^3)\times(0.8\times10^3)^2\} - \{(0.9\times10^3)\times(0.5\times10^3)^2\}\right]$

$= \frac{1}{6}\{(768\times10^6) - (225\times10^6)\} = 90.5\times10^6$ N.mm.

Maximum stress intensity in the section,

$$\sigma_{max} = \frac{P}{A} + \frac{M}{Z} = \frac{2\times10^6}{0.51\times10^6} + \frac{(2\times10^6)\times100}{90.5\times10^6}$$

$= 3.92 + 2.21 = 6.13$ N/mm^2 $= 6.13$ mPa. (Ans)

Minimum stress intensity in the section,

$$\sigma_{min} = \frac{P}{A} - \frac{M}{Z} = \frac{2\times10^6}{0.51\times10^6} - \frac{2\times10^6\times100}{90.5\times10^6}$$

$= 3.92 - 2.21 = 1.71$ N/mm^2 $= 1.71$ mpa (Ans)

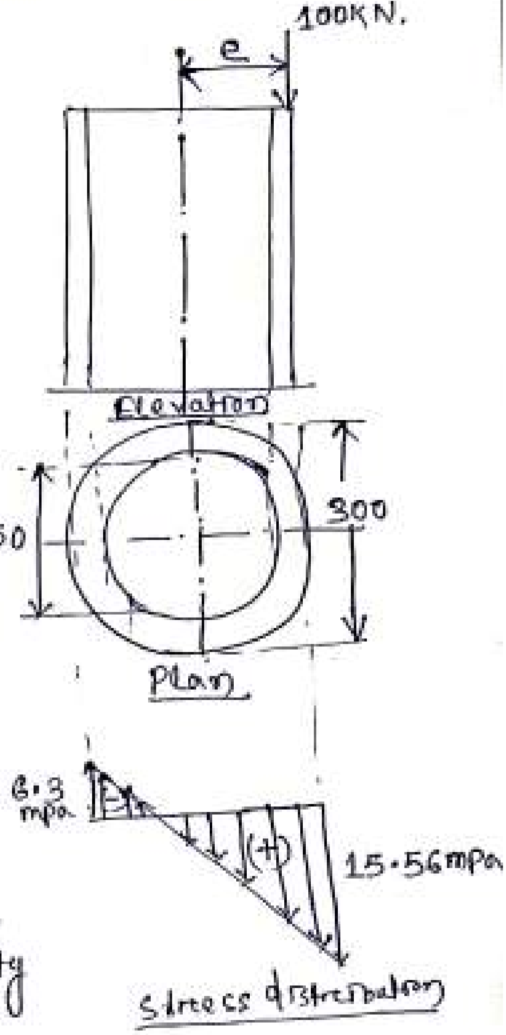
5. A hollow circular column having external and internal diameters of 300 mm and 250 mm respectively, carries a vertical load of 100KN at the outer edge of the column. Calculate the maximum and minimum intensities of stress in the section.

Sol^n:- Given data

External diameter (D) = 300 mm

Internal diameter (d) = 250mm

load (P) = 100 KN = 100×10^3 N.



The area of the column = $A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(300^2 - 250^2)$

$= 21.6 \times 10^3 \text{ mm}^2$

Section Modulus $= Z = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$

$= \frac{\pi}{32}\left(\frac{300^4 - 250^4}{300}\right)$

$= 1372 \times 10^3 \text{ mm}^3$

Since the column carries the vertical load at its outer edge, therefore eccentricity 'e' = 300/2 = 150 mm.

Maximum intensity of stress in the section,

$\sigma_{max} = \frac{P}{A} + \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} + \frac{100 \times 10^3 \times 150}{1372 \times 10^3}$

$= 4.63 + 10.93 = 15.56 \text{ N/mm}^2 = 15.56 \text{ mPa}$. (Ans)

Minimum intensity of stress in the section,

$\sigma_{min} = \frac{P}{A} - \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} - \frac{100 \times 10^3 \times 150}{137.2 \times 10^3}$

$= 4.63 - 10.93 = -6.3 \text{ N/mm}^2$

$= 6.3 \text{ N/mm}^2$ (Tension)

$= 6.3$ Mpa (Tension) (Ans)

Chapter-4

Columns & Struts

Strut

→ A structural member subjected to axial compression or compressive force is called strut.

→ Strut may be vertical, horizontal or inclined.

→ The cross-sectional dimensions of strut are small.

→ Normally, struts carry smaller compressive loads.

→ Struts are used in roof truss and bridge trusses.

Column

→ When strut is vertical is known as column.

→ The cross-sectional dimensions of column are large.

→ Normally, columns carry heavy compressive loads.

→ Columns are used in concrete and steel buildings.

Mainly, columns & struts are compression members in buildings, bridges, supporting systems of tanks, factories and many more structures.

These are used to transfer a load of superstructure to the foundation safely.

Radius of Gyration (K)

The distance from an axis upto a point where, the entire area of a plane is assumed to be concentrated, is known as radius of gyration.

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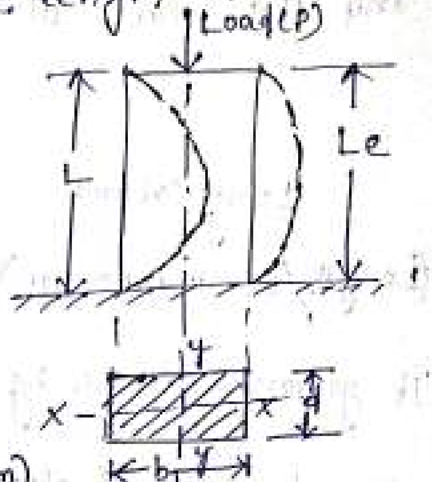
→ It is denoted by 'K'.

→ $K = \sqrt{\left(\frac{I}{A}\right)}$ or $I = AK^2$

where, K = radius of gyration (mm)
I = Moment of Inertia (mm^4)
A = Area of the plane or section (mm^2)

Slenderness Ratio

It is defined as the ratio of effective length of column to the minimum radius of gyration.



→ It is denoted by 'λ'.

Mathematically, $\boxed{\lambda = \frac{l_e}{K_{min}}}$

where, λ = slenderness ratio
l_e = effective length (mm or m)
K_{min} = Minimum Radius of gyration (mm or m) $= \sqrt{\frac{I_{min}}{A}}$ ← (Minimum of I_{xx} & I_{yy})

→ If λ is more, its load carrying capacity will be less.

→ It does not have any units.

Classification of column

Columns are classified according to nature of failure as:

1. Long column
2. Short column

Long column

→ When length of column is more as compared to its c/s dimension, it is called long column.

→ In this case λ or $l_e/K_{min} > 50$

→ Failure occurs mainly due to bending stress and the role of direct compressive stress is negligible.

Short column

→ When length of column is less as compared to its C/S dimension, it is called short column.

→ In this case λ or $l_e/K_{min} < 50$

→ Failure occurs mainly due to direct compressive stress only and the role of bending stress is negligible.

Difference

Long column	Short column
1. (Length/Least dimension) > 12	1. (Length/Least dimension) < 12
2. It generally fails by buckling/bending	2. It generally fails by crushing.
3. Slenderness Ratio > 50.	3. Slenderness Ratio < 50.
4. Subjected to buckling or bending stress	4. Subjected to direct stress (compressive/tensile)
5. Radius of gyration is less	5. Radius of gyration is more.
6. Load carrying capacity is less.	6. Load carrying capacity is more.

Euler's theory of long columns:

The first logical attempt, to study the stability of long columns, was made by ~~Euler's~~ Mr. Euler.

→ He derived an equation, for the buckling load of long columns based on the bending stress.

→ While deriving this equation, the effect of direct stress is neglected and this may be justified with the statement that the direct stress induced in a long column is negligible as compared to the bending stress.

→ It may be noted that the Euler's formula cannot be used in the case of short columns, because the direct stress is considerable and hence cannot be neglected.

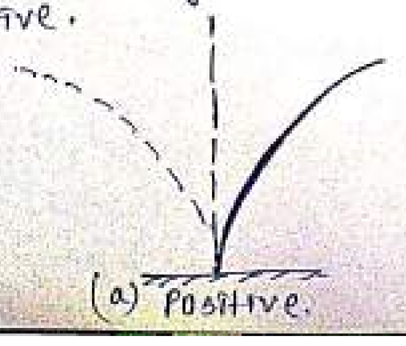
Assumptions in the Euler's Column Theory:

The following assumptions are made in the Euler's Column theory:

1. Initially the column is perfectly straight and the load applied is truly axial.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic and thus obeys Hooke's law.
4. The length of column is very large as compared to its cross-sectional dimensions.
5. The shortening of column, due to direct compression (being very small) is neglected.
6. The failure of column occurs due to buckling alone.

Sign Conventions:

1. A moment, which tends to bend the column with convexity towards its initial central line as shown in figure (a) is taken as positive.



(a) Positive.

2. A moment, which tends to bend the column with its concavity towards its initial central line as shown in figure (b) is taken as negative.

(b) Negative

Euler's formula for long column & Equivalent/Effective Length of a column: (for various end conditions)

Euler's formula for all cases: (critical load for all end conditions)

$$\boxed{P_E = \frac{\pi^2 EI}{l_e^2}}$$

where,

P_E = Euler's load (crippling / buckling load) in N

E = Modulus of Elasticity or Young's Modulus in N/mm^2

I = Moment of Inertia of the cross-section (MI) in mm^4

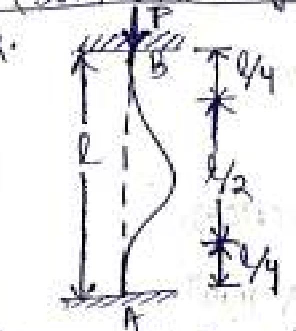
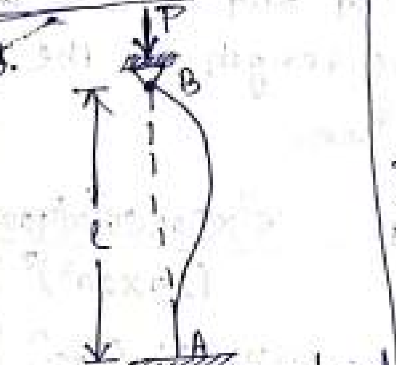
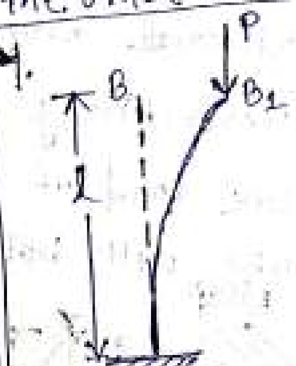
l_e = Effective or Equivalent length in mm.

Effective length or Equivalent length for end conditions

In actual practice, there are a number of end conditions, for columns.

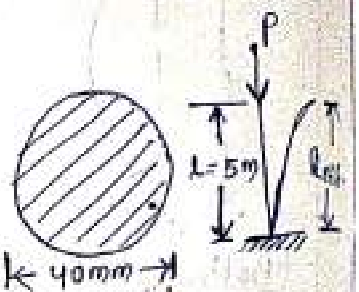
Euler's column theory on the following four types of end conditions are studied always:

1. Both ends hinged
2. Both ends fixed
3. One end is fixed and the other hinged, and
4. One end is fixed and the other free.

End condition	Critical load (P)	Effective length
1.  (Both ends hinged)	$P = \frac{\pi^2 EI}{l^2}$	$l_{eff} = l$
2.  (Both ends fixed.)	$P = \frac{\pi^2 EI}{(l/2)^2}$	$l_{eff} = l/2$
3.  (One end is fixed and the other end hinged)	$P = \frac{\pi^2 EI}{(\frac{l}{\sqrt{2}})^2}$	$l_{eff} = l/\sqrt{2}$
4.  (One end fixed and the other is free)	$P = \frac{\pi^2 EI}{(2l)^2}$	$l_{eff} = 2l$

Problem-1

Q. A steel rod 5m long and of 40 mm. diameter is used as a column, with one end fixed and the other free. Determine the crippling load by Euler's formula. Take E as 200 GPa.



Sol^n: Given Data:

Length $(l) = 5m = 5 \times 10^3 mm$

Diameter of column $(d) = 40mm$

Modulus of elasticity $(E) = 200\ GPa = 200 \times 10^3 N/mm^2$

The moment of inertia of the column section,

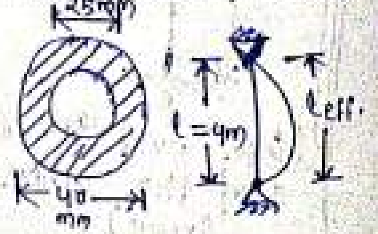
$$I = \frac{\pi}{64} \times (d)^4 = \frac{\pi}{64} \times (40)^4 = 40,000\,\pi\ mm^4$$

Since the column is fixed at one end and free at the other, therefore equivalent or effective length of the column, $L_{eff.} = 2l = 2 \times 5 \times 10^3 = 10 \times 10^3\ mm.$

$$\therefore \text{Euler's crippling load} = P_E = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times (200 \times 10^3) \times (40000\pi)}{(10 \times 10^3)^2}$$

$$= 2480\,N = 2.48\,KN \text{ (Ans.)}$$

2. Q. A hollow alloy tube 4 m. long with external and internal diameters of 40mm and 25mm. respectively was found to extend 4.8 mm under a tensile load of 60KN. Find the buckling load for the tube with both ends pinned.



Sol^n:- Given data :

$l = 4m = 4 \times 10^3\ mm$

External diameter of column $(D) = 40\ mm.$

Internal diameter of column $(d) = 25mm.$

Deflection $(\delta l) = 4.8\ mm$

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Tensile load = 60 KN = 60×10^3 N

The area of the tube, $A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(40^2 - 25^2) = 765.8 \; mm^2$

Moment of Inertia of the tube,

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}\left[(40)^4 - (25)^4\right] = 106500 \; mm^4$$

We also know the strain in the alloy tube,

$$e = \frac{\delta l}{l} = \frac{4.8}{4 \times 10^3} = 0.0012$$

Modulus of elasticity for the alloy,

$$E = \frac{\sigma}{e} = \frac{(Load/Area)}{strain} = \frac{60 \times 10^3 / 765.8}{0.0012} = 65{,}290 \; N/mm^2$$

Since, The column is pinned at its both ends, therefore equivalent or effective length of the column,

$$l_e = l = 4 \times 10^3 \; mm.$$

Euler's buckling load, $P_E = \frac{\pi^2 E I}{l_e^2} = \frac{\pi^2 \times 65{,}290 \times 106500}{(4 \times 10^3)^2}$

$= 4290 \; N = 4.29 \; KN$ (Ans.)

3. Q. A steel bar of rectangular section 40mm × 50mm. Pinned at each end is subjected to axial compression. The bar is 2m long. Determine the buckling load and the corresponding axial stress using Euler's formula. Take $E = 2 \times 10^5 \; N/mm^2$

Soln:- Given data:

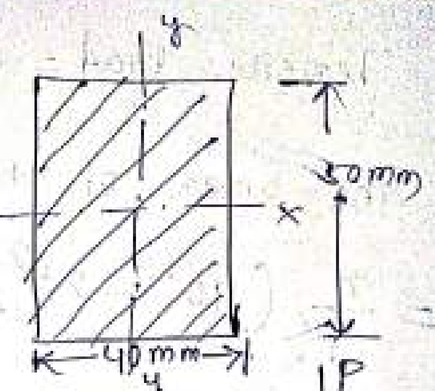
Rectangular section: $A = 40mm \times 50mm$

Pinned at each end.

$L = 2m = 2000 \; mm.$

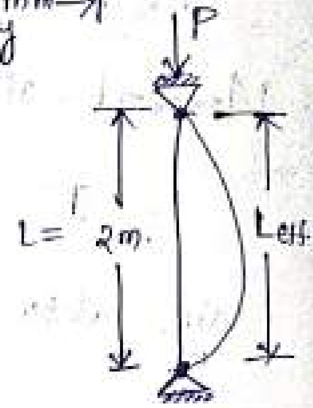
$E = 2 \times 10^5 \; N/mm^2$

Both the ends of the bar is pinned or hinged.



$\therefore$ Effective length $= l_{eff.} = l = 2000$ mm

Moment of inertia of given rectangular section is given by:-



$$I_{xx} = \frac{bd^3}{12} = \frac{40 \times 50^3}{12} = 416.67 \times 10^3 \, mm^4$$

$$I_{yy} = \frac{db^3}{12} = \frac{50 \times 40^3}{12} = 266.67 \times 10^3 \, mm^4$$

$I_{yy} < I_{xx}$.

So, I_{yy} will be taken as I_{min} for calculation of Euler's load.

$\therefore$ Euler's buckling load $= P_E = \frac{\pi^2 EI}{l_e^2}$

$$\Rightarrow P_E = \frac{\pi^2 \times 2 \times 10^5 \times 266.67 \times 10^3}{(2000)^2}$$

$$\Rightarrow P_E = 131.60 \times 10^3 \, N \text{ or, } 131.60 \text{ KN (Ans.)}$$

Axial stress induced in the column becaused of Euler's buckling /crippling load:

$$\sigma = \frac{P_E}{A} \Rightarrow \sigma = \frac{131.60 \times 10^3}{40 \times 50}$$

$$\Rightarrow \sigma = 65.79 \, N/mm^2 \text{ (Ans.)}$$

Chapter - 5

Shear force and Bending Moment

Types of loads and beams:

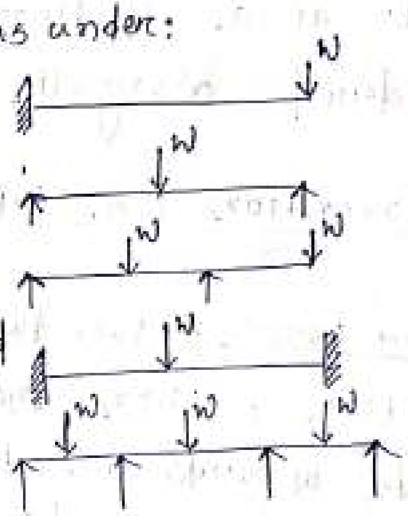
Load:- Load may be defined as a force tending to effect and produce deformations, stresses or displacements in the structure.

Beam:- A horizontal structural member which is acted upon by a system of external loads at right angle to its axis is known as beam.

Types of Beams

The types of beams are classified as under:

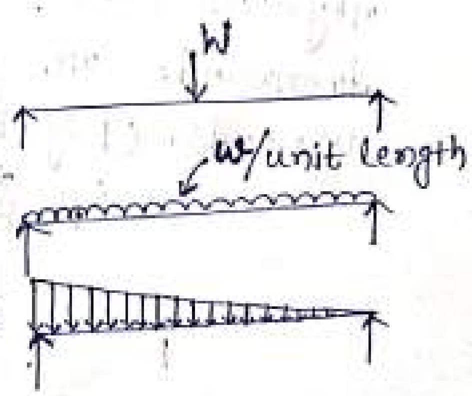
1. Cantilever beam →
2. Simply supported beam →
3. Overhanging beam →
4. Rigidly fixed or built-in-beam and →
5. Continuous beam →



Types of Loadings

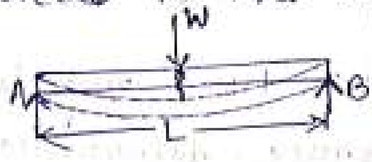
A beam may be subjected to either or in combination of the following types of loads:

1. Concentrated or point load →
2. Uniformly distributed load and →
3. Uniformly varying load →



Shear force (S.F.)

The shear force at the cross-section of a beam, is defined as the algebraic sum of all unbalanced vertical forces to the right or left of the section.



→ It's unit will be 'N' or 'KN'.

Bending Moment (B.M.)

The bending moment at the cross-section of a beam, is defined as the algebraic sum of the moments of the forces, to the right or left of the section.

→ It's unit will be 'N.mm' or 'KN.m'.

Note :- while calculating the shear force or bending moment at a section, the end reactions must also be considered alongwith other external loads.

Sign Convention for S.F. and B.M. :-

1. Shear force :- we take shear force at a section as positive, when the left hand portion tends to slide upwards or the right hand portion tends to slide downwards. (fig-a)

→ Similarly, we take shear force at a section as negative, when the left hand portion tends to slide downwards or the right hand portion tends to slide upwards. (fig-b)

+ (Positive)

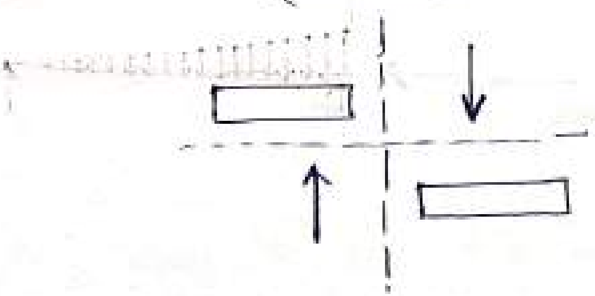


fig. (a)

(Negative)

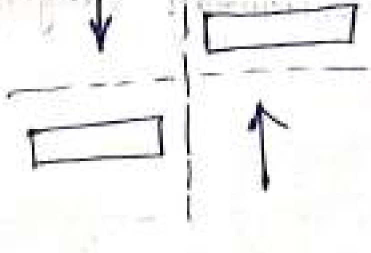


fig. (b)

2. Bending Moment : We take bending moment at a section as positive, if it tends to bend the beam at that point to a curvature having concavity at the top (fig. a)

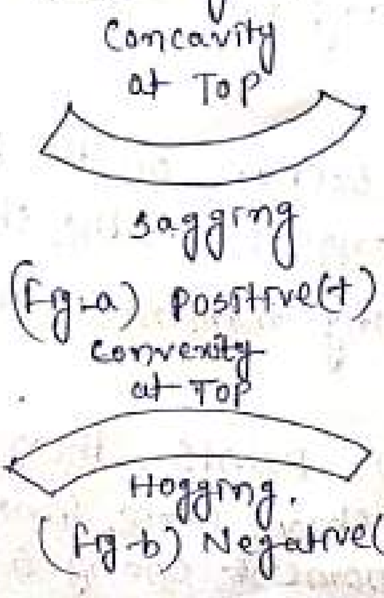
→ Similarly, we take bending moment at a section as negative, if it tends to bend the beam at that point to a curvature having convexity at the top (fig-b)

→ We often call the positive bending moment as sagging moment and negative bending moment as hogging moment.

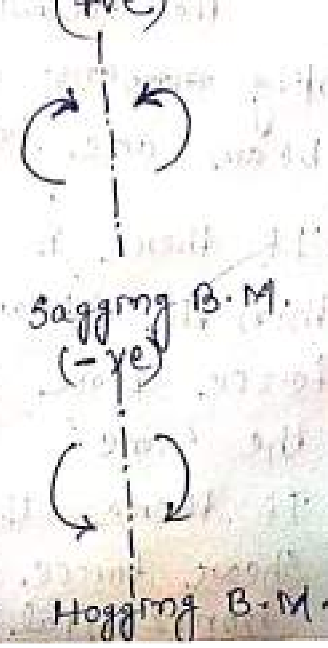
Another way of assigning the sign convention to the bending moment is by the direction in which it acts at a section.

→ We take the bending moment at a section as positive, when it is acting in clockwise direction to the left or in anticlockwise direction to the right.

→ Similarly, we take the bending moment at a section as negative, when it is acting in anticlockwise direction to the left or in clockwise direction to the right.



Or,



Note: While calculating bending moment or shear force at a section the beam will be assumed to be weightless.

Shear force and Bending Moment Diagrams:

Shear force Diagram (S.F.D)

A shear force diagram is the graphical representation of the variation of shear force along the length of the beam and is abbreviated as S.F.D.

Bending Moment Diagram (B.M.D.)

A bending moment diagram is the graphical representation of the variation of the bending moment along the length of the beam and is abbreviated as B.M.D..

These diagrams can be done by plotting the shear force or the bending moment as ordinate and the position of the cross as abscissa.

Note: While drawing the shear force or bending moment diagrams, all the positive values are plotted above the base line and negative values below it.

Relation between intensity of load, S.F. and B.M. :-

The following relations between loading, shear force and bending moment at a point or between any two sections of a beam are important:

1. If there is a point load at a section on the beam, then the shear force suddenly changes (i.e., the shear force line is vertical). But Bending moment remains the same.
2. If there is no load between two points, then the shear force does not change (i.e., Shear force line is horizontal). But the bending moment changes

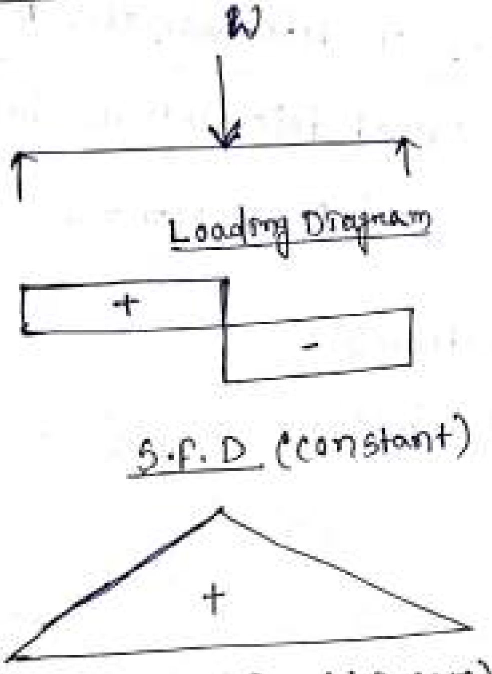
linearly (i.e., bending moment line is an inclined straight line).

3. If there is a uniformly distributed load between two points, then the shear force changes linearly (i.e. shear force line is an inclined straight line). But the bending moment changes according to the parabolic law. (i.e., bending moment line will be a parabola).

4. If there is a uniformly varying load between two points then the shear force changes according to the parabolic law (i.e., shear force line will be a parabola). But the bending moment changes according to the cubic law.

Relations

1.

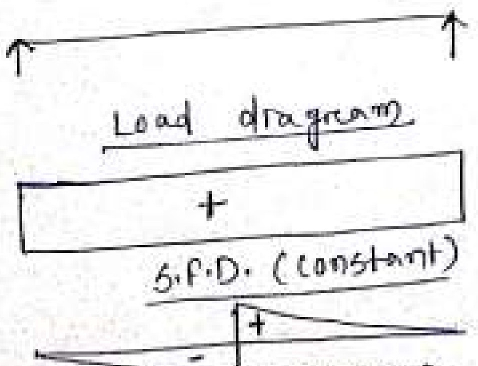


Loading Diagram

S.F.D (constant)

B.M.D. (Linear)

2.

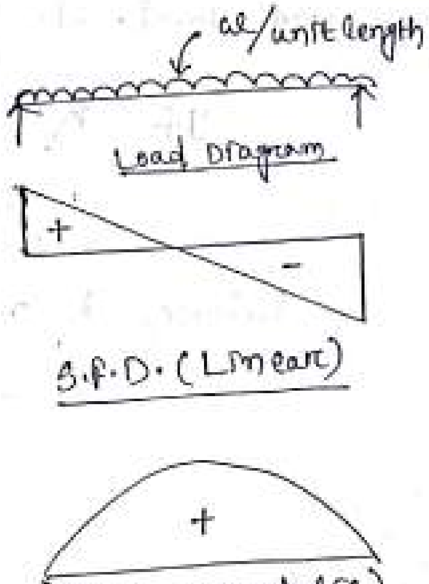


Load diagram

S.F.D. (constant)

B.M.D. (linear)

3.

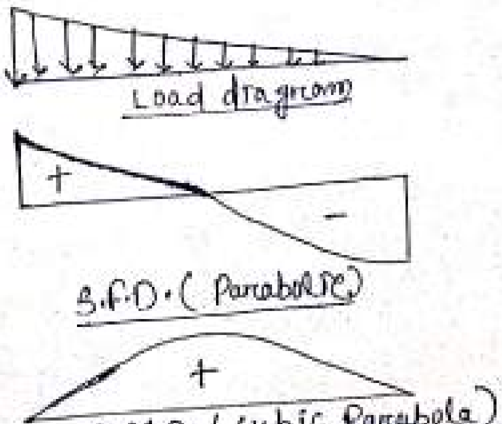


Load Diagram

S.F.D. (Linear)

B.M.D. (Parabolic)

4.



Load diagram

S.F.D. (Parabolic)

B.M.D. (Cubic Parabola)

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Important points to be noted while drawing S.F.D. & B.M.D.

1. Length of S.F.D. & B.M.D. must be equal to the span of the beam.

2. S.F.D. is drawn below the loaded beam & B.M.D. is drawn below S.F.D.

3. For simply supported beam, B.M. is zero at the supports.

4. For cantilever beam, B.M. will be zero at free end.

5. Calculate S.F. & B.M. at all critical points.

6. If no load is present between two points, then S.F. will be constant.

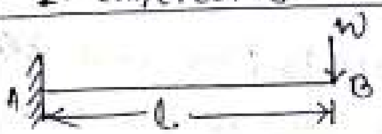
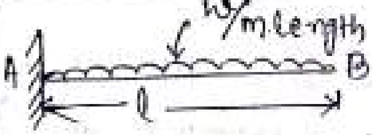
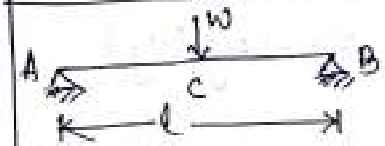
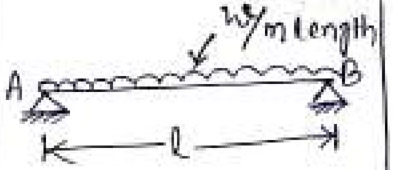
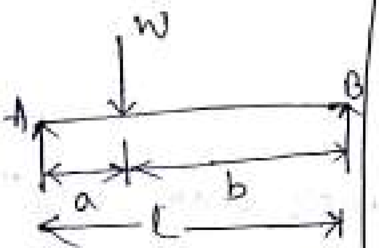
S.F. and B.M. of general cases of determinate beams

If $R = 3n$, It is called determinate beam

$R > 3n$, It is called Indeterminate beam

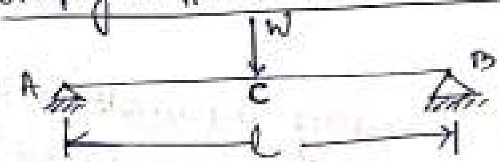
where, R = no. of reactions

n = no. of members.

Beam Cases	S.F.	B.M.
1. Cantilever		
	W	Wl (Fixed end)
	wl	$\frac{wl^2}{2}$ (Fixed End)
2. Simply Supported		
	$\frac{W}{2}$	$\frac{Wl}{4}$ (centre)
	$\frac{wl}{2}$	$\frac{wl^2}{8}$ (centre)
	$\frac{Wb}{l}$	$\frac{Wab}{L}$ (Load)

S.F.D. & B.M.D. for various types of beams with different loading

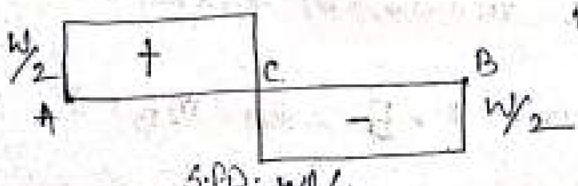
1. Simply supported beam with point load at centre.



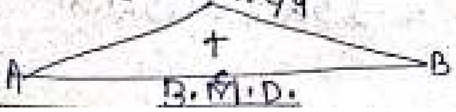
Here, AB beam is simply supported at the ends A and B. Let its span be l and carry a concentrated load W at the mid span.

→ As the load is symmetrically placed on the span, reaction at each support is $W/2$.

$\therefore R_A = R_B = W/2$



S.F.D.



B.M.D.

→ The shear force at any section between A & C (i.e. up to the point just before the load W) is constant and is equal to the unbalanced vertical force, i.e. $+W/2$.

→ Shear force at any section between C and B (i.e. just after the load W) is also constant and is equal to the unbalanced vertical force, i.e. $-W/2$. $(\because +\frac{W}{2} - W = -W/2)$

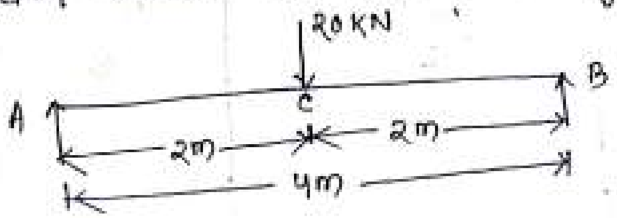
→ The bending moment at A & B is zero. It increases by a straight line law or linearly and is maximum at centre of beam.

This bending moment at C,

$M_C = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$ (+ve sign for sagging)

Problem-1

Draw shear force and bending moment diagram of the simply supported beam as shown in figure.



Sol^n:- S.F. (Shear Force)

Reactions Calculation

Both the reactions will be equal, since beam is symmetric

i.e. $R_A = R_B = \frac{W}{2} = \frac{20}{2} = 10\,KN$

$\therefore$ Shear force between A & C $= +10\,KN$.

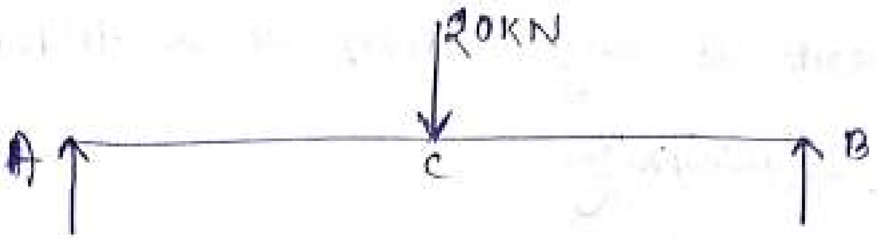
Shear force between C & B $= +10 - 20 = -10\,KN$.

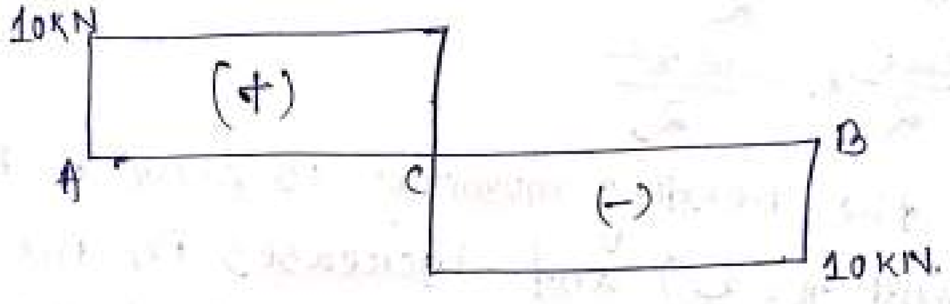
B.M. (Bending Moment)

In case of simply supported beam, bending moment will be zero at supports. And it will be maximum where shear force is zero

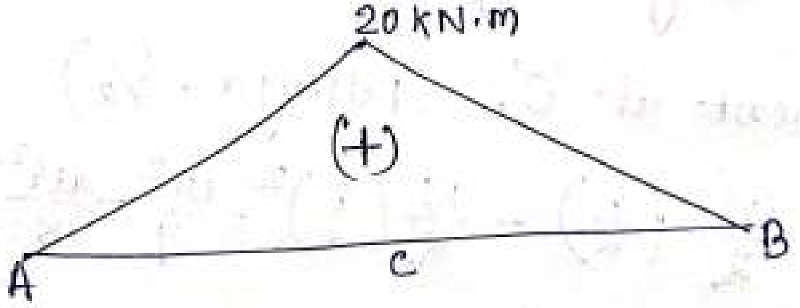
$\therefore$ Bending moment at point A & B $= M_A = M_B = 0$

Bending moment at point C $= 10 \times 2 = 20\,KNm$





S.F.D.

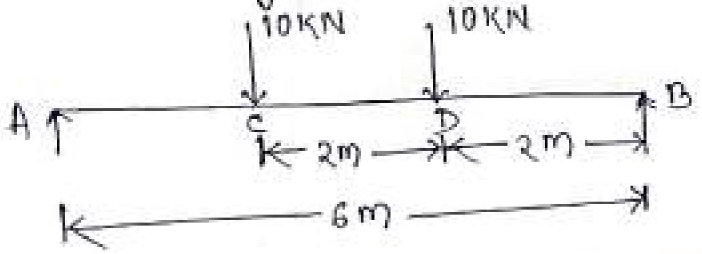


B.M.D.

~~Problem-2~~

Problem-2

A simply supported beam AB is shown in figure. Draw the shear force and bending moment diagrams for the beam.



Solⁿ:- Shear Force

$R_A + R_B = 10 + 10$

$\Rightarrow R_A + R_B = 20$

Taking moment about 'A',

$R_B \times 6 = (10 \times 4) + (10 \times 2)$

$\Rightarrow R_B = 60/6 = 10$ KN.

$\therefore R_A = 20 - 10 = 10$ KN

Shear force between A & C = +10KN

Shear force between C & D = $+10 - 10 = 0$ KN.

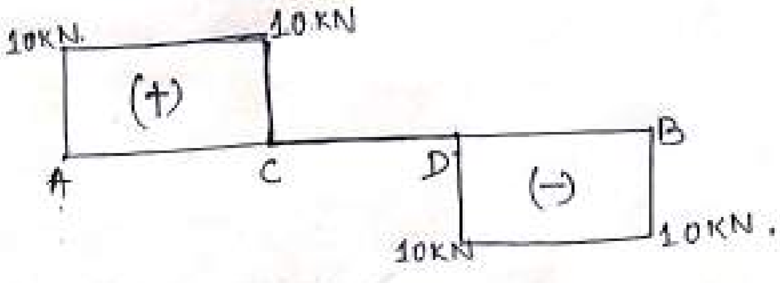
Shear force between D & B = $0 - 10 = -10$ KN.

Bending Moment.

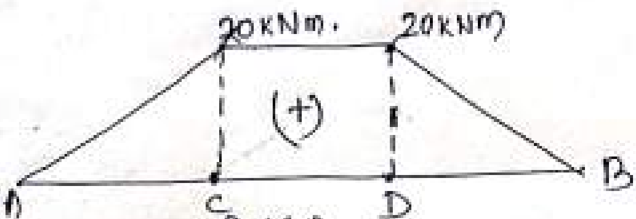
Due to simple support, $M_A = M_B = 0$

$M_D = \overset{R_B}{10} \times 2 = 20$ KNm.

$M_C = \overset{R_A}{10} \times 2 = 20$ KNm.



S.F.D



B.M.D

2. Simply supported with a uniformly distributed load

A simply supported beam AB is carrying a uniformly distributed load (u.d.l.) of w per unit length over the span.

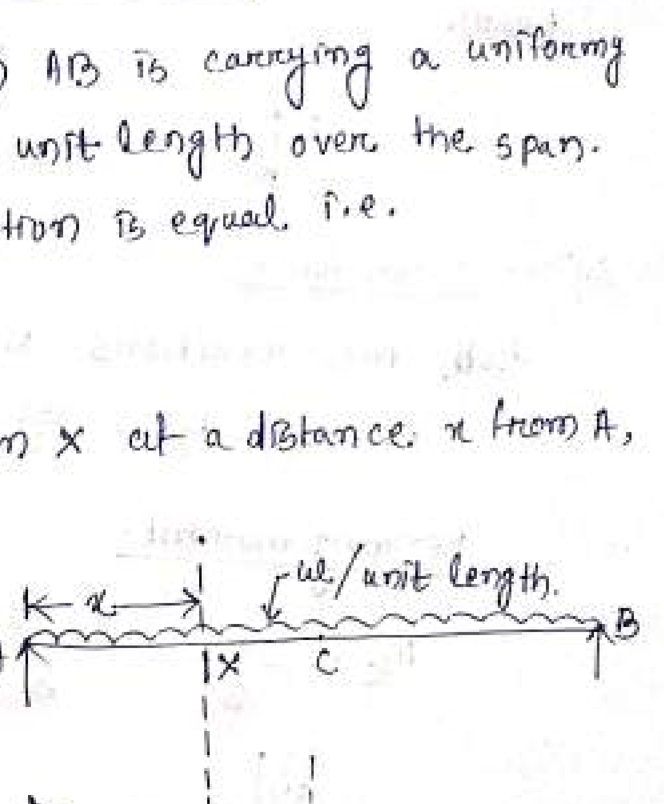
→ By symmetry each support reaction is equal i.e.

$$R_A = R_B = \frac{wl}{2}$$

The shear force at any section X at a distance x from A,

$$F_x = R_A - wx = \frac{wl}{2} - wx$$

The shear force at A is equal to $wl/2$, where $x = 0$ and decreases uniformly by a straight line to zero at the mid-point of the beam; beyond which it continues to decrease uniformly to $-wl/2$ at B i.e., R_B.



The bending moment at any section at a distance x from A,

$$M_x = R_A x - (wx) \times \frac{x}{2}$$

$$= R_A x - \frac{wx^2}{2}$$

$$= \frac{wl}{2} x - \frac{wx^2}{2}$$

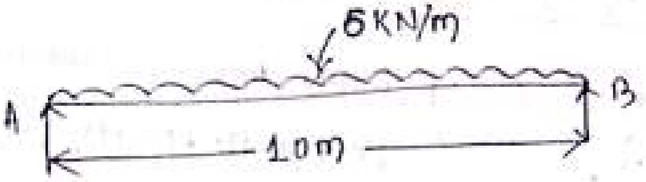
→ In this case, the bending moment is zero at A & B (where $x = 0$ and $x = l$) and increases in the form of a parabolic curve at C, i.e. mid-point of the beam where shear force changes sign.

∴ Bending moment at 'C', (putting $x = l/2$)

$$M_C = \frac{wl}{2}\left(\frac{l}{2}\right) - \frac{w}{2}\left(\frac{l}{2}\right)^2 = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$$

Problem-1

Draw the S.F.D. & B.M.D. for the given simply supported beam.



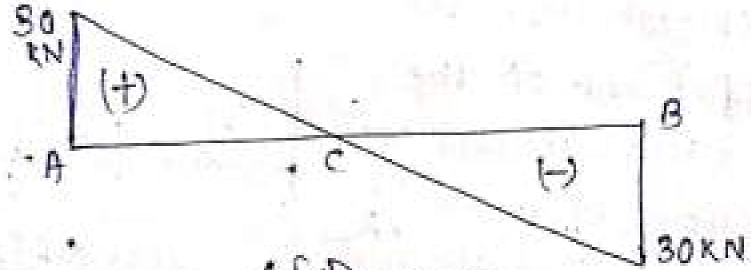
Solⁿ:— Shear force.

Both the reactions will be equal, since beam is symmetric

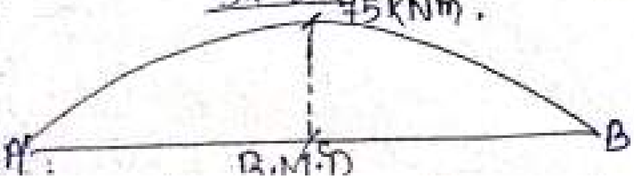
i.e. $R_A = R_B = \frac{wl}{2} = \frac{6 \times 10}{2} = 30\ KN$

Bending moment.

$$M_C = \frac{wl^2}{8} = \frac{6 \times 10^2}{8} = \frac{600}{8} = 75\ KNm.$$



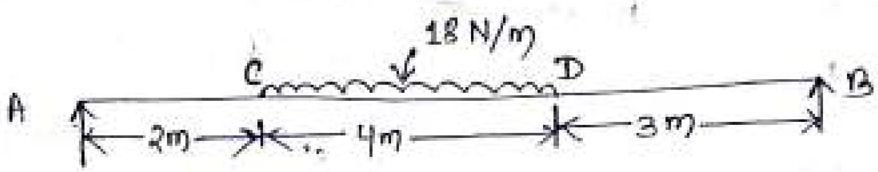
S.F.D.



B.M.D.

A simply supported beam AB of span 9 m carrying a uniformly distributed load of 18 N/m on the part CD of the span, so that AC = 2 m, CD = 4 m & BD = 3 m. Draw the shear-force and bending moment diagram for the beam indicating the values.

Soln:-



Shear force

Reaction Calculation:

R_A & R_B , $R_A + R_B = 18 \times 4$

$\Rightarrow R_A + R_B = 72\,N$

Taking moments about A and equating

$$R_B \times 9 = (18 \times 4) \times (2+2)$$

$$\Rightarrow R_B \times 9 = 72 \times 4$$

$$\Rightarrow R_B = 32\,N$$

$$\therefore R_A = (18 \times 4) - 32 = 72 - 32 = 40\,N$$

For S.F.D.

F_{A-C} = Shear force at A-C = +40 N

F_D = Shear force at D = $+40 - (18 \times 4) = +40 - 72 = -32\,N$

F_{D-B} = Sheare force at D-B = $-32\,N$

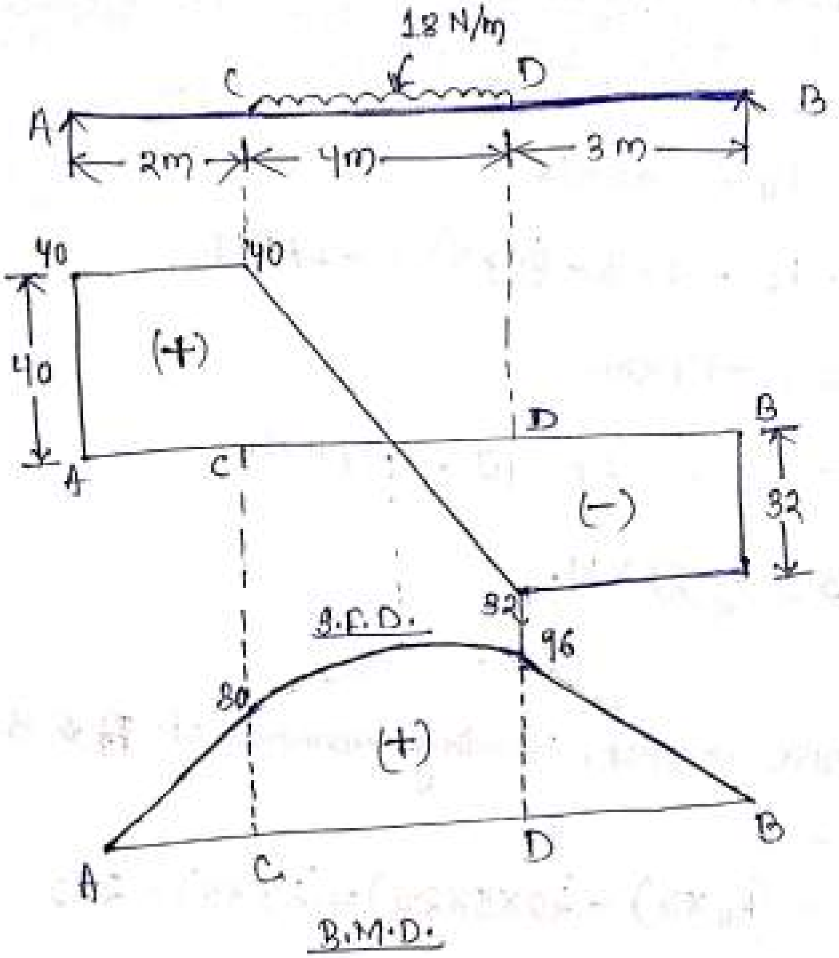
Bending Moment

Due to simple support, Bending moment at A & B = 0

$$\therefore M_A = M_B = 0.$$

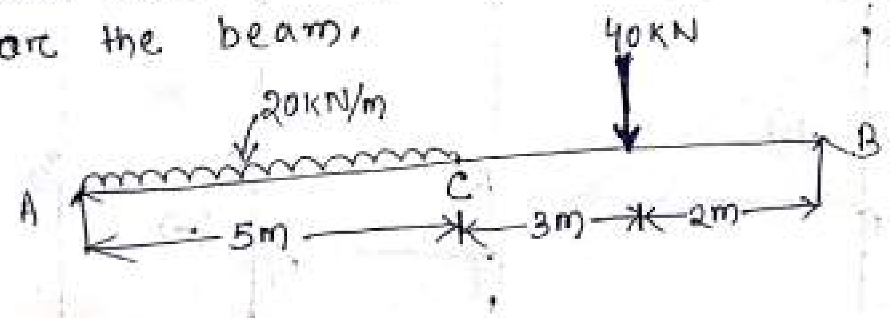
$$M_C = R_A \times 2 = 40 \times 2 = 80\,N\cdot m$$

$$M_D = R_B \times 2 = 32 \times 3 = 96\,N\cdot m.$$



Problem-3

A simply supported beam AB of span 10 metres carrying an uniformly distributed load of 20 KN/m for a distance of 5m from the left end A and a concentrated load of 40 KN at a distance of 2m from the right end B. Draw S.F. and B.M. diagrams for the beam.



Sol^n:-

Shear force:

Reaction calculation

$$R_A + R_B = (20\times5) + 40 = 140\,KN$$

Taking moment about 'A'

$$R_B \times 10 = (40\times8) + (20\times5\times2.5)$$

$$\Rightarrow R_B = (320+250)/10 \Rightarrow R_B = 57\,KN$$

$\therefore R_A = 140 - 57 = 83\,KN.$

For S.F.D.

S.F. at 'A' $= F_A = +83\,KN$

S.F. at 'C' $= F_c = +83 - (20 \times 5) = -17\,KN.$

S.F. at C-D $= -17\,KN.$

S.F. at D $= F_D' = -17 - 40 = -57\,KN.$

S.F. at D-B $= -57\,KN.$

Bending Moment.

Due to simple support, Bending moment at A & B = 0

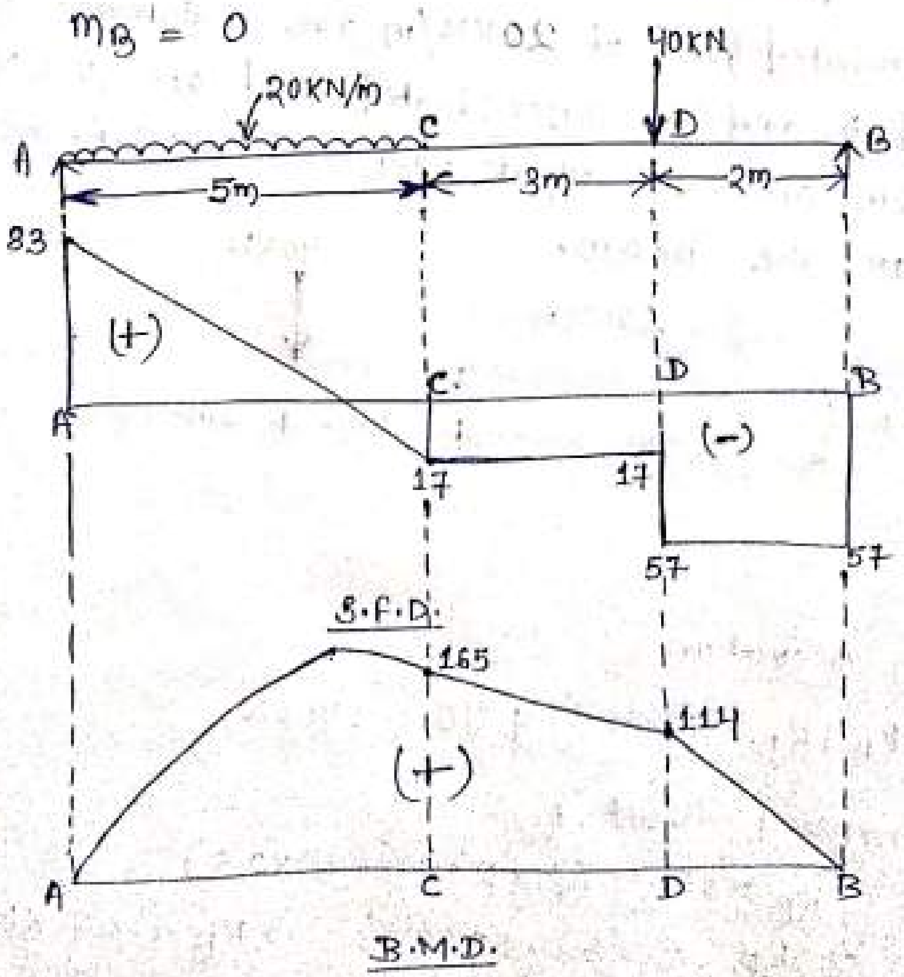
$M_A = 0$

$M_c = (R_A \times 5) - (20 \times 5 \times 2.5) = (83 \times 5) - 250$

$= 415 - 250 = 165\,KN.m.$

$M_D = R_B \times 2 = 57 \times 2 = 114\,KN.m.$

$M_B = 0$

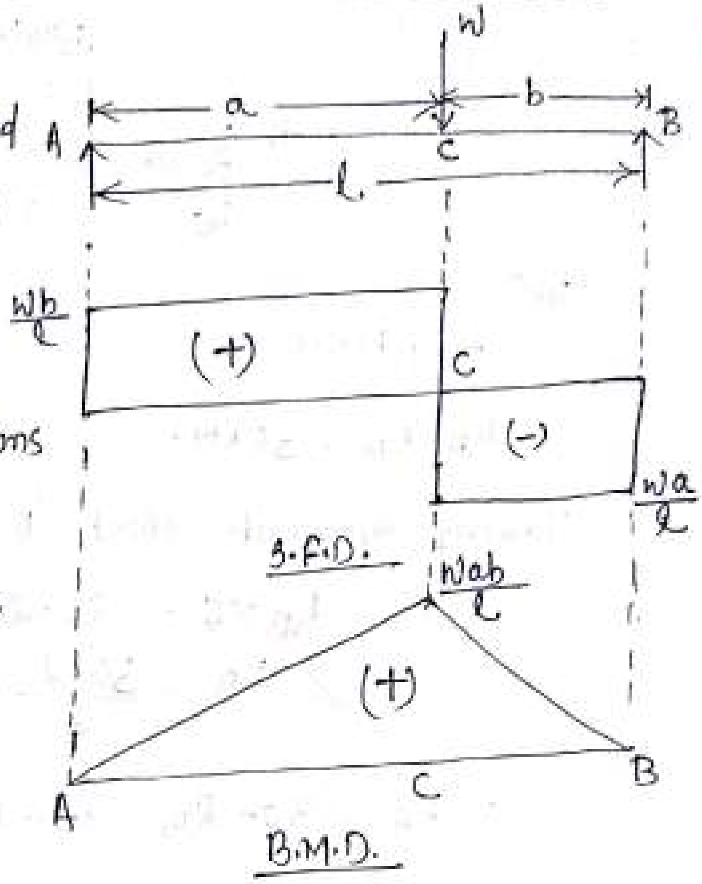


S.F.D.

B.M.D.

3. Simply supported beam carrying a concentrated load placed eccentrically on the span.

Here a simply supported beam AB, is carrying a concentrated load W at 'C' eccentrically on the span.



Let AC = a & CB = b

Shear Force

Let R_A & R_B be the vertical reactions at A & B.

For the equilibrium of the beam,

Taking moment of the forces on the beam about A,

$R_B \times l = W \times a$

$\Rightarrow R_B = \frac{Wa}{l}$

$R_A + R_B = W$

$\Rightarrow R_A = W - \frac{Wa}{l} = \frac{Wl - Wa}{l}$

$\Rightarrow R_A = \frac{W(l-a)}{l} = \frac{Wb}{l}$ ($\because a+b=l$)

Shear Force Diagram

For any section between A & C, the shear force $= R_A = +\frac{Wb}{l}$

& for any section between B & C, the shear force $= R_B = -\frac{Wa}{l}$

Bending Moment:

Bending moment at 'C', $M_C = R_A$ or $R_B \times (a$ or $b) = \frac{Wab}{l}$

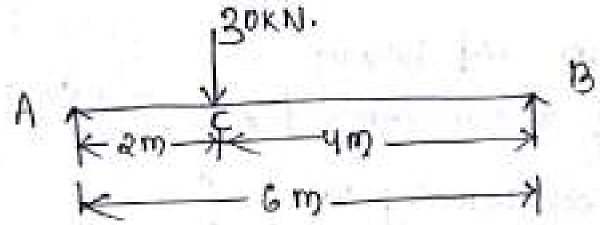
Bending Moment Diagram

Due to sagging nature, the B.M. at 'C' will be $+\frac{Wab}{l}$

and due to simple support $M_A = M_B = 0$.

Problem-1

Draw the B.M.D & S.F.D for the given beam.



Solⁿ:–

Shear force

$R_A + R_B = 30\,kN.$

Taking moment about 'A'

$R_B \times 6 = 20 \times 2$

$\Rightarrow R_B = \frac{30 \times 2}{6} = 10\,kN.$

$\therefore R_A = 30 - R_B = 30 - 10 = 20\,kN.$

S.F.D.

Shear force at A–C = +20 KN.

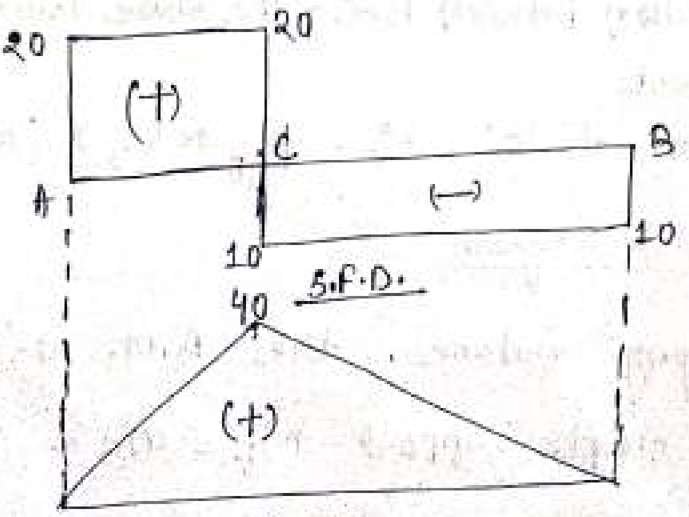
Shear force at C–B = −10 KN.

Bending Moment.

Bending moment at 'C' = $M_C = R_A \times 2 = 20 \times 2 = 40\,KNm.$

B.M.D.:

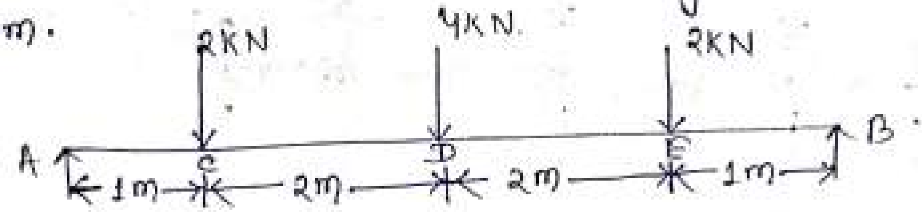
Bending moment at 'C' will be +40KNm due to sagging.



B.M.D.

Problem-2

Draw the shear force and Bending moment diagram for the beam.



Shear force

$R_A + R_B = 2+4+2$

$\Rightarrow R_A + R_B = 8$ KN.

Taking moment about 'A'.

$R_B \times 6 = (2\times5)+(4\times3)+(2\times1)$

$\Rightarrow R_B = 24/6 = 4$ KN.

$\therefore R_A = 8-4 = 4$ KN.

S.F.D.

Shear force at 'A-C' = +4 KN.

Shear force at 'C-D' = +4−2 = +2 KN.

Shear force at 'D-E' = +2−4 = −2 KN.

Shear force at 'E-B' = −2−2 = −4 KN.

Bending Moment.

Bending moment at 'A' = $M_A = 0$

at 'C' = $M_C = R_A \times 1 = 4\times1 = 4$ KNm

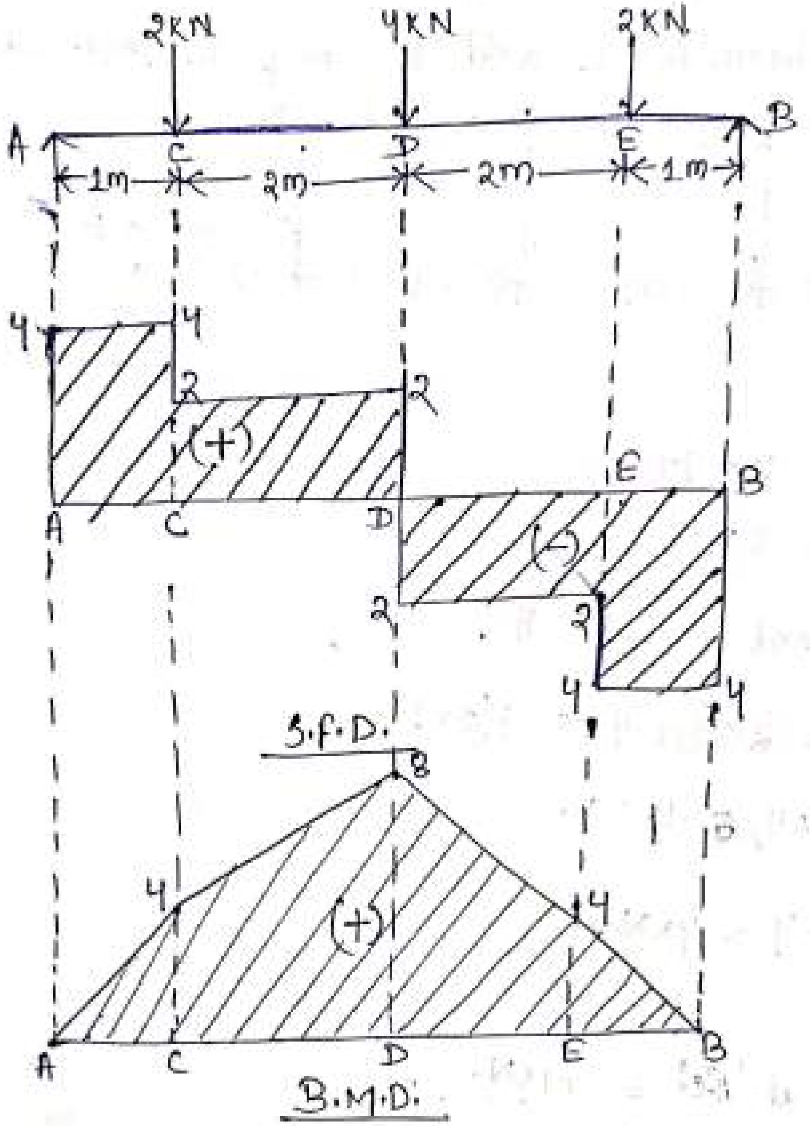
at 'D' = $M_D = (4\times3)-(2\times2) = 8$ KNm

at 'E' = $M_E = R_B \times 1 = 4\times1 = 4$ KNm

at 'B' = $M_B = 0$

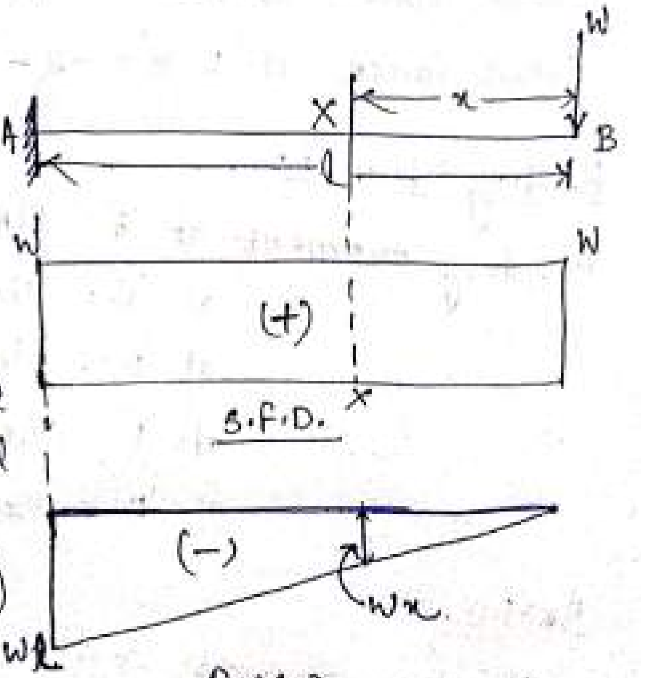
B.M.D.

Bending moment values at all points are positive due to sagging.



Case-4 A Cantilever beam with a point load at it's free end.

Here., a cantilever beam. AB of length 'L' B carrying a point load 'w' at it's free end 'B'.



Shear force.

The shear force at any section 'x', at a distance x from the free end, is equal to the total unbalanced vertical force i.e.

$$F_x = +W \quad (\uparrow\downarrow \text{ is } +ve)$$

and

Bending moment

Bending moment at this section 'x'

$= M_x = -Wx$ (−ve for hogging)

S.F.D. & B.M.D.

From the equation of shear force, the shear force is constant and is equal to +W at all sections between B and A.

→ It is shown by a horizontal line.

From bending moment equation, the bending moment is zero at B (where $x=0$) and increased by a straight line, linearly to $-Wl$ (where $x=l$).

Problem-1

A cantilever beam AB, is carrying a point load of 4KN at it's free end. The length of the beam is 2m. Draw the S.F.D. & B.M.D. for the beam.

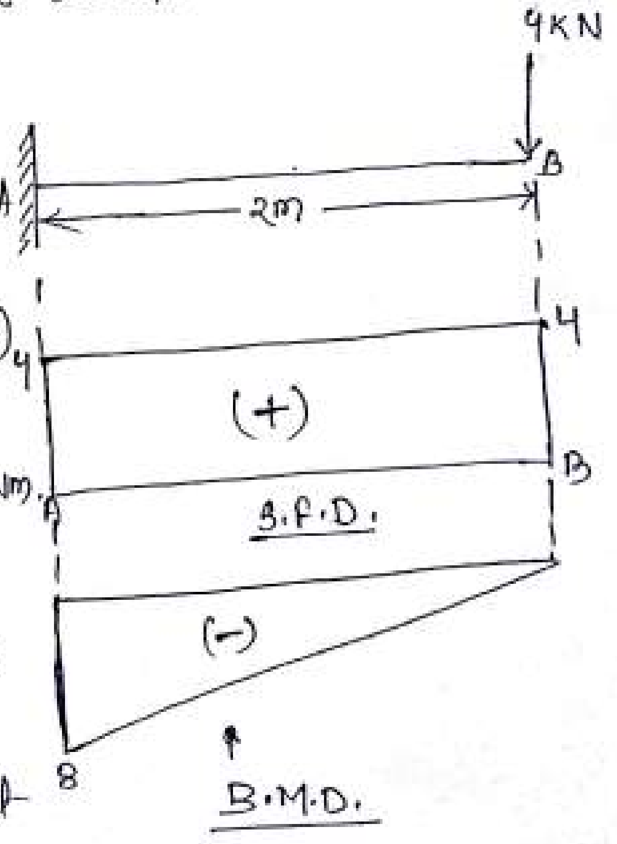
Sol^n:-

Here the S.F. will be (at any section) $= +W = +4KN$.

And the B.M. will be (at any section)

$= -Wx$

At $A = M_A = -W \times 2 = -4 \times 2 = -8KNm$.

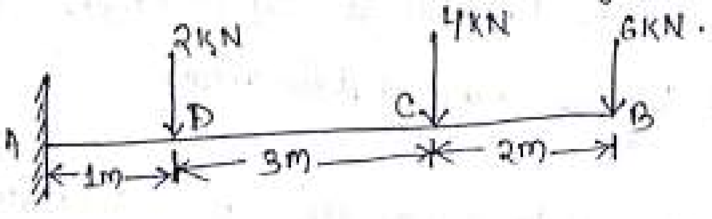


→ The shear force diagram will be horizontal having constant shear force of +4KN throughout the beam.

→ The bending moment diagram will be linear and the bending moment at 'A' it will be $-8KNm$.

Problem-2

Draw S.F.D. & B.M.D. for the given beam.



Soln:- Shear Force.

S.f. at B = 6 KN.

S.f. at C = 4KN + 6KN = 10KN.

S.f. at D = 2KN + 10KN = 12 KN.

S.f. at A = 12 KN.

For shear force diagram all the shear forces are +ve. ($\because$ |↓)

Bending Moment.

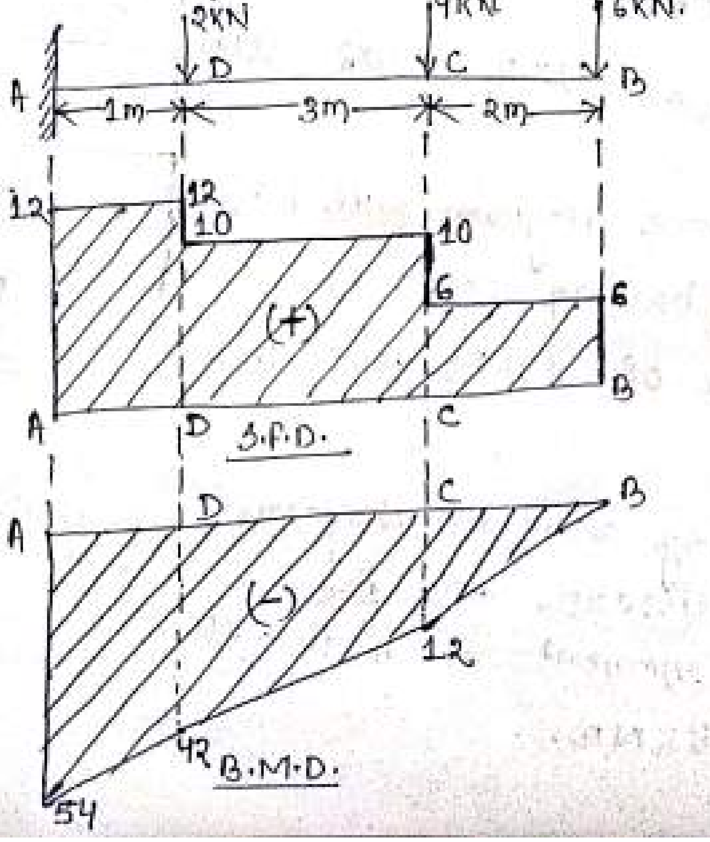
M_B = B.M. at 'B' = 0

M_C = B.M. at 'C' = 6 × 2 = 12 KNm.

M_D = B.M. at 'D' = (6×5) + (4×3) = 30+12 = 42 KNm

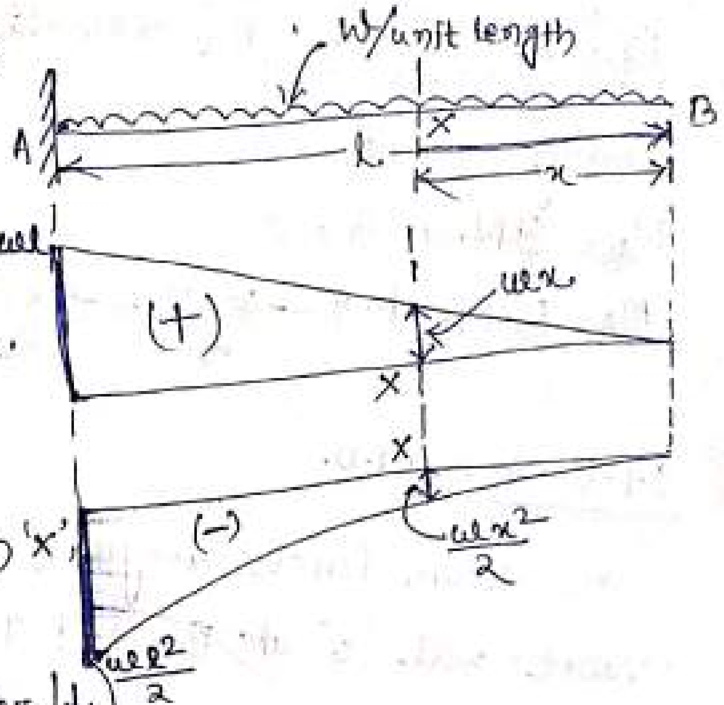
M_A = B.M. at 'A' = (6×6) + (4×4) + (2×1) = 54 KNm

For Bending Moment diagram all the bending moments are -ve. ($\because$ Hogging)



Case-5

A cantilever with a uniformly Distributed load.



Here, a cantilever beam AB of length l is carrying a uniformly distributed load of w per unit length over the entire length of the cantilever.

Shear Force.

The shear force at any section 'x', at a distance x from B,

$F_x = +wx$ (+ve sign for |↓)

So, the shear force at B = 0 ($\because x=0$)

& increases by a straight line to $+wl$ at A.

Bending Moment.

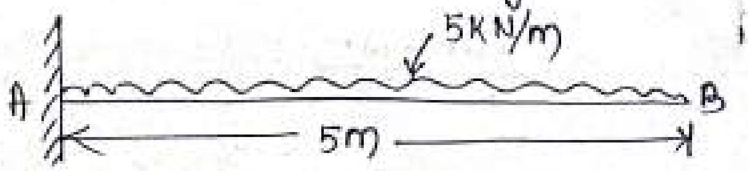
The bending moment at 'x',

$M_x = -wx \times \frac{x}{2} = -\frac{wx^2}{2}$ (−ve sign for hogging)

So, the bending moment at B = 0 (where x = 0) and increases in the form of a parabolic curve to $-\frac{wl^2}{2}$ at B (where x = l)

Problem-1

Draw S.F.D. & B.M.D. for the given beam.



Shear force.

f_B = S.F. at B = 0

f_A = S.F. at A = wl = $5 \times 5 = +25$ KN

Bending Moment:

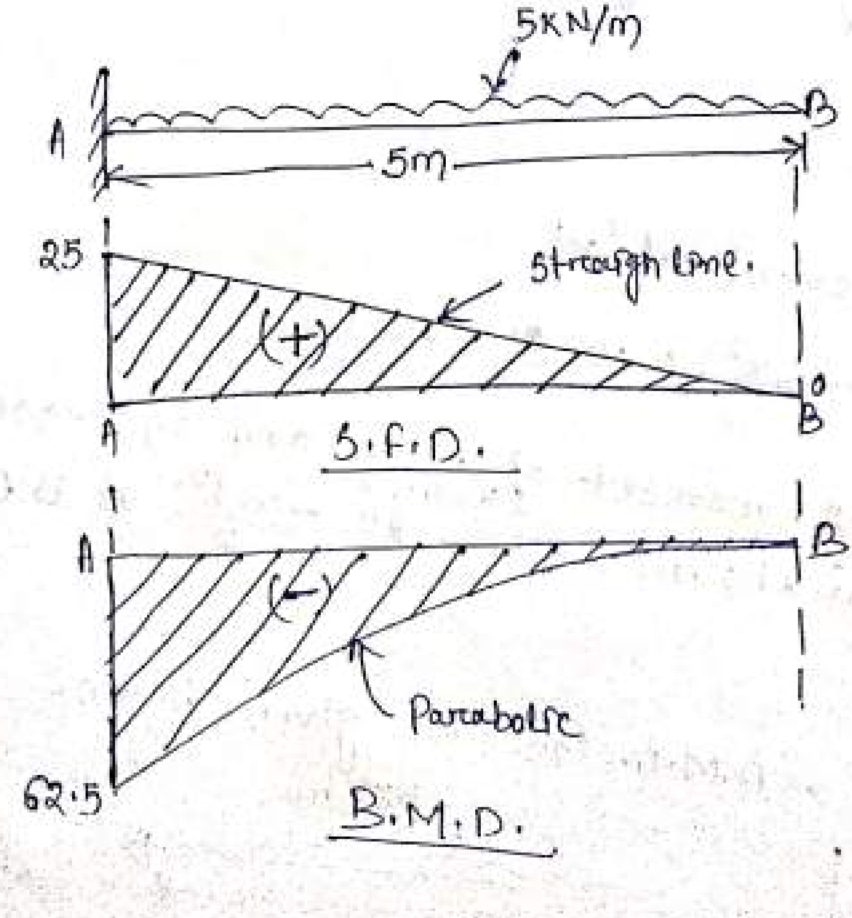
M_B = B.M. at B = 0

M_A = B.M. at A $= -\frac{wl^2}{2} = \frac{-5 \times 5^2}{2} = -62.5$ KNm.

S.F.D. & B.M.D.

The shear force diagram will be linear and it's value will '0' at B and +25KN at 'A'. ($\because$ ↑↓ is +ve)

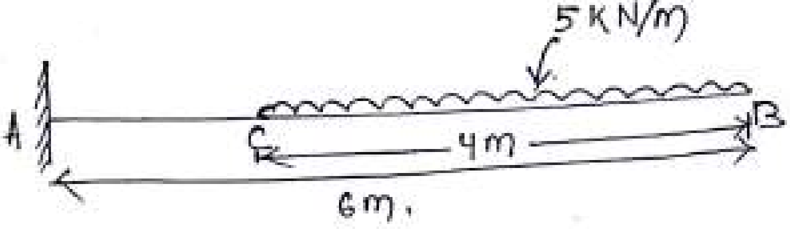
The Bending moment diagram will be parabolic and it's value will be '0' at B and -62.5KNm at 'A'. ($\because$ -ve for hogging.



Problem-2

A cantilever beam AB, 6 m long carries a uniformly distributed load of 5 KN/m over a length of 4 m from the free end. Draw the shear force and bending moment diagrams for the beam.

Solⁿ:-



Shear force

F_B = S.F. at B = 0

$F_C = w \times l_c = 5 \times 4 = +20$ KN

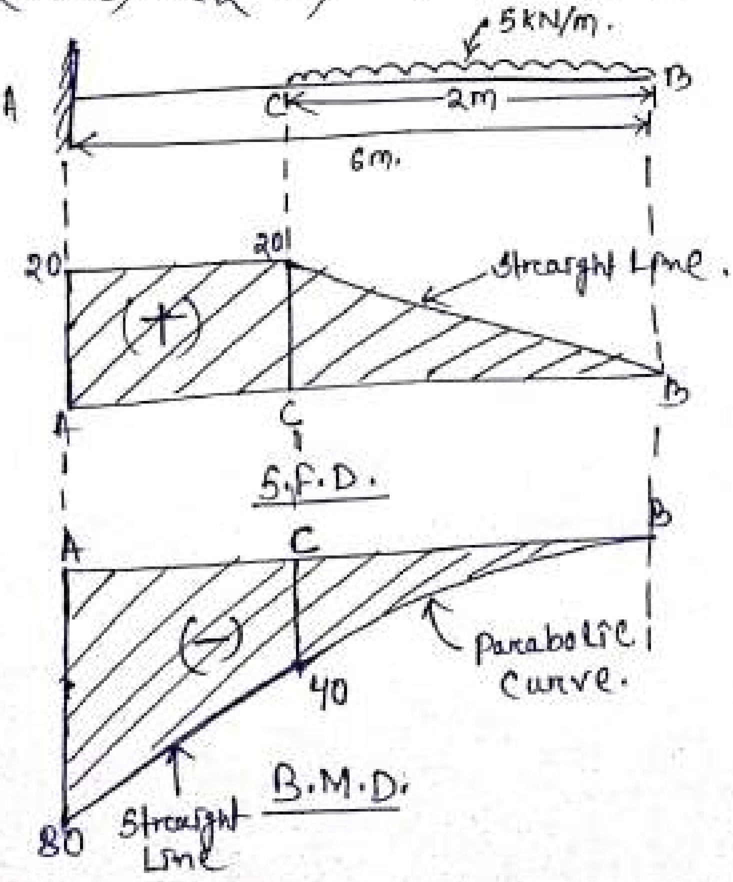
$F_A = +20$ KN.

Bending Moment.

M_B = B.M. at B = 0

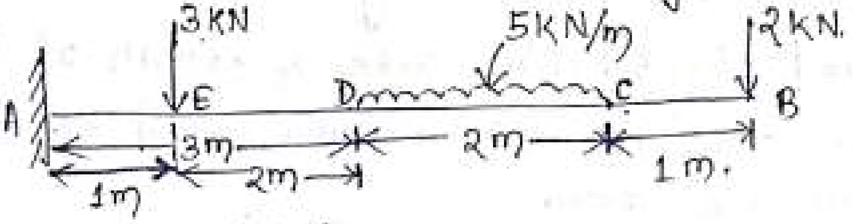
$M_C = -\frac{w \times l_c^2}{2} = -\frac{5 \times 4^2}{2} = -40$ KNm

$M_A = -(w \times l_c) \times \left(\frac{l_c}{2} + 2\right) = -(5 \times 4) \times \left(\frac{4}{2} + 2\right) = -80$ KNm.



Problem-8

Draw the S.F.D. & B.M.D. for the given beam.



Shear force

$R_A = 3 + (5\times2) + 2 = 15 KN.$

S.f. at B-C = 2KN.

S.f. at D = $2 + (5\times2) = 12$ KN

S.f. at D-E = 12 KN.

S.f. at E-A = $12 + 3 = 15$ KN.

S.F.D.

Shear force diagram will be +ve. (∵ ↑↓ is +ve)

Bending Moment

$M_B = 0$

$M_C = 2\times1 = 2$ KNm.

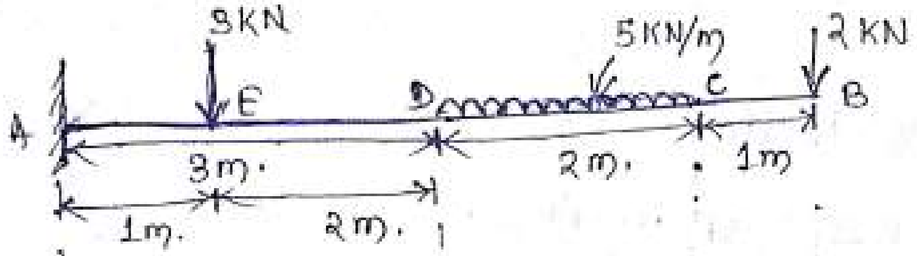
$M_D = (2\times3) + (5\times2\times1) = 16$ KNm.

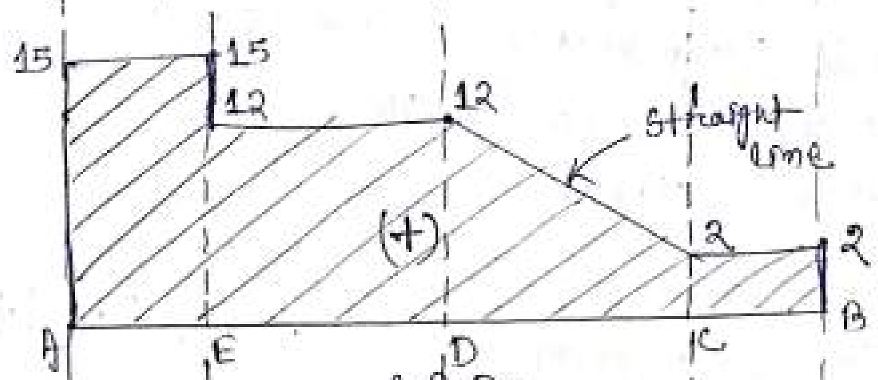
$M_E = (2\times5) + (5\times2)(1+2) = 10 + 30 = 40$ KNm.

$M_A = (2\times6) + (5\times2)(1+3) + (3\times1) = 12 + 40 + 3 = 55$ KNm.

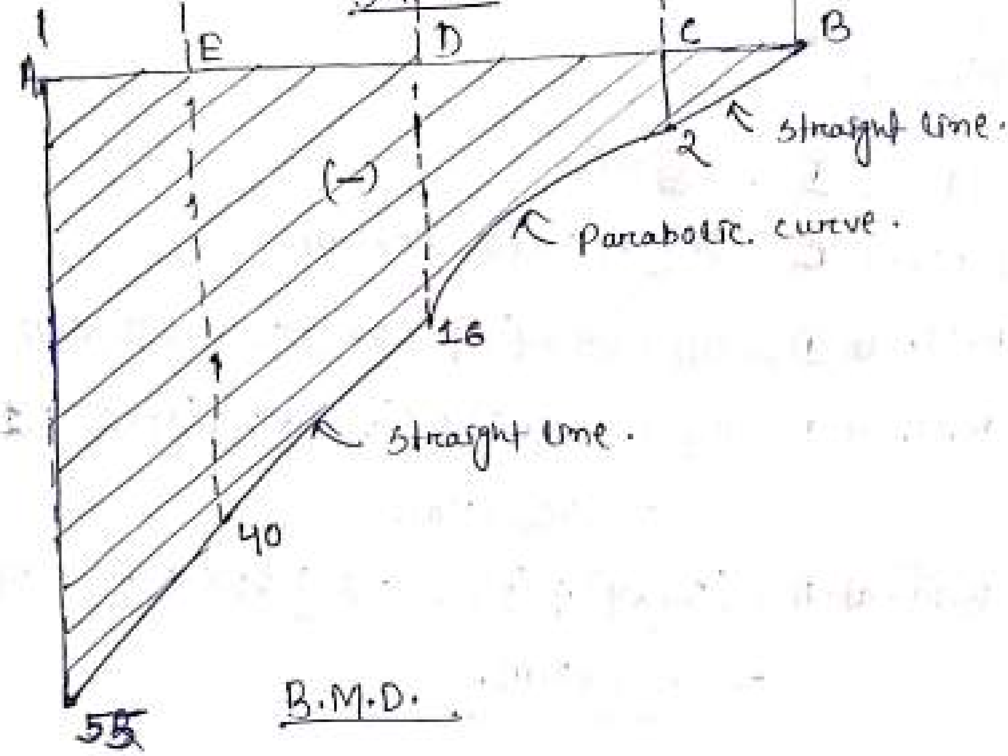
B.M.D.

Bending moment diagram will be -ve (∵ -ve is for hogging)





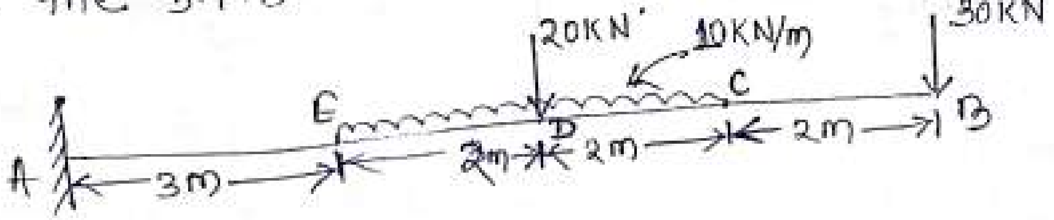
S.F.D.



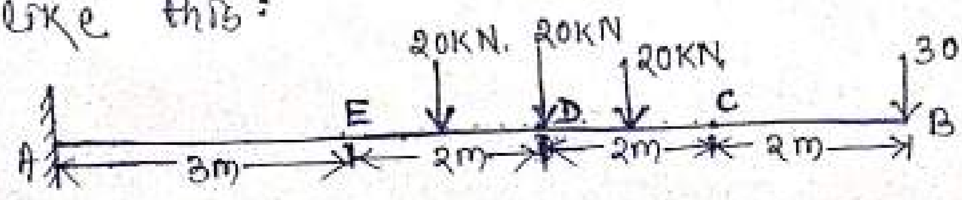
B.M.D.

Problem-4

Draw the S.F.D. & B.M.D. for the given beam.



Solⁿ:— By simplifying the diagram, we can draw it like this:



Shear force.

S.F. at B – C = +30 KN

S.F. at D (Just right) = 30 + 20 = 50 KN.

S.F. at D (Just left) = 50 + 20 = 70 KN

S.F. at E = 70 + 20 = 90 KN.

S.F. at A = 90 KN.

B.F.D.

The shear force diagram will be +ve (∵ |↓ is +ve)

Bending Moment.

B.M. at B = $M_B = 0$

B.M. at C = $M_C = 30 \times 2 = 60$ KNm.

B.M. at D = $M_D = (30 \times 4) + (10 \times 2) \times 1 = 140$ KNm.

B.M. at E = $M_E = (30 \times 6) + (10 \times 2) \times 3 + (20 \times 2) + (10 \times 2) \times 1$

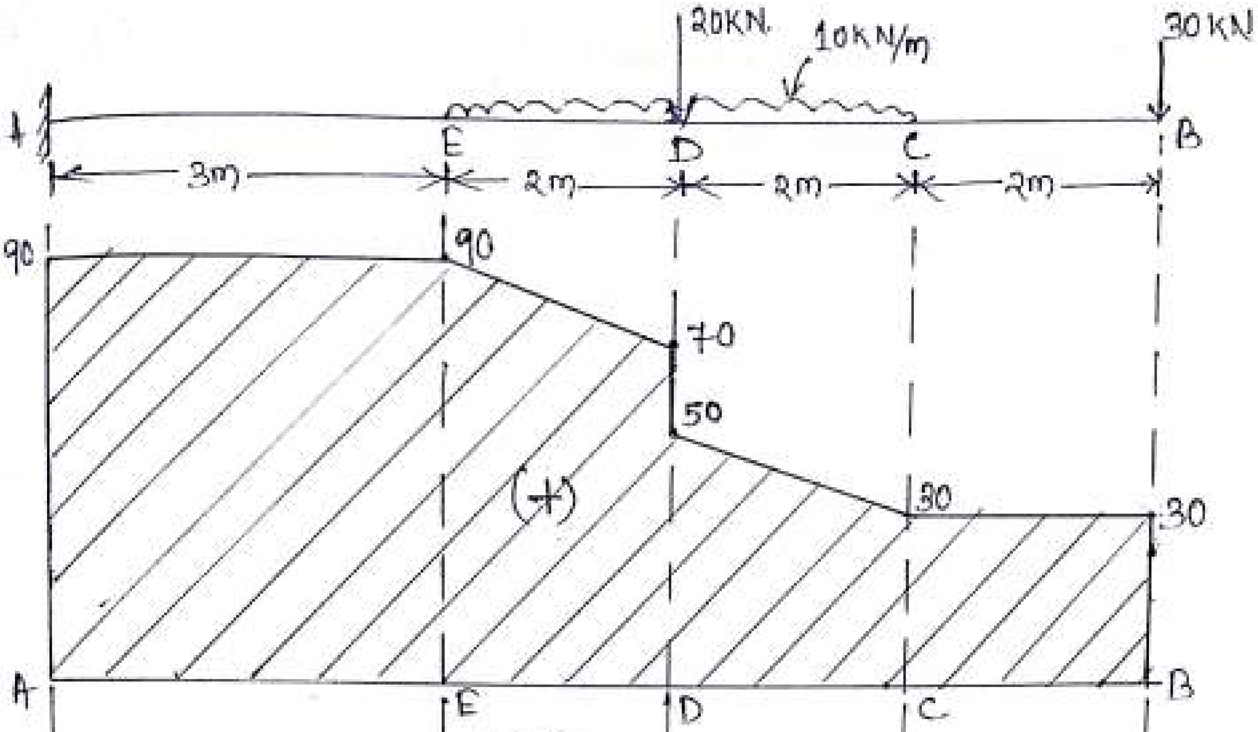
$= 300$ KNm.

B.M. at A = $(30 \times 9) + (10 \times 2) \times 6 + (20 \times 5) + (10 \times 2) \times 4$

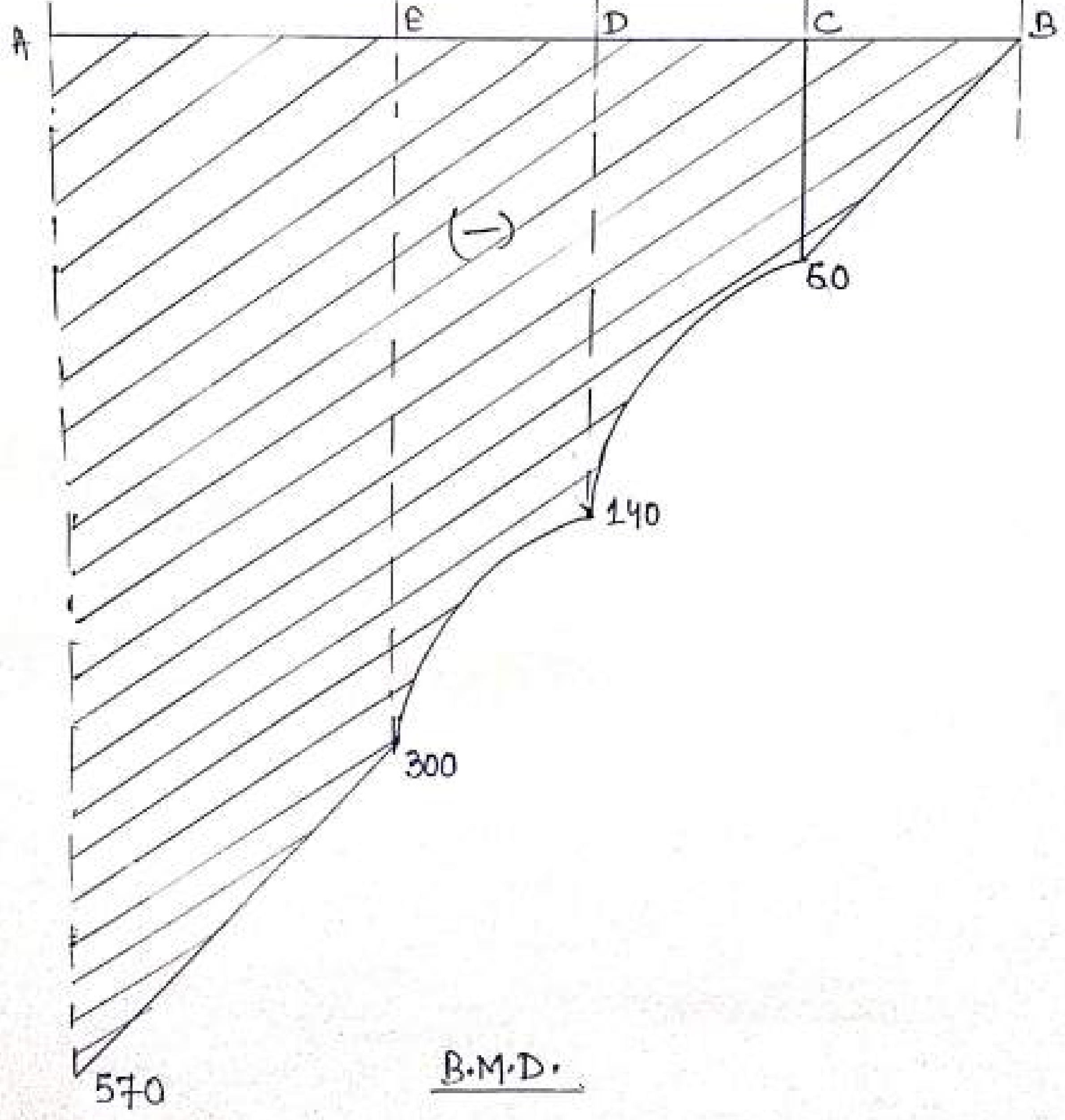
$= 570$ KNm.

B.M.D.

The Bending moment diagram will be –ve (∵ –ve for Hogging)


20KN
10KN/m
30KN
A
E
D
C
B
3m
2m
2m
2m
90
90
70
50
(+)
30
30
A
E
D
C
B

S.F.D.

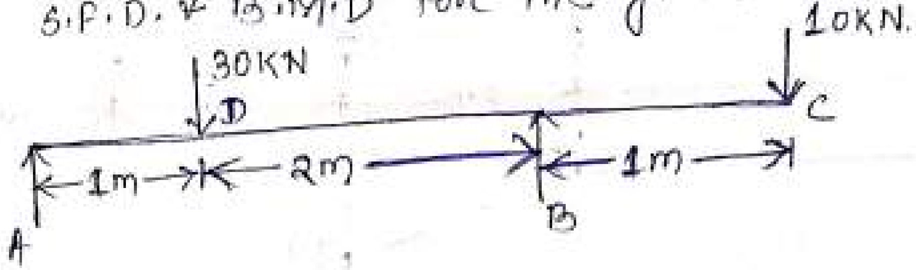

A
E
D
C
B
(−)
60
140
300
570

B.M.D.

Overhanging Beam

Problem-1

Draw the S.F.D. & B.M.D for the given beam.



Solⁿ:—

Shear force

↑ = ↓

$R_A + R_B = 30 + 10 = 40$ KN.

Taking moment about 'A'

$R_B \times 3 = 30 \times 1 + 10 \times 4$

$\Rightarrow R_B = 70/3 = 23.33$ KN.

$\therefore R_A = 40 - 23.33 = 16.67$ KN.

S.F. at A-D = +16.67 KN.

S.F. at D-B = +16.67 − 30 = −13.33 KN.

S.F. at B-C = −13.33 + 23.33 = +10 KN.

S.F. at C = +10 KN.

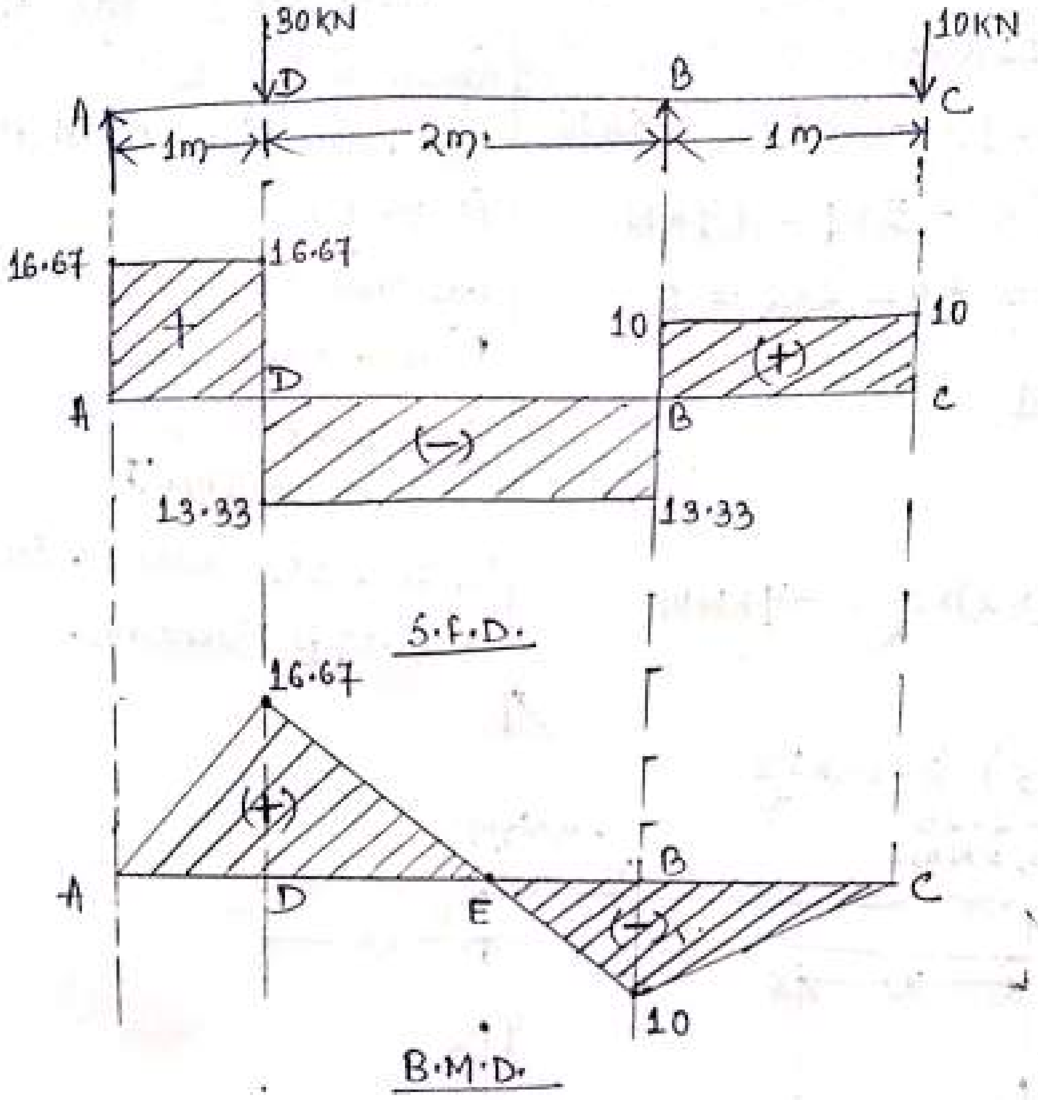
B.M.

M_A = B.M. at A = 0

M_D = B.M. at D = $16.67 \times 1 = 16.67$ KN.m (+ve for sagging)

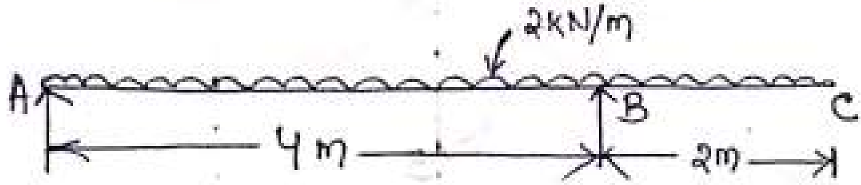
M_B = B.M. at B = $-10 \times 1 = -10$ KNm (−ve for Hogging)

Scanned with CamScanner



Problem-2

Draw the S.F.D. and B.M.D. for the given beam.



Soln:-

Shear force.

Total Load = 2×6 = 12 KN

$\therefore R_A + R_B = 12 KN$

Taking moment about 'A'

$R_B \times 4 = 2 \times 6 \times \frac{6}{2}$

$\Rightarrow R_B = 36/4 = 9 KN.$

$\therefore R_A = 12 - 9 = 3 KN.$

S.F. at $A = +3$ KN.

S.F. at $B(L) = +3 - (2\times4) = -5$ KN.

S.F. at $B(R) = -5+9 = +4$ KN.

S.F. at $C = +4 - 2\times2 = 0$

Maximum Bending Moment.

From S.F.D., the shear force at D = 0

Let point 'D' is 'x' distance from 'A'.

The shear force equation at 'D' will be $= 3-2x$

But $3-2x = 0$

$\Rightarrow x = 1.5$ m.

So, at 1.5m from 'A', S.F. = 0, & B.M is Maximum.

Bending Moment.

$M_A = 0$

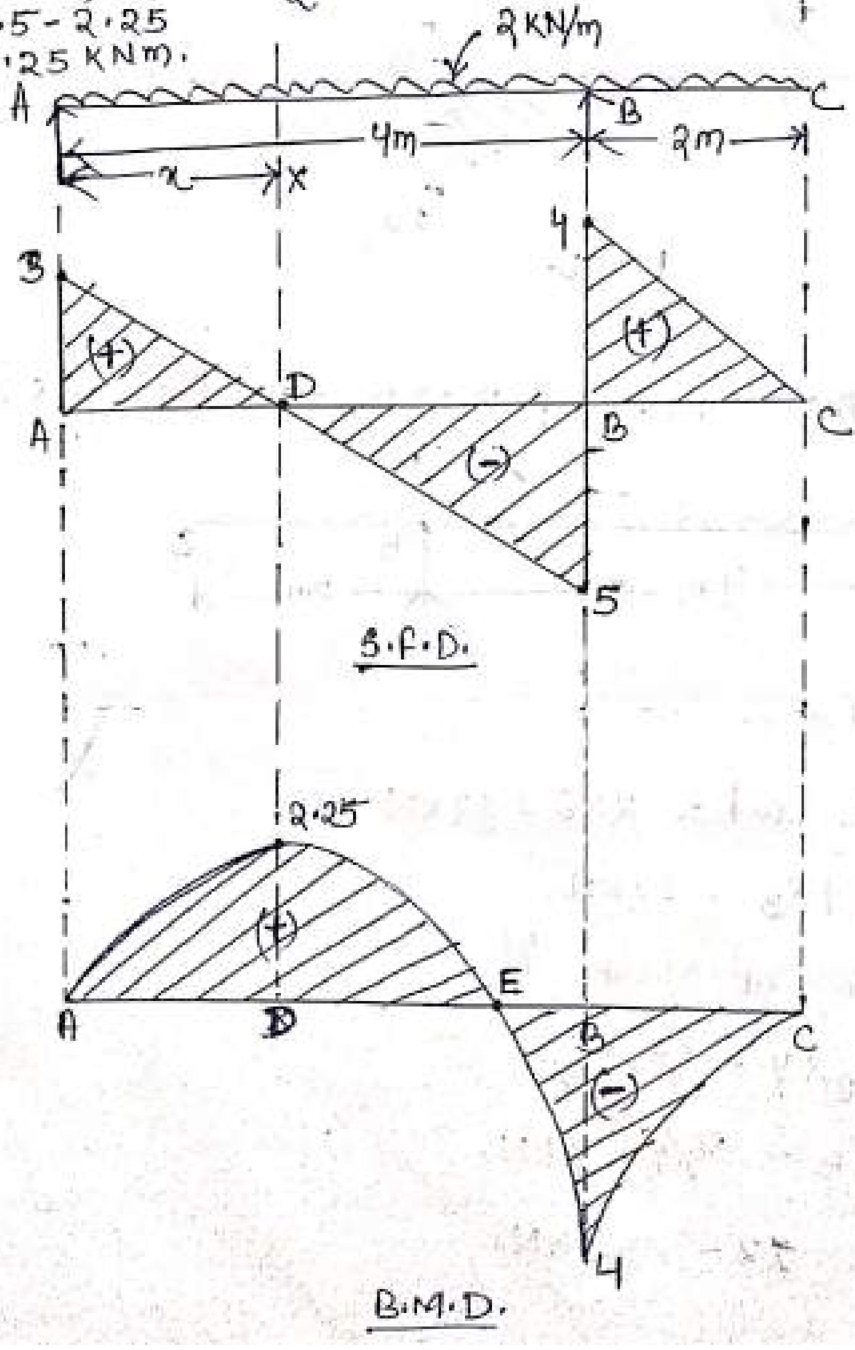
$M_B = -(2\times2)\times\frac{2}{2} = -4$ KNm.

$M_C = 0$

$M_D = (3\times1.5) - (2\times1.5)\times\frac{1.5}{2}$

$= 4.5 - 2.25$

$= 2.25$ KNm.



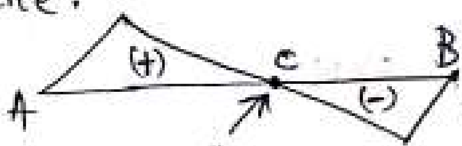
Point of Contraflexure

→ Contra-flexure = Contra (opposite) + flexure. (bending)

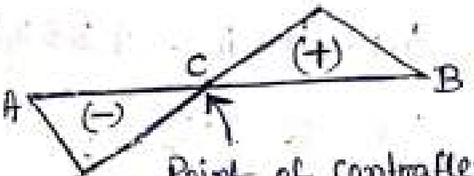
→ Point of contraflexure is the point, where bending moment changes its sign i.e. from positive value to a negative value or vice versa.

→ At point of contraflexure, the value of bending moment is zero.

→ To find where point of contraflexure exists, we need to draw bending moment diagram. The point where diagram meets zero line i.e., sign changes, it is the point of contraflexure.



Point of contraflexure. B.M.D.

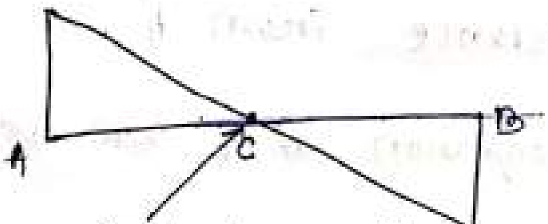


Point of contraflexure

Point of zero Shear

→ Point of zero shear is that point where shear force is zero.

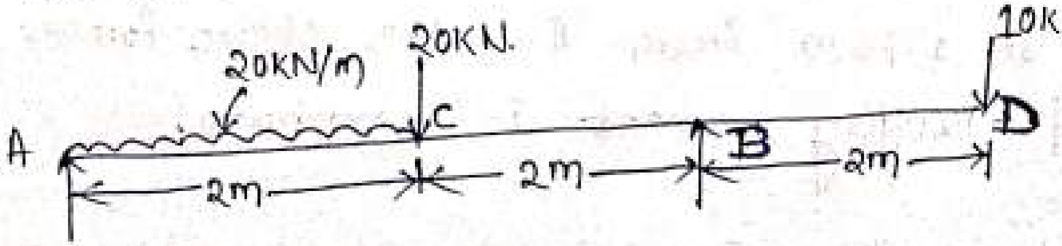
→ At this point, the bending moment is maximum.



Point of zero shear.

Problem-3

Draw the S.F.D. & B.M.D. for the given beam.



Shear force

$$R_A + R_B = (20\times2) + 20 + 10$$
$$= 70\,KN.$$

Taking moment about 'A'

$$R_B \times 4 = (20\times2\times1) + (20\times2) + (10\times6)$$

$$\Rightarrow R_B = 140/4 = 35\,KN.$$

$$\therefore R_A = 70 - 35 = 35\,KN.$$

S.F. at A = +35 KN.

S.F. at C(L) = +35 − (20×2) = −5 KN.

S.F. at C(R) = −5 − 20 = −25 KN

S.F. at B(L) = −25 KN

S.F. at B(R) = −25 + 35 = +10 KN.

S.F. at D = +10 KN.

Maximum Bending Moment:

From S.F.D., the shear force is '0' at 'E'. Let Point 'E' is at 'x' distance from 'A'.

The shear force equation at 'E' will be = $35 - 20\times x$

$$\therefore 35 - 20x = 0$$

$$\Rightarrow 20x = 35 \Rightarrow x = 35/20 = 1.75\,m.$$

So, at 1.75 m from 'A'; the shear force is zero and bending moment is maximum.

Bending Moment.

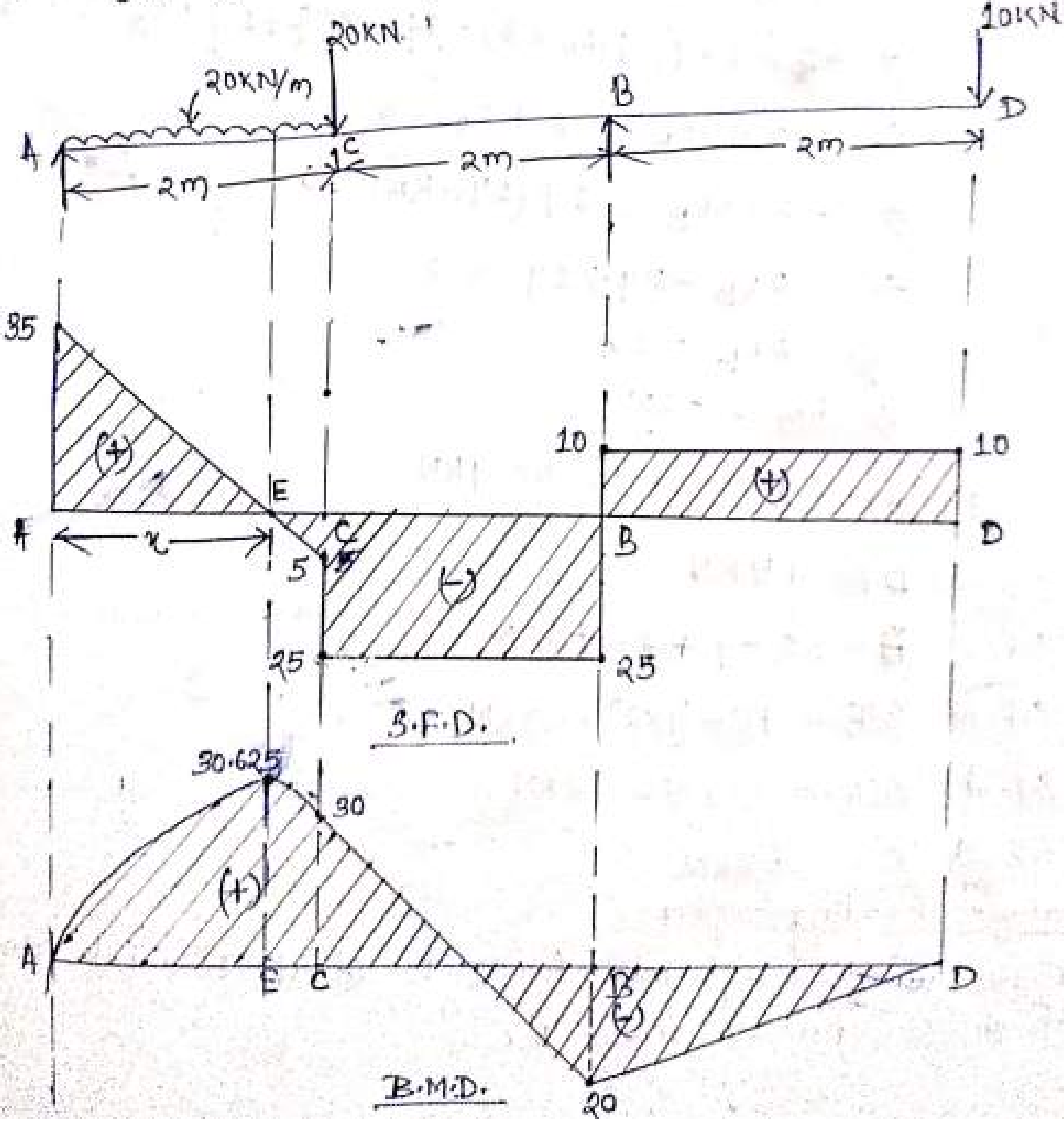
$$M_A = 0$$

$$M_C = R_A \times 2 - 20 \times 2 \times 1$$
$$= 35 \times 2 - 40$$
$$= 30 \text{ KNm}.$$

$$M_B = -10 \times 2 = -20 \text{ KNm}$$

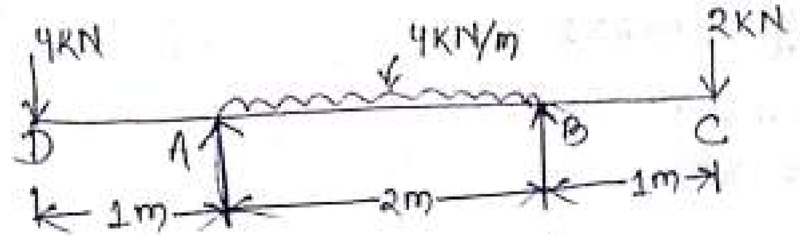
$$M_E = R_A \times 1.75 - 20 \times 1.75 \times \frac{1.75}{2}$$
$$= (35 \times 1.75) - 30.625$$
$$= 30.625 \text{ KNm}.$$

$$M_D = 0$$



Problem-4

Draw the S.F.D. and B.M.D. for the given beam.



Soln:— Shear force.

$R_A + R_B = 4 + (4\times2) + 2 = 14$ KN.

$\Rightarrow R_A = 14 - R_B$

Taking moment about 'D'

$\Sigma M_D = 0$

$\Rightarrow (-2\times4) + (R_B\times3) - \{4\times2\times(\frac{2}{2}+1)\} + (R_A\times1) = 0$

$\Rightarrow -8 + 3R_B - 16 + R_A = 0$

$\Rightarrow -8 + 3R_B - 16 + (14 - R_B) = 0$

$\Rightarrow 2R_B - 24 + 14 = 0$

$\Rightarrow 2R_B = 10$

$\Rightarrow R_B = 5$ KN.

$\therefore R_A = 14 - R_B = 14 - 5 = 9$ KN

S.F. at D-A = -4 KN

S.F. at A = $-4 + 9 = +5$ KN

S.F. at B(L) = $+5 - (4\times2) = -3$ KN

S.F. at B(R) = $-3 + 5 = +2$ KN

S.F. at C = $+2$ KN.

Maximum Bending Moment.

From S.F.D.; the shear force is '0' at 'E'. Let point 'E' is at x distance from 'A'.

So, the shear force equation at 'E' will be

$$-4+9-(4\times x)=0$$

$$\Rightarrow 5-4x=0$$

$$\Rightarrow x=5/4=1.25\text{ m}.$$

So, at 1.25 m from 'A', the shear force is zero and bending moment is maximum.

Bending moment.

$M_D=0$

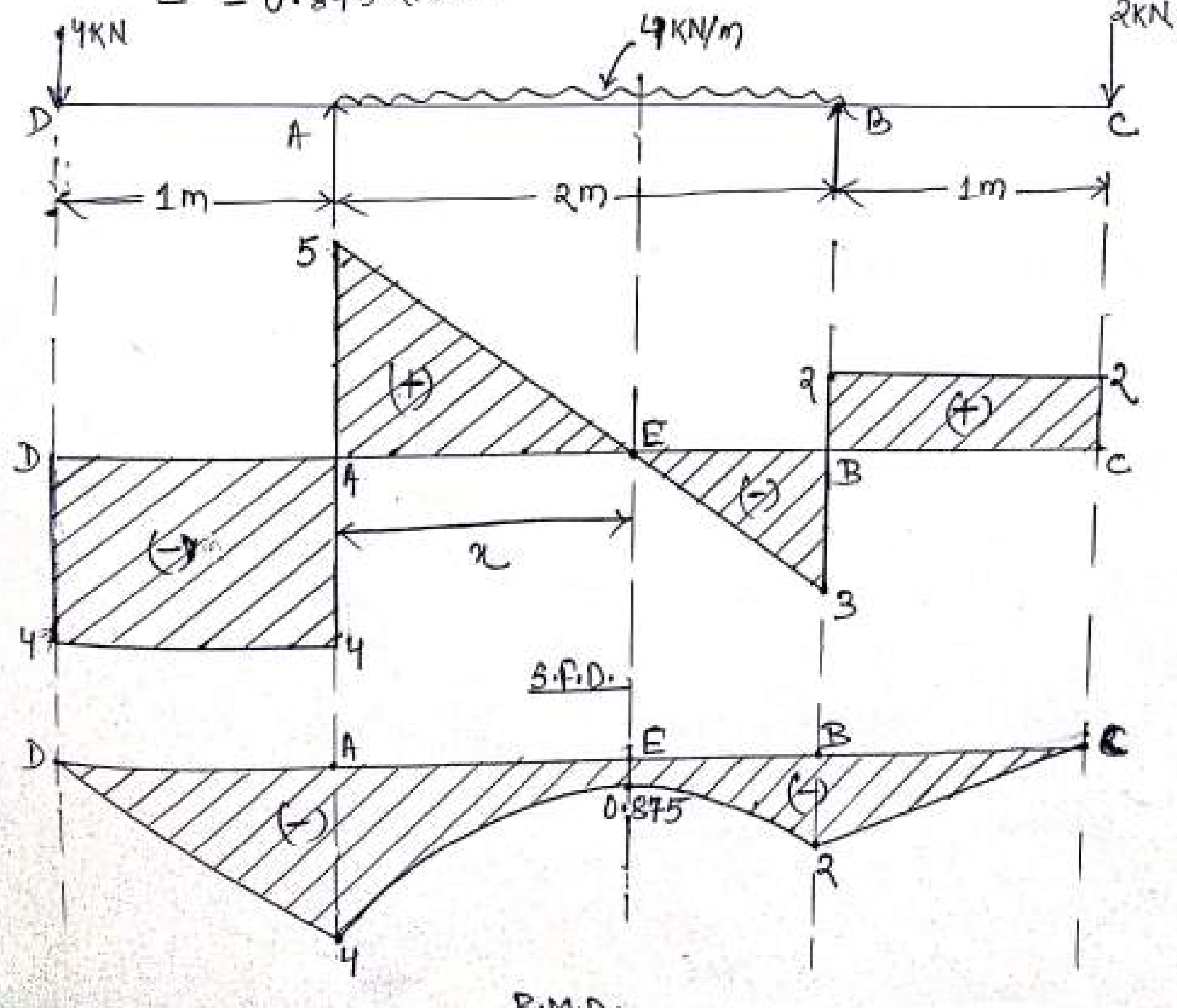
$M_A=-4\times 1=-4\text{ KNm}$

$M_B=-2\times 1=-2\text{ KNm}$

$M_C=0.$

$M_E=-2\times(1+0.75)+5\times 0.75-\left\{(4\times 0.75)\times\frac{0.75}{2}\right\}$

$=-0.875\text{ KNm}.$



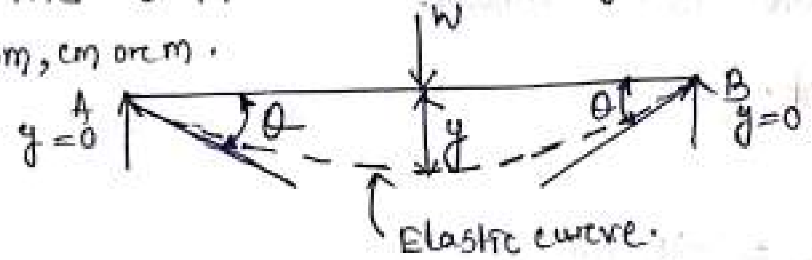
B.M.D.

Chapter-6

Slope and Deflection

Deflection :- It is the vertical distance of the beam measured before & after loading.

→ It is denoted by 'y'.
→ Deflection at the support is always zero.
→ It's unit is mm, cm or m.



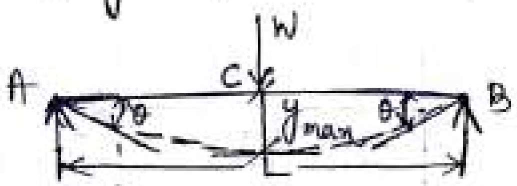
Slope :- It is the angle in radians measured between the tangent to the elastic curve & original axis of the beam.

→ slope is denoted by 'θ' or '$\frac{dy}{dx}$'.

→ It's unit is 'radians'.

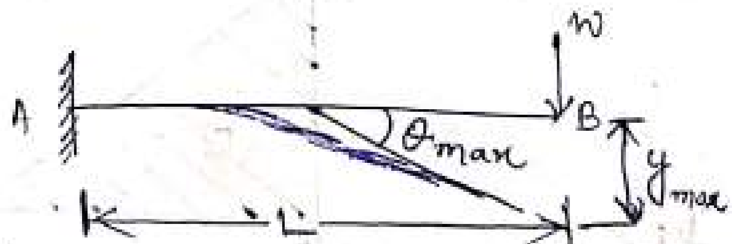
Boundary - Conditions:

(i) Simply supported beam



1. At A & B ; deflection is zero.
2. At C ; deflection is max^m.
3. At C ; slope is zero.
4. At A & B ; slope is max^m.

(ii) Cantilever beam

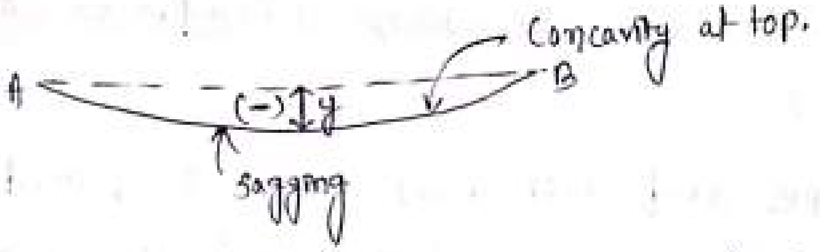


1. At A ; deflection is zero.
2. At A ; slope is zero
3. At B ; deflection is max^m.
4. At B ; slope is max^m.

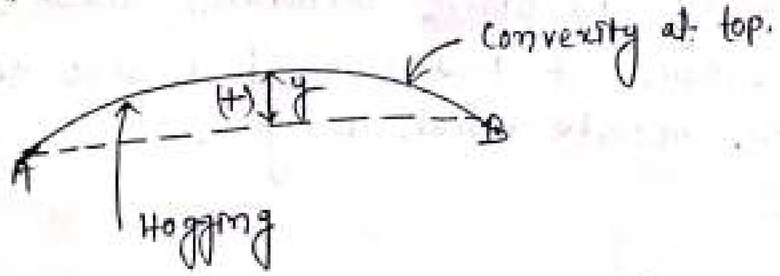
Shape and nature of elastic curve

Under load, the neutral axis becomes a curved line and is called the elastic curve.

(i) If the elastic curve of a beam is like concavity at top, it is taken as sagging and corresponding deflection is −ve.



(ii) If the elastic curve of a beam is like convexity at top, it is taken as hogging and corresponding deflection is +ve.



Relationship between slope, deflection and curvature

$$\boxed{M = EI \frac{d^2y}{dx^2}}$$

where, M = Bending Moment.

E = Young's Modulus of beam material

I = Moment of Inertia of the beam section

dy/dx = slope of the beam, d^2y/dx^2 = Curvature.

y = deflection.

Importance of slope and deflection

→ Calculation of slope and deflection gives us theoretical representation of beam after the application of load.

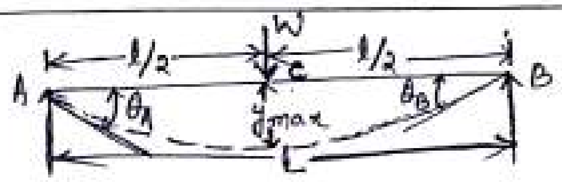
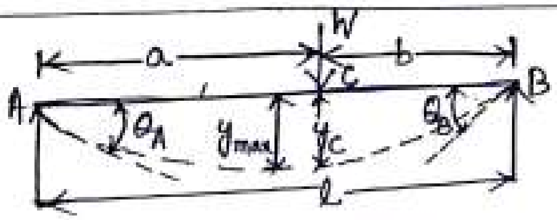
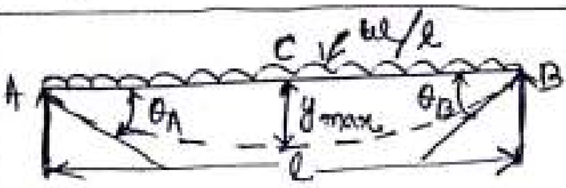
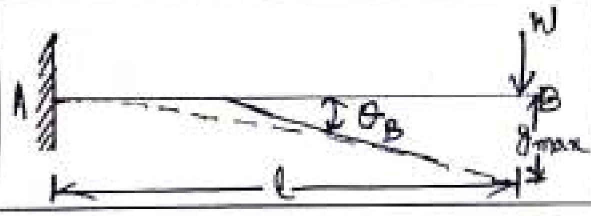
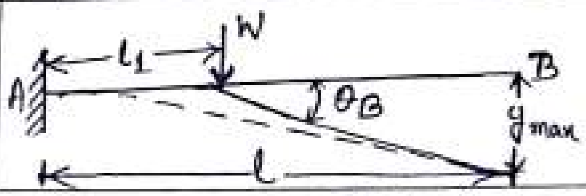
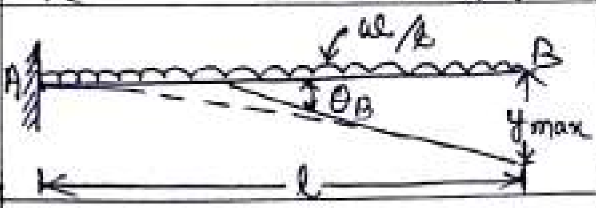
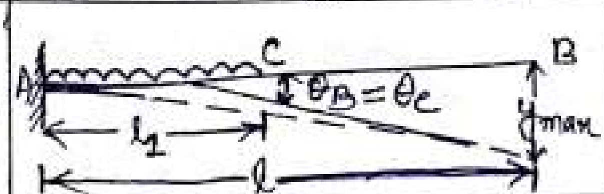
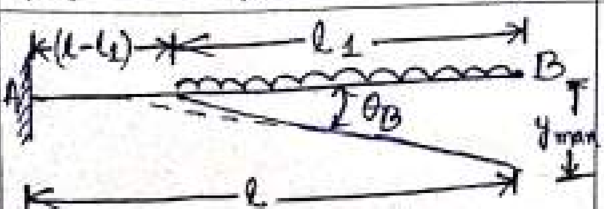
→ Deflection gives us the value of distance to which the beam will deflect after the application of load.

→ Slope gives the data about how the beam is going to deflect after application of load. i.e. shape of bend/deflection.

→ So, slope and deflection are required to know how the beam will bend and to which extent.

→ Except the design purpose, the slope and deflection helps us get to know how the beam will behave after load application and decide its position relatively to other members according to the behaviour of beam. And it also helps in deciding other architectural designs.

Slopes and Deflections for different Loadings

Sl. No.	Type of Loading	Slope	Maximum Deflection
01.		$\theta_A = -\frac{Wl^2}{16EI}$ $\theta_B = +\frac{Wl^2}{16EI}$	$y_{max} = y_C = -\frac{Wl^3}{48EI}$
02.		$\theta_A = -\frac{Wb(l^2-b^2)}{6EI}$ $\theta_B = +\frac{Wa(l^2-a^2)}{6EI}$	$y_{max} = \frac{-Wb(l^2-b^2)^{3/2}}{9\sqrt{3}\,EIl}$ $y_C = \frac{Wab(l^2-a^2-b^2)}{6EIl}$
03.		$\theta_A = -\frac{wl^3}{24EI}$ $\theta_B = +\frac{wl^3}{24EI}$	$y_{max} = y_C = -\frac{5wl^4}{384EI}$
04.		$\theta_B = -\frac{Wl^2}{2EI}$	$y_{max} = y_B = -\frac{Wl^3}{3EI}$
05.		$\theta_B = -\frac{Wl_1^2}{2EI}$	$y_{max} = y_B = -\frac{Wl_1^2}{6EI}(3l-l_1)$
06.		$\theta_B = -\frac{wl^3}{6EI}$	$y_{max} = y_B = -\frac{wl^4}{8EI}$
07.		$\theta_B = \theta_C = \frac{-wl_1^3}{6EI}$	$y_{max} = y_B =$ $\frac{-wl_1^4}{8EI} + \frac{wl_1^3}{6EI}(l-l_1)$
08.		$\theta_B = \frac{wl^3}{6EI} - \frac{w(l-l_1)^3}{6EI}$	$y_{max} = y_B$ $\left[\frac{wl^4}{8EI}\right] - \left[\frac{w(l-l_1)^4}{8EI} + \frac{w(l-l_1)^3 l_1}{6EI}\right]$

Methods for slope and Deflection at a section

There are two important methods to find out the slope and deflection at a section.

1. Double integration method.
2. Macaulay's method.

1. Double Integration Method

In this method, the differential equation for deflection at any c/s. is integrated twice to get the deflection

$$M = EI \frac{d^2y}{dx^2}$$

Integrating the above equation once,

$$EI \frac{dy}{dx} = \int(M \cdot dx) + C_1$$ → from which slope can be calculated.

$$EI \cdot y = \iint(M \cdot dx) + C_1 x + C_2$$ → from which deflection can be calculated.

→ The constants of integration are found by applying the end conditions i.e. x value.

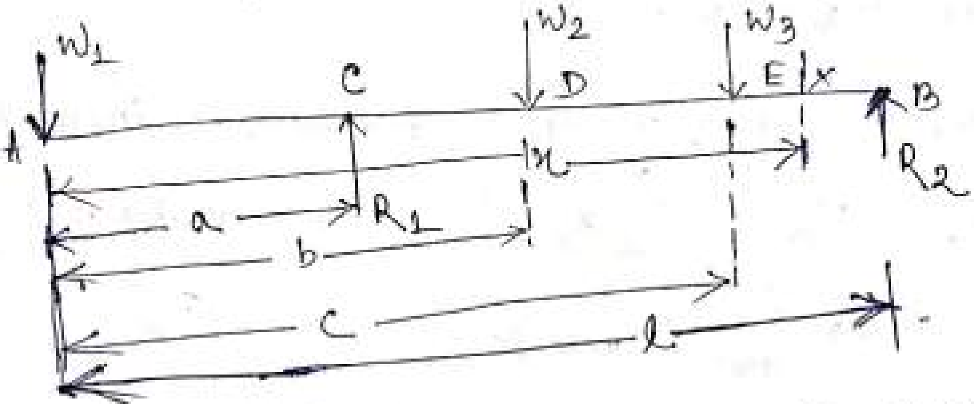
→ This method is suitable for single load case or simple case as a separate expression for the bending moment is needed to be written for each section of the beam, each producing a different equation with it's own constants of integration.

Macaulay's Method

→ This method is similar to double integration method but improved.

→ In this method, a single equation is written for the bending moment for all the portions of the beam.

→ The equation is formed in such a way that the same constants of integration are applicable to all portions.



$$M = EI\frac{d^2y}{dx^2} = -W_1x \,|\, + R_1(x-a) \,|\, - W_2(x-b) \,|\, - W_3(x-c)$$

In the above expression, there are separation lines.

1→ The portion to the left of the first separation line is valid for the portion AC.

2→ The portion to the left of the second separation line is valid for the portion CD.

3→ The portion to the left of the third separation line is valid for the portion DE.

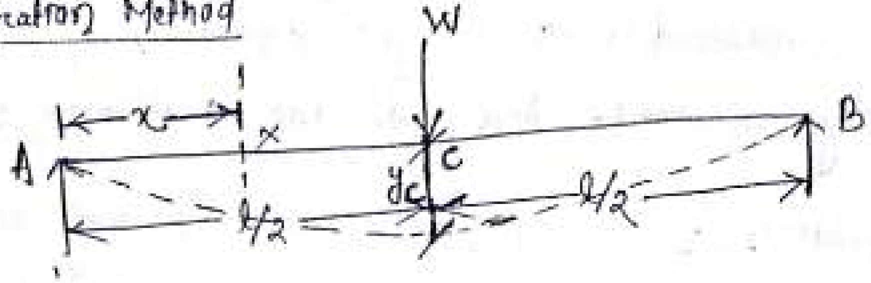
4→ The whole of the expression is valid for the portion EB.

→ So, this method is suitable for several loads acting on a beam.

→ This method was original proposed by Mathematician Mr. A. Clebsch, which was further developed by Mr. William Herrick Macaulay.

Simply Supported Beam with a Central Point load

By Double Integration Method



Here $R_a = R_b = W/2$

Consider a section 'x' at a distance x from A.

B.M. at x = $M_x = \frac{W}{2}x$ (+ve moment due to sagging)

$$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{W}{2}x \text{ ------ } (1)$$

Integrating the above equation,

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} + C_1 \text{ ---- } (2)$$

where, C_1 is the constant of integration.

We know, at centre, slope = 0 or $\frac{dy}{dx} = 0$.

Substituting these ∴ at $x = l/2$, $\frac{dy}{dx} = 0$ in eq^n (2)

$$0 = \frac{Wl^2}{16} + C_1$$

$$\Rightarrow C_1 = -\frac{Wl^2}{16}$$

Putting this value of C_1 in eq^n (2)

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} \text{ --- } (3)$$

Eq^n (3) is the required equation for slope at any section.

Integrating again, $EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x + C_2$ -- (4)

We know, at support, deflection = 0 or $y = 0$.

Substituting these

$\therefore$ at $x=0$, $y=0$ in eqn (4)

$\Rightarrow c_2=0$

$\therefore$ putting $c_2=0$ in eqn (4)

$$EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x \quad \cdots (5)$$

Eqn (5) is the required equation for deflection at any section.

Therefore, slope and deflection equation for the beam are given by,

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{W}{16EI}(l^2-4x^2) \quad \cdots \text{(Slope Equation)}$$

$$EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x$$

$$\Rightarrow y = -\frac{W}{48EI}(3l^2x - 4x^3) \quad \text{— (deflection Equation)}$$

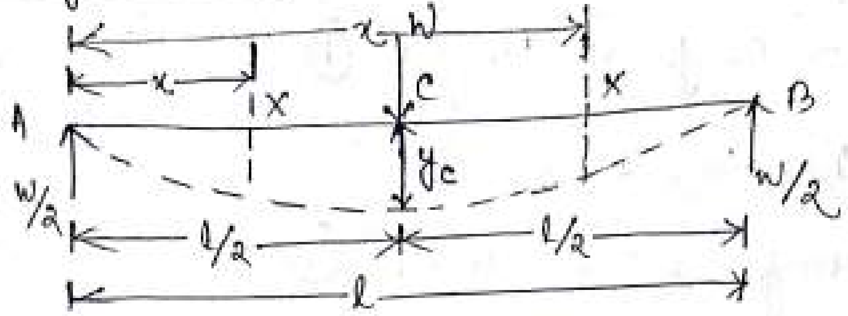
Maximum slope & deflection

At A, $x=0$ $\therefore$ slope $(\theta_A) = -\frac{Wl^2}{16EI}$, $\theta_B = \frac{Wl^2}{16EI}$ (due to symmetry)

At C, $x=l/2$ $\therefore$ Deflection $(y_c) = -\frac{W}{48EI}\left(3l^2\cdot\frac{l}{2} - 4\cdot\frac{l^3}{8}\right) = \frac{-Wl^3}{48EI}$

(-ve sign Due to sagging)

By Macaulay's Method



Taking A as origin, Bending moment at any point, in section AC at a distance x from A,

$$M_x = \frac{W}{2}x$$

and the bending moment at any point in section CB and at a distance x from A.

$$M_x = \frac{W}{2}x - W\left(x-\frac{l}{2}\right)$$

Thus, the bending moment for all sections of the beam can be expressed as

$$M_x = \frac{W}{2}x \;\Big|\; - W(x - l/2)$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = \frac{Wx}{2} \;\Big|\; - W(x-l/2) \quad \text{------ (1)}$$

Integrating the above equation,

$$\Rightarrow EI\frac{dy}{dx} = \frac{Wx^2}{4} + C_1 \;\Big|\; - \frac{W}{2}(x-l/2)^2 \quad \text{-- (2)}$$

$(x - l/2)$ is taken as a whole number

Integrating eqn (2) again,

$$\Rightarrow EIy = \frac{Wx^3}{12} + C_1x + C_2 \;\Big|\; - \frac{W}{6}(x-l/2)^3 \quad \text{---- (3)}$$

We know, when $x=0,\ y=0$ & $x=l,\ y=0$

Putting these values in equation (3)

for AB > $C_2 = 0$

for CB > $0 = \frac{Wl^3}{12} + C_1 l - \frac{W}{6}\left(\frac{l}{2}\right)^3$

$$\Rightarrow C_1 l = \frac{Wl^3}{48} - \frac{Wl^3}{12}$$

$$\Rightarrow C_1 l = \frac{-3Wl^3}{48} = -\frac{Wl^3}{16}$$

$$\Rightarrow C_1 = -\frac{Wl^2}{16}$$

For slope

Putting this value of C_1 in eqn (2)

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} \,\Big|\, - \frac{W}{2}(x - l/2)^2$$

This is the required equation for slope at any section.

We know that maximum slope occurs at A & B.

So, putting the values of $x = 0$ & l at A & B,

For AB $\rightarrow$ $EI\left(\frac{dy}{dx}\right)_A = -\frac{Wl^2}{16}$

$$\Rightarrow \boxed{\theta_A = -\frac{Wl^2}{16EI}}$$

For CB $\rightarrow$ $EI\left(\frac{dy}{dx}\right)_B = \frac{Wl^2}{4} - \frac{Wl^2}{16} \,\Big|\, - \frac{W}{2}(l - l/2)^2$

$$\Rightarrow EI\,\theta_B = \frac{Wl^2}{4} - \frac{Wl^2}{16} - \frac{Wl^2}{8}$$

$$\Rightarrow \boxed{\theta_B = \frac{Wl^2}{16EI}}$$

For deflection putting the value of C_1 & C_2 in eqn (3)

$$EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x \,\Big|\, - \frac{W}{6}(x - l/2)^3$$

This is the required equation for deflection at any section.

We know that maximum deflection occures at centre 'C'.

So, putting $x = l/2$. (for AC part/CB part)

$$EI\,y_C = \frac{W}{12}\left(\frac{l}{2}\right)^3 - \frac{Wl^2}{16}\left(\frac{l}{2}\right) = \frac{Wl^3}{96} - \frac{Wl^3}{32} = -\frac{Wl^3}{48}$$

$$\Rightarrow \boxed{y_C = \frac{-Wl^3}{48EI}}$$ (−ve sign for sagging)

Problem-1 Simply supported beam with central load

A simply supported beam of span 2.4 m is subjected to a central point load of 15 KN. What is the maximum slope and deflection at the centre of the beam? Take EI for the beam as 6×10^{10} N.mm².

Solⁿ:– Given data: Span $(l) = 2.4\,m = 2.4\times10^{3}\,mm$

$W =$ Central load $= 15\,KN = 15\times10^{3}\,N$

$EI = 6\times10^{10}\,N.mm^2$

Maximum slope.

Maximum slope of the beam $= \theta = \dfrac{Wl^2}{16EI}$

$$= \frac{15\times10^{3}\times(2.4\times10^{3})^2}{16\times6\times10^{10}} = 0.09 \text{ rad (Ans.)}$$

Maximum deflection

Maximum deflection of the beam $= y = \dfrac{Wl^3}{48EI}$

$$= \frac{15\times10^{3}\times(2.4\times10^{3})^3}{48\times6\times10^{10}} = 72\,mm. \text{ (Ans.)}$$

Problem-2

A beam 3 m long, simply supported at its ends, is carrying a point load at its centre. If the slope at the ends of the beam is not to exceed 1°, find the deflection at the centre of the beam.

Solⁿ:– Given data: Span $(l) = 3\,m = 3\times10^{3}\,mm$.

Slope $= \theta = 1^\circ = 1\times(\pi/180) = 0.0175$ Rad.

For this type of beam with central load,

$$\text{Deflection} = \text{Slope}\times\frac{l}{3} = 0.0175\times\frac{3\times10^{3}}{3} = 17.5\,mm \text{ (Ans.)}$$

Problem-3

A wooden beam 140 mm wide and 240 mm deep has a span of 4m. Determine the load, that can be placed at its centre to cause the beam a deflection of 10 mm. Take E as 6GPa.

Soln:- Given data:-

width of beam (b) = 140 mm.

depth of beam (d) = 240 mm.

length of beam (l) = 4m = 4×10^3 mm.

$y = 10$ mm.

$E = 6$ GPa $= 6 \times 10^3$ N/mm^2

Let, the magnitude of load = W

The moment of Inertia of the beam section,

$$I = \frac{bd^3}{12} = \frac{140 \times 240^3}{12} = 161.3 \times 10^6 \text{ mm}^4$$

The deflection of the beam at its centre = 10mm.

$$\therefore y = \frac{Wl^3}{48EI}$$

$$\Rightarrow 10 = \frac{W \times (4 \times 10^3)^3}{48 \times 6 \times 10^3 \times 161.3 \times 10^6}$$

$$\Rightarrow W = 7.25 \times 10^3 \text{N} = 7.25 \text{ KN (Ans.)}$$

Problem-4 Simply supported beam with uniformly distributed load

A simply supported beam of span 4 m is carrying a uniformly distributed load of 2KN/m over the entire span. find the maximum slope and deflection of the beam. Take EI for the beam as 80×10^9 N-mm^2.

Soln:- Given data:

Span (l) = 4m = 4×10^3 mm

$w = 2$ KN/m $= 2$ N/mm

$EI = 80 \times 10^9$ N.mm^2

Maximum slope.

$$\theta = \frac{wl^3}{24EI} = \frac{2\times(4\times10^3)^3}{24\times80\times10^9} = 0.067 \text{ rad. (Ans.)}$$

Maximum deflection

$$y = \frac{5wl^4}{384EI} = \frac{5\times2\times(4\times10^3)^4}{384\times80\times10^9} = 83.3 \text{ mm. (Ans.)}$$

Problem-5

A beam simply supported at its both ends carries a uniformly distributed load of 16 KN/m. If the deflection of the beam at its centre is limited to 2.5 mm, find the span of the beam. Take EI for the beam as 9×10^{12} N.mm².

Soln.:- Given data.

$w = 16 \text{ KN/m} = 16 \text{ N/mm}$

$y = 2.5 \text{ mm}.$

$EI = 9\times10^{12} \text{ N.mm}^2$

for this case, $y = \frac{5wl^4}{384EI}$

$$\Rightarrow 2.5 = \frac{5\times16\times l^4}{384\times9\times10^{12}}$$

$$\Rightarrow l^4 = \frac{2.5\times384\times9\times10^{12}}{5\times16}$$

$$\Rightarrow l^4 = 108\times10^{12}$$

$$\Rightarrow l = \sqrt[4]{108\times10^{12}}$$

$$\Rightarrow l = 3223.7 \text{ mm}$$

$$\Rightarrow l = 3.22 \text{ m (Ans.)}$$

Problem-6

A simply supported beam of span 6m is subjected to a uniformly distributed load over the entire span. If the deflection at the centre of the beam is not to exceed 4mm, find the value of the load. Take E = 200 Gpa and $I = 300\times10^6 \text{mm}^4$.

Solⁿ:- Given data :-

Span $(l) = 6m = 6 \times 10^3 mm$.

Deflection at the centre $= y_c = 4\ mm$.

Modulus of elasticity $= E = 200\ GPa = 200 \times 10^3\ N/mm^2$

Moment of Inertia $= I = 300 \times 10^6\ mm^4$

Let, w = value of uniformly distributed load in N/mm or KN/m.

Deflection at the centre for this type of beam

$$y = \frac{5wl^4}{384EI}$$

$$\Rightarrow w = \frac{4 \times 384 \times 200 \times 10^3 \times 300 \times 10^6}{5 \times (6 \times 10^3)^4}$$

$$\Rightarrow w = 14.2\ KN/m \quad \text{(Ans.)}$$

Problem-7 (Cantilever with a point load at the free end)

A cantilever beam 120mm wide and 150 mm deep is 1.8 m. long. Determine the slope and deflection at the free end of the beam, when it carries a point load of 20KN at its free end. Take E for the cantilever beam as 200 GPa.

Solⁿ:- Given data

width $(b) = 120\ mm$.

depth $(d) = 150\ mm$.

Span $(l) = 1.8m = 1.8 \times 10^3\ mm$.

Point load $(W) = 20\ KN = 20 \times 10^3 N$.

Modulus of elasticity $= E = 200\ GPa = 200 \times 10^3\ N/mm^2$.

Slope at the free end

Moment of inertia of the beam $= I = \frac{bd^3}{12} = \frac{120 \times (150)^3}{12} = 33.75 \times 10^6\ mm^4$

slope at the free end $= \theta = \frac{Wl^2}{2EI} = \frac{(20 \times 10^3) \times (1.8 \times 10^3)^2}{2 \times 200 \times 10^3 \times 33.75 \times 10^6}$

$= 0.0048$ rad. (Ans.)

Deflection at the free end

The deflection at the free end for this beam,

$$y = \frac{Wl^3}{3EI} = \frac{(20\times10^3)\times(1.8\times10^3)^3}{3\times(200\times10^3)\times(33.75\times10^6)} = 5.76\,mm \text{ (Ans.)}$$

Problem-8

A cantilever beam of 160 mm width and 240 mm depth is 1.75 m long. What load can be placed at the free end of the cantilever, if its deflection under the load is not to exceed 4.5 mm. Take E for the beam material as 180 GPa.

Soln:- Given data,

span (l) = 1.75 m = 1.75×10^3 mm.

width (b) = 160 mm

depth (d) = 240 mm.

deflection (y) = 4.5 mm.

modulus of elasticity (E) = 180 GPa = $180\times10^3\ N/mm^2$

Let = W = Load which can be placed at the free end.

Deflection for this type of beam,

$$y = \frac{Wl^3}{3EI}$$

I = moment of inertia of the beam $= \frac{bd^3}{12}$

$$= \frac{160\times(240)^3}{12} = 184.32\times10^6\ mm^4$$

$$\therefore\ y = \frac{Wl^3}{3EI}$$

$$\Rightarrow 4.5 = \frac{W\times(1.75\times10^3)^3}{3\times180\times10^3\times184.32\times10^6}$$

$$\Rightarrow W = \frac{4.5\times3\times180\times184.32\times10^9}{(1.75)^3\times10^9} = 83{,}572\,N = 83.57\,KN \text{ (Ans)}$$

Problem-9 (Cantilever with a uniformly Distributed Load)

A cantilever beam 2m long is subjected to a uniformly distributed load of 5KN/m over its entire length. Find the slope and deflection of the cantilever beam at its free end. Take $EI = 2.5 \times 10^{12} Nmm^2$.

Solⁿ:- Given data

Span (l) = 2m = 2×10^3 mm.

Uniformly distributed load = 5 KN/m = 5 N/mm.

$EI = 2.5 \times 10^{12} Nmm^2$

Slope of the beam at its free end

$$\theta = \frac{wl^3}{6EI} = \frac{5 \times (2 \times 10^3)^3}{6 \times 2.5 \times 10^{12}} = 0.0027 \text{ rad. (Ans.)}$$

Deflection of the beam at its free end.

$$y = \frac{wl^4}{8EI} = \frac{5 \times (2 \times 10^3)^4}{8 \times 2.5 \times 10^{12}} = 4.0 \text{ mm (Ans.)}$$

Problem-10

A cantilever beam 100 mm wide and 180 mm deep is Projecting 2m from a wall. Calculate the uniformly distributed load, which the beam should carry, if the deflection of the free end should not exceed 3.5 mm. Take E as 200 GPa.

Solⁿ:- Given data

width (b) = 100mm, depth (d) = 180 mm, Span (l) = 2m = 2×10^3 mm

deflection at the free end (y) = 3.5 mm, E = 200GPa = 200×10^3 N/mm²

Let w = Uniformly distributed load, which the beam should carry

The moment of Inertia of the beam = $I = \frac{bd^3}{12}$

$$= \frac{100 \times 180^3}{12} = 48.6 \times 10^6 \text{ mm}^4$$

Deflection $y = \frac{wl^4}{384EI}$

$$\Rightarrow 3.5 = \frac{w \times (2 \times 10^3)^4}{384 \times 200 \times 10^3 \times 48.6 \times 10^6} \Rightarrow w = \frac{3.5}{0.206} = 17 \text{ N/mm}$$

$$= 17 \text{ KN/m (Ans.)}$$

Problem-11

A cantilever 3 metres long carries a uniformly distributed load over the entire length. If the slope at the free end is 1°, find the deflection at the free end.

Soln:- Span $(l) = 3\,m = 3 \times 10^3\,mm$.

Slope $= (\theta) = 1^\circ = \frac{\pi}{180}$ rad.

For this case slope $= \frac{wl^3}{6EI}$

So, $\frac{wl^3}{6EI} = \frac{\pi}{180}$

$\Rightarrow \frac{wl^3}{EI} = \frac{\pi}{30}$

Deflection at the free end $= y = \frac{wl^4}{8EI} = \frac{wl^3}{EI} \times \frac{l}{8}$

$\Rightarrow y = \frac{\pi}{30} \times \frac{3 \times 10^3}{8} = 39.27\,mm$ (Ans.)

—x—

Chapter -7

Indeterminate Beams

Indeterminacy of Beam :- If the no. of unknown reactions are more than the no. of equilibrium equations available then the structure is called indeterminate structure.

→ Indeterminacy of beam can be calculated from degree of indeterminacy.

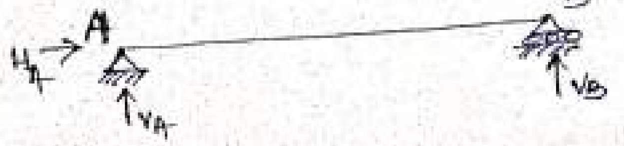
→ Indeterminacy of beam is of 2 types.

1. Static indeterminacy (D_s) 2. Kinetic Indeterminacy (D_K)

Static indeterminacy means to know the external & internal forces in any structure.

Kinetic indeterminacy means to know the rotation and displacement in any structure.

Statically determinate structure	Statically Indeterminate structure
1. Number of unknowns can be found using conditions of equilibrium alone.	2. Number of unknowns can not be found using condition of equilibrium.
2. Here no. of unknowns ≤ available condition of equilibrium.	2. Here no. of unknowns > available condition of equilibrium.
3. Degree of indeterminacy is zero	3. Degree of indeterminacy is required to find out.
4. ex:- Simply supported	4. ex:- Propped Cantilever



$D_s = R - 3 = 3 - 3 = 0$

$D_s = 4 - 3 = 1$.

Degree of static indeterminacy (D_s)

D_s = Total number of unknown − Total no. of equation of equilibrium

Stability of structure

If $D_s < 0$, it is unstable

If $D_s = 0$, Just stable / Just rigid

If $D_s > 0$, Over stable / over rigid.

Examples

1.

$D_s = R$ − no. of equilibrium eq^{n}s

$= 2 - 3 = -1 \Rightarrow$ unstable.

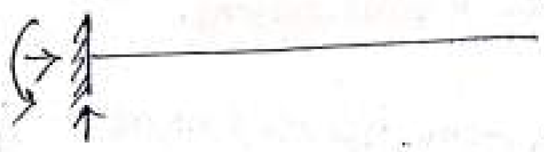
2.

$D_s = 3 - 3 = 0 \Rightarrow$ Just stable.

3.

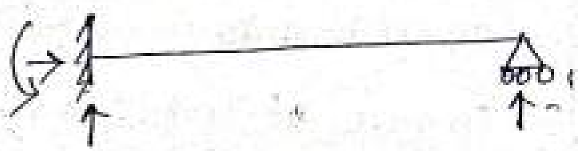
$D_s = 4 - 3 = 1 \Rightarrow$ Over stable

1.



$D_s = 3 - 3 = 0 \Rightarrow$ Just stable.

2.



$D_s = 4 - 3 = 1 \Rightarrow$ Over stable.

Principle of Consistent Deformation / compatibility

If the structure is determinate, then equilibrium conditions are enough to analyse the structure.

- Equilibrium conditions do not involve c/s properties (A, E, I)
- for Indeterminate structure, equilibrium conditions and compatibility conditions are required to analyse the structure.
- Analysis of indeterminate structure depends upon the c/s properties (A, E, I)

Analysis of indeterminate structure is done by Consistent deformation method or force method or compatibility method.

Consistent Deformation method

- This method is used for analysis of indeterminate beam. In this method along with equilibrium eqns, compatibility conditions are used to find deformation of structure with respect to support condition.
- Reactive forces are taken as unknowns, hence this method is a type of force method.

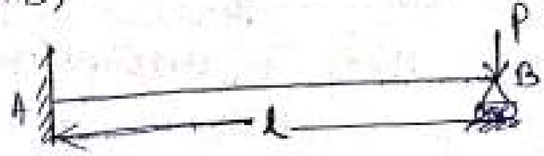
Compatibility condition

Compatibility condition is a relationship between forces and known deformations.

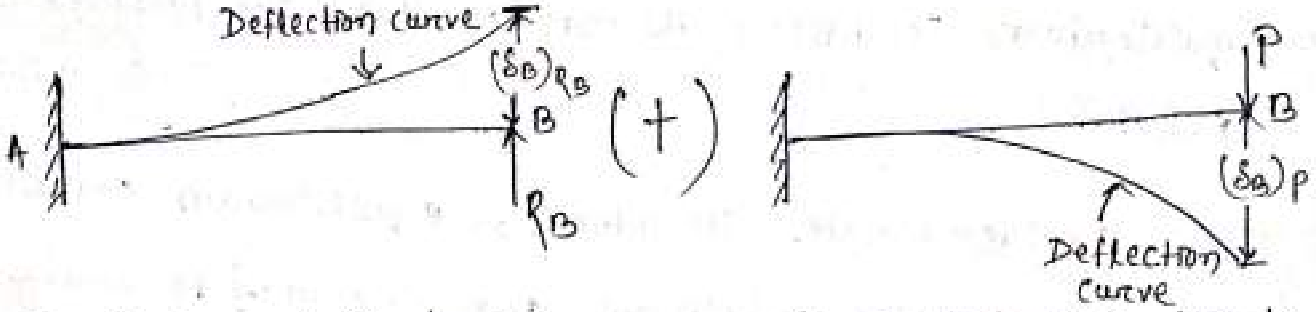
Eg.

For this propped cantilever beam, compatibility condition is Deflection at B = 0 i.e. $\delta_B = 0$

Means,



can be written as



Deflection at B due to R_B (Reaction at Propped end B)

$$y_B = (\delta_B)_{R_B} = \frac{R_B l^3}{3EI} \ (+ve)$$

Deflection at B due to external point load 'P',

$$y_B = (\delta_B)_P = \frac{Pl^3}{3EI} \ (-ve)$$

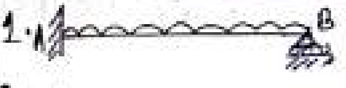
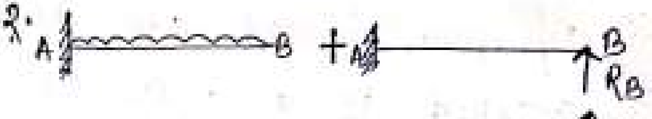
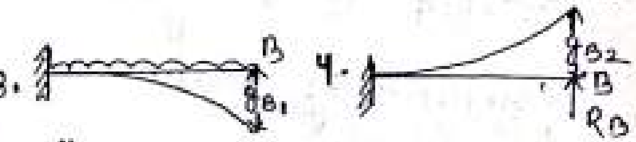
So, compatibility eqn is:

$$\delta_B = 0 \ (\text{net deflection at } B = 0)$$

$$\Rightarrow (\delta_B)_{R_B} + (\delta_B)_P = 0 \quad (\text{without considering the sign})$$

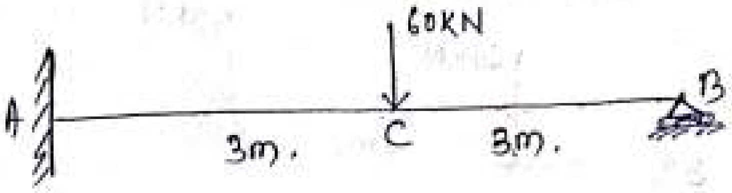
Steps for Analysing Consistent Deformation method

By Consistent deformation method; propped cantilever and fixed beams are analysed.

Process	Demonstration
1. Find D_S	1.  $D_S = 4-3 = 1$
2. Choose the Redundant or the force to be removed so that the structure will become determinate and remove this redundant.	2. 
3. Determine (Δ_L) = displacement due to external load	3.  $y_{B_1} = ?$ or Δ_L
4. Determine (Δ_R) = displacement due to redundants	4. $y_{B_2} = ?$ or Δ_R
5. Use compatibility conditions of displacement to determine the redundants.	5. Compatibility eqn that is $y_B = 0$ $\therefore (y_{B_1} \text{ or } \Delta_L) + (y_{B_2} \text{ or } \Delta_R) = 0$ or, $\Delta = \Delta_L + \Delta_R$
6. Find unknown by static eqn	6. R_B, R_A, M .....
7. Draw S.F.D / B.M.D.	

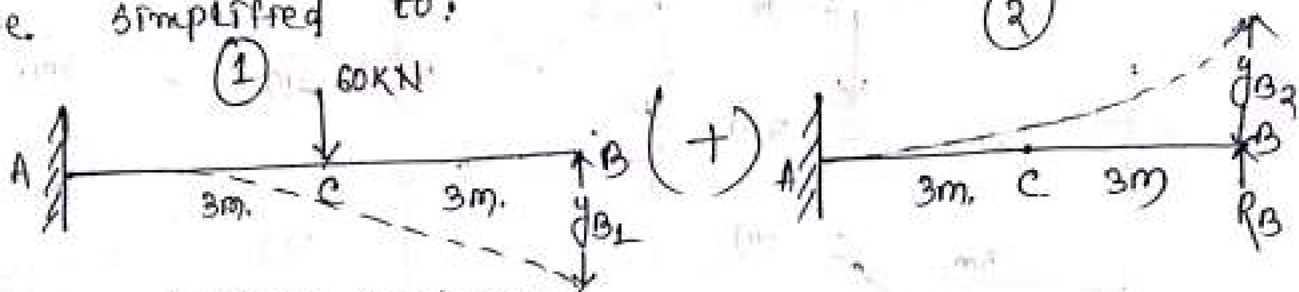
Problem-1

Analyse the Propped Cantilever beam shown using Consistency deformation method. Draw S.F.D, B.M.D



Soln:- Degree of static indeterminacy $= D_s = 4-3 = 1$
or
no. of unknown

Let, the redundant force as R_B., the figure can be simplified to:



Using compatibility condition,

$$y_B = 0 \text{ or } \delta_B = 0$$

$$\Rightarrow y_{B_1} + y_{B_2} = 0$$

$$\Rightarrow \frac{-5Wl^3}{48EI} + \frac{R_B l^3}{3EI} = 0$$

$$\Rightarrow \frac{R_B l^3}{3EI} = \frac{5Wl^3}{48EI}$$

$$\Rightarrow R_B = \frac{5 \times 3 \times W}{48} = \frac{5W}{16} = \frac{5 \times 60}{16} = 18.75\,KN$$

Using Equilibrium eqn.

$\Sigma V = 0$ or $\Sigma f_x = 0$

$$\Rightarrow R_A + R_B = 60$$

$$\Rightarrow R_A = 60 - 18.75 = 41.25\,KN.$$

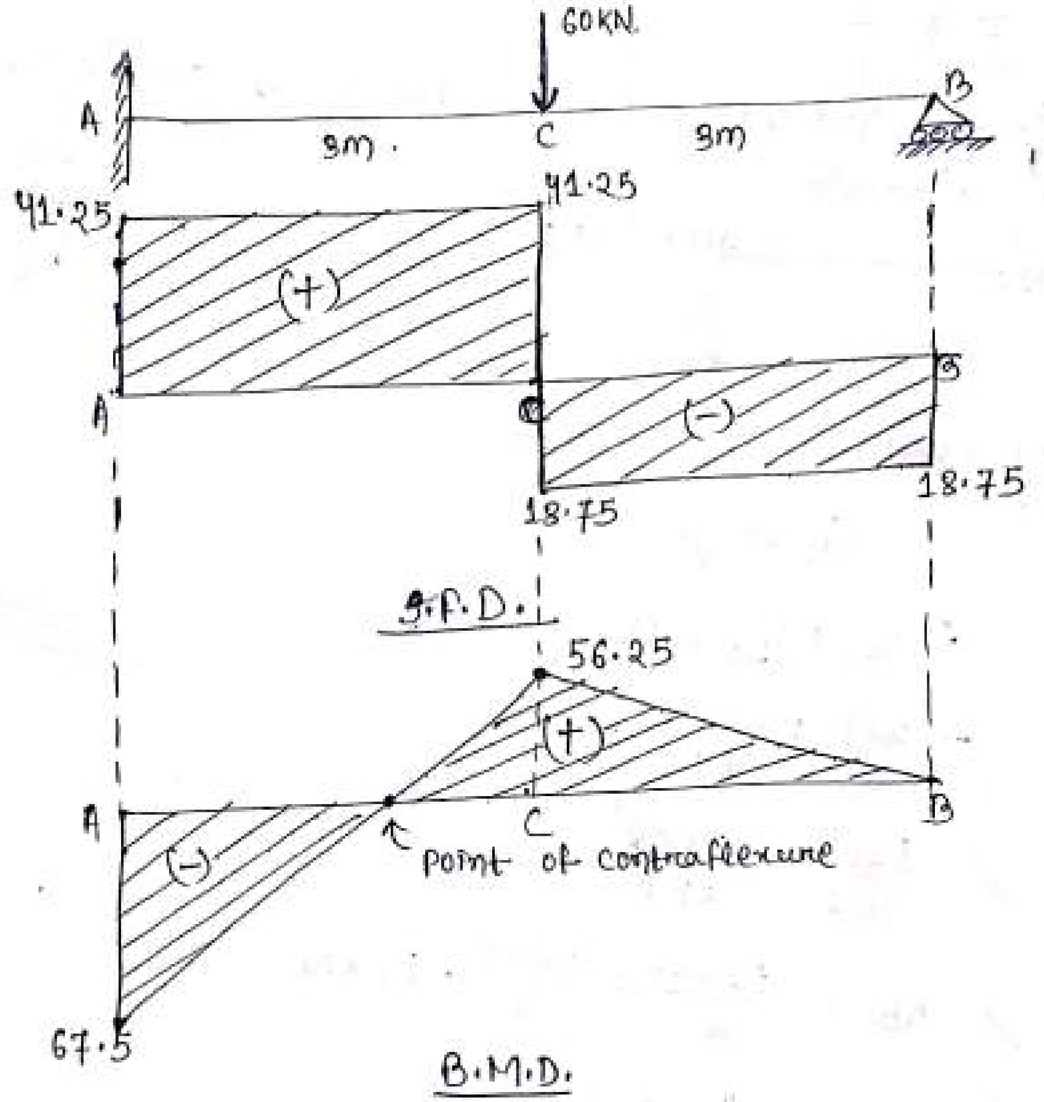
Calculating Bending moment.

M_B = B.M. at B = 0

$M_C = R_B \times 3 = 56.25$ KNm (+ve for sagging)

$M_A = (R_B \times 6) - (60 \times 3) = (\underbrace{18.75 \times 6}_{\text{Sagging}}) - \underbrace{60 \times 3}_{\text{Hogging}}$

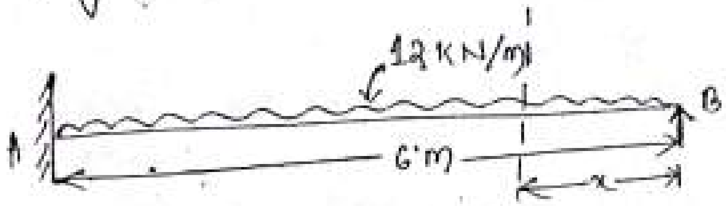
$= -67.5$ KNm (-ve for Hogging)



Problem-2

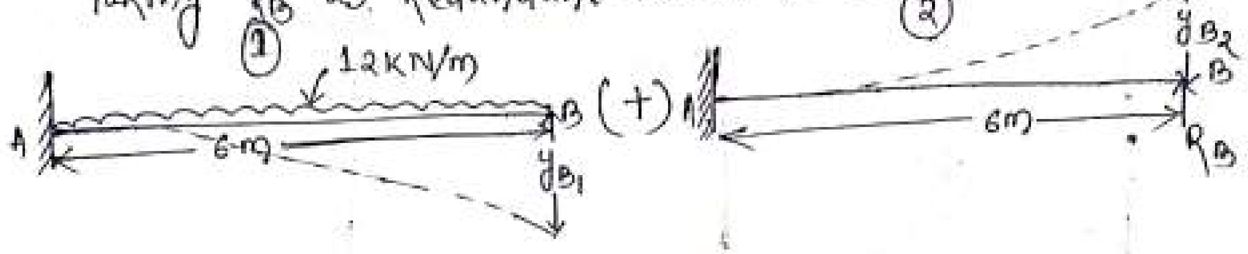
A propped cantilever beam 6m span is subjected to udl over its entire span. If it is propped at the free end at the level of fixed end. Calculate the reaction of the prop, when the cantilever carries a udl of 12 KN/m. And draw the shear force and Bending moment diagrams showing all the salient values.

Solⁿ:- According to given data, the beam can be drawn as



Here $D_s = 4-3 = 1$

Taking R_B as Redundant force, the beam can be simplified.



Using Compatibility condition,

$$\delta_B \text{ or } y_B = 0$$

$$\Rightarrow y_{B_1} + y_{B_2} = 0$$

$$\Rightarrow -\frac{wl^4}{8EI} + \frac{wl^3}{3EI} = 0$$

$$\Rightarrow \frac{R_B l^3}{3EI} = \frac{12 \times l^4}{8EI}$$

$$\Rightarrow R_B = \frac{12 l \times 3}{8} = \frac{36 \times 6}{8} = 27 \text{ KN}$$

Using Equilibrium equation,

$$\Sigma V = 0 \text{ or } \Sigma f_x = 0$$

$$\Rightarrow R_A + R_B = 12 \times 6 = 72 \text{ KN}$$

$$\Rightarrow R_A = 72 - R_B = 72 - 27 = 45 \text{ KN}.$$

Due to uniformly distributed load, the shear force will ~~change~~ change its sign at a point and let that point is at a distance 'x' from 'B'.

The shear force at that point will be '0'

So, $R_B - 12x = 0$

$$\Rightarrow 27 - 12x = 0 \Rightarrow x = 2.25 \text{ m}.$$

Calculating the Bending moment,

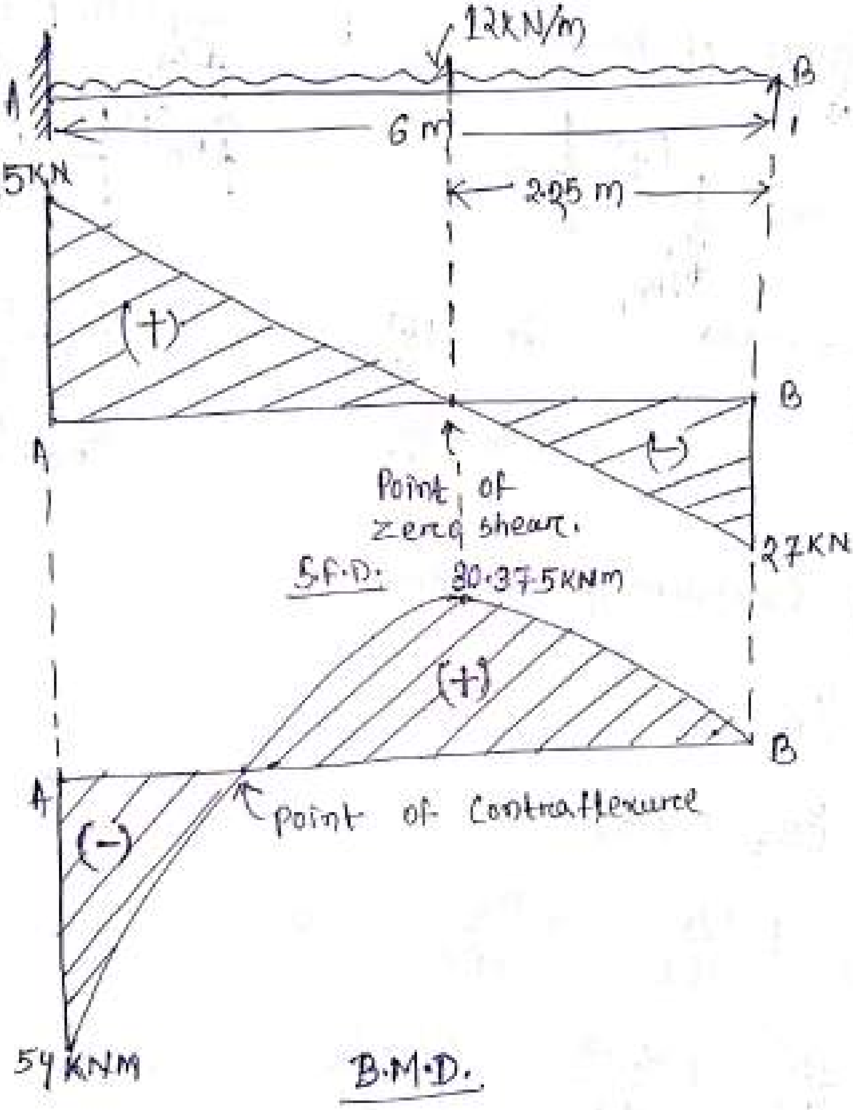
$M_B = 0$

$M_A = R_B \times 6 - 12 \times 6 \times \frac{6}{2} = 27 \times 6 - 12 \times 6 \times 3 = -54$ KN.m.

The maximum Bending moment will be at zero shear point.

Max^m Bending moment $= R_B \times 2.25 - 12 \times 2.25 \times \frac{2.25}{2}$

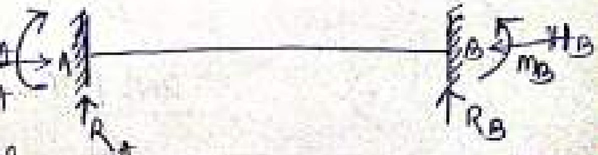
$= 27 \times 2.25 - 6 \times (2.25)^2 = 30.375$ KNm



Fixed Beam

A fixed beam is a beam whose end supports are such that the end slopes remain zero (or unaltered) and is also called a built-in or encastre beam. as it is extended into the supports.

→ It is fixed at both ends.
→ Due to fixidity, the slope & deflection at both ends is zero.
→ It is statically indeterminate.



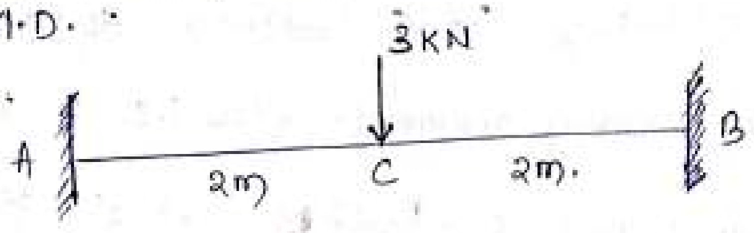
4-2 = 2 (neglecting Horizontal reactions)
as there is no horizontal loading.

so, $H_A = H_B = 0$

Indeterminacy more than 1

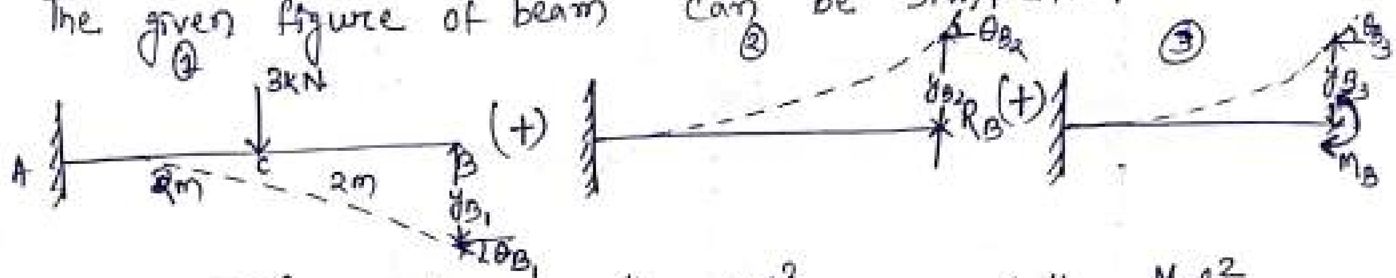
Problem-1

Analyse the fixed beam shown in the figure. Draw S.F.D. and B.M.D.



Soln.— for this beam $D_s = 4-2 = 2$

The given figure of beam can be simplified to



$y_{B_1} = \frac{-5Wl^3}{48EI}$ $\qquad$ $y_{B_2} = +\frac{Wl^3}{3EI}$ $\qquad$ $y_{B_3} = \frac{M_B l^2}{2EI}$

$\theta_{B_1} = \frac{-Wl^2}{8EI}$ $\qquad$ $\theta_{B_2} = \frac{Wl^2}{2EI}$ $\qquad$ $\theta_{B_3} = \frac{M_B l}{EI}$

Using Compatibility Condition -1

$$y_B = 0$$

$$\Rightarrow y_{B_1} + y_{B_2} + y_{B_3} = 0$$

$$\Rightarrow \frac{-5Wl^3}{48EI} + \frac{Wl^3}{3EI} + \frac{M_B l^2}{2EI} = 0$$

$$\Rightarrow \frac{Wl^3}{3EI} + \frac{M_B l^2}{2EI} = \frac{5Wl^3}{48EI}$$

$$\Rightarrow \frac{l^2}{EI}\left(\frac{Wl}{3} + \frac{M_B}{2}\right) = \frac{l^2}{EI} \times \frac{5Wl}{48}$$

$$\Rightarrow \frac{Wl}{3} + \frac{M_B}{2} = \frac{5Wl}{48}$$

$$\Rightarrow \frac{2Wl + 3M_B}{6} = \frac{5Wl}{48} \Rightarrow 2Wl + 3M_B = \frac{5Wl}{48} \times 6 = \frac{5Wl}{8}$$

$$\Rightarrow 2R_B l + 3M_B = \frac{5Wl}{8} \Rightarrow 8R_B + 3M_B = \frac{60}{8} \Rightarrow 64R_B + 24M_B = 60 \quad \text{—(1)}$$

Using compatibility condition - 2

$\theta_B = 0$

$\Rightarrow \theta_{B_1} + \theta_{B_2} + \theta_{B_3} = 0$

$\Rightarrow -\frac{wl^2}{8EI} + \frac{wl^2}{2EI} + \frac{M_B l}{EI} = 0$

$\Rightarrow \frac{wl^2}{2EI} + \frac{M_B l}{EI} = \frac{wl^2}{8EI}$

$\Rightarrow \cancel{\frac{l}{EI}}\left(\frac{wl}{2} + M_B\right) = \cancel{\frac{l}{EI}} \times \frac{wl}{8}$

$\Rightarrow \frac{Wl}{2} + M_B = \frac{wl}{8}$

$\Rightarrow \frac{R_B l}{2} + M_B = \frac{wl}{8}$

$\Rightarrow 2R_B + M_B = \frac{3}{2}$

$\Rightarrow 4R_B + 2M_B = 3$ ---- (2)

Solving eqn (1) & (2)

$$\begin{aligned}
&64 R_B + 24 M_B = 60 \text{ ---- (1)} \\
-\ &48 R_B + 24 M_B = 36 \text{ ---- (2)} \times 12 \\
&(-) \qquad\quad (-) \qquad\quad (-) \\
\hline
&16 R_B = 24
\end{aligned}$$

$\Rightarrow R_B = 24/16 = 1.5$ kN.

In the given beam,

$R_A + R_B = 3$ kN

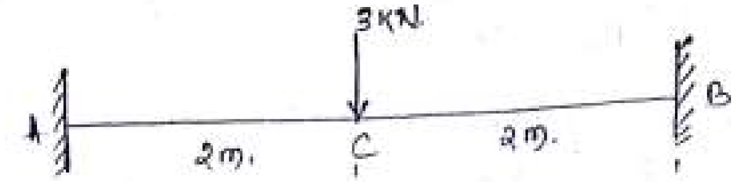
$\Rightarrow R_A = 3 - 1.5 = 1.5$ kN.

Putting R_B value in eqn (2)

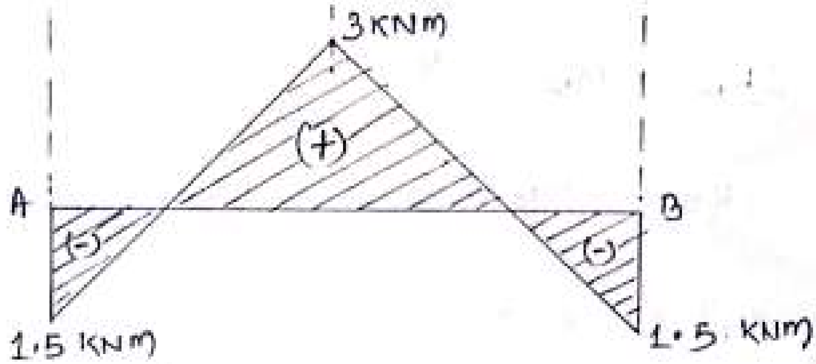
$M_B = -1.5$ kNm (-ve due to hogging moment at fixed end)

By symmetry, $M_A = M_B = -1.5$ kNm.

$M_C = R_B \times 2 = 1.5 \times 2 = +3\,KNm$ (+ve for sagging moment at centre)



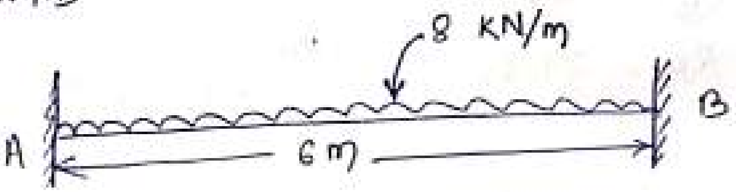
S.F.D.



B.M.D.

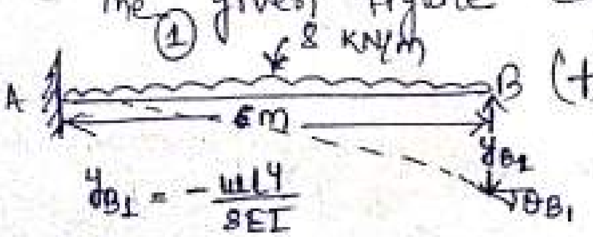
Problem-2

Analyse the fixed beam shown in the figure. Draw S.F.D. and B.M.D.

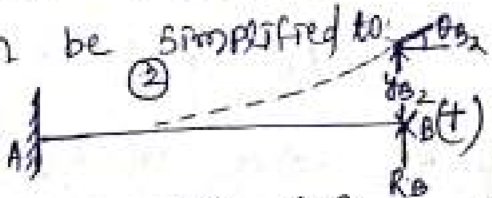


Sol:— For this beam $D_s = 4-2 = 2$

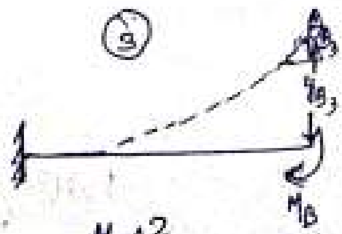
The given figure can be simplified to:



(+)



(+)



$y_{B_1} = -\dfrac{wl^4}{8EI}$

$\theta_{B_1} = \dfrac{-wl^3}{6EI}$

$y_{B_2} = \dfrac{Wl^3}{3EI} = \dfrac{R_B l^3}{3EI}$

$\theta_{B_2} = \dfrac{Wl^2}{2EI} = \dfrac{R_B l^2}{2EI}$

$y_{B_3} = \dfrac{M_B l^2}{2EI}$

$\theta_{B_3} = \dfrac{M_B l}{EI}$

Using Compatibility Condition - 1

$y_B = 0$

$\Rightarrow y_{B_1} + y_{B_2} + y_{B_3} = 0$

$\Rightarrow -\frac{wl^4}{8EI} + \frac{R_B l^3}{3EI} + \frac{M_B l^2}{2EI} = 0$

$\Rightarrow \frac{R_B l^3}{3EI} + \frac{M_B l^2}{2EI} = \frac{wl^4}{8EI}$

$\Rightarrow \cancel{\frac{l^2}{EI}}\left(\frac{R_B l}{3} + \frac{M_B}{2}\right) = \cancel{\frac{l^2}{EI}} \times \frac{wl^2}{8}$

$\Rightarrow \frac{R_B l}{3} + \frac{M_B}{2} = \frac{wl^2}{8}$

$\Rightarrow \frac{2R_B l + 3M_B}{\cancel{6}\,3} = \frac{wl^2}{\cancel{8}\,4}$

$\Rightarrow 2R_B l + 3M_B = \frac{3wl^2}{4}$

$\Rightarrow 12R_B + 3M_B = \frac{3 \times \cancel{8}^{2} \times 6 \times 6}{\cancel{4}}$

$\Rightarrow 12R_B + 3M_B = 216$

$\Rightarrow 4R_B + M_B = 72$ ------ (1)

Using Compatibility Condition - 2

$\theta_B = 0$

$\Rightarrow \theta_{B_1} + \theta_{B_2} + \theta_{B_3} = 0$

$\Rightarrow -\frac{wl^3}{6EI} + \frac{R_B l^2}{2EI} + \frac{M_B l}{EI} = 0$

$\Rightarrow \frac{R_B l^2}{2EI} + \frac{M_B l}{EI} = \frac{wl^3}{6EI}$

$\Rightarrow \cancel{\frac{l}{EI}}\left(\frac{R_B l}{2} + M_B\right) = \cancel{\frac{l}{EI}} \times \frac{wl^2}{6}$

$\Rightarrow \frac{R_B l + 2M_B}{\cancel{2}} = \frac{wl^2}{\cancel{6}\,3} \Rightarrow 6R_B + 2M_B = \frac{8 \times 6 \times \cancel{6}^{2}}{\cancel{3}}$

$\Rightarrow 6R_B + 2M_B = 96 \Rightarrow 3R_B + M_B = 48$ --- (2)

Scanned with CamScanner

Solving eq^n 1 & eq^n 2

$$4R_B + M_B = 72 \quad \text{------ eq}^n\ (1)$$
$$(-)\ 3R_B + M_B = 48 \quad \text{--- eq}^n\ (2)$$
$$(-)\quad (-)\quad (-)$$

$\Rightarrow R_B = 24\ KN$

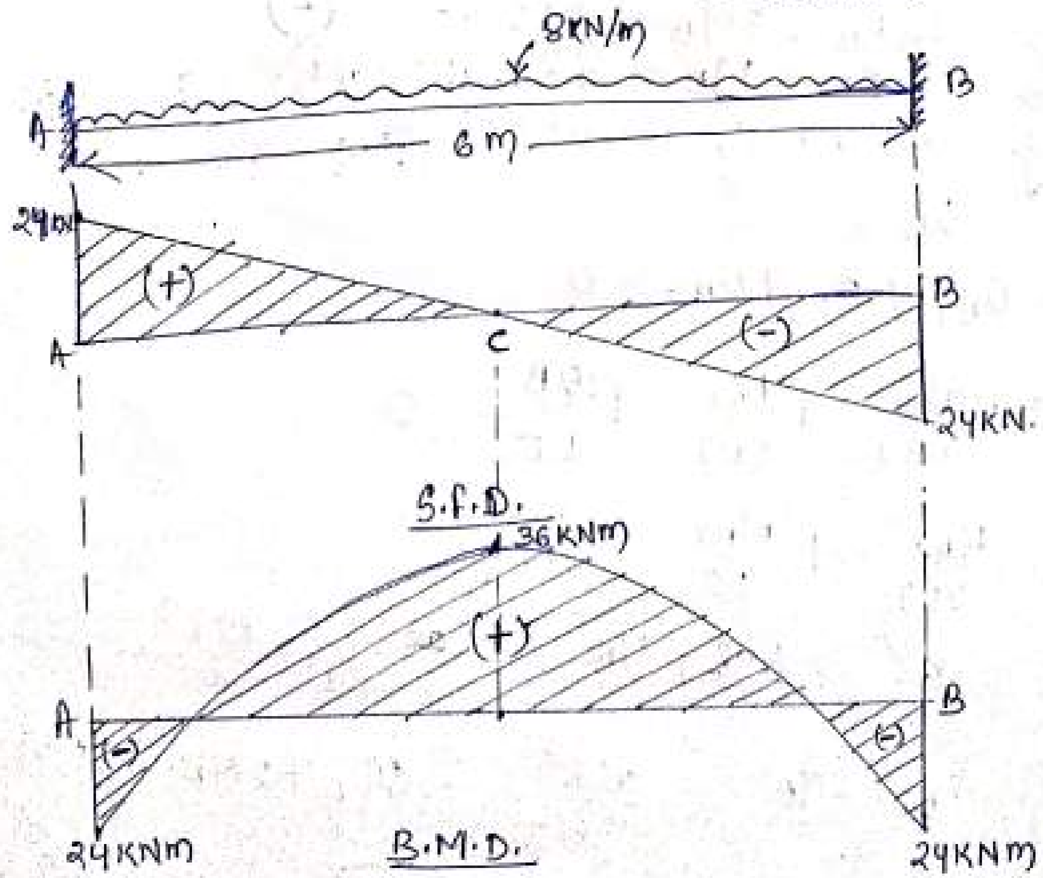
$\Rightarrow R_A = \text{Total Load} - R_B = (8\times6) - 24 = 24KN.$

Putting the R_B value in eq^n 2

$$3R_B + M_B = 48$$
$$\Rightarrow 3\times24 + M_B = 48$$
$$\Rightarrow M_B = 48-72 = -24\ KNm = M_A \text{ (Due to symmetry)}$$

The point of contraflexure will be at centre. due to symmetry. and let that point be 'c'.

The bending moment at 'c' $= M_C = R_B\times3 - 8\times3\times\frac{3}{2}$
$$= (24\times3) - (36)$$
$$= 36\ KNm$$



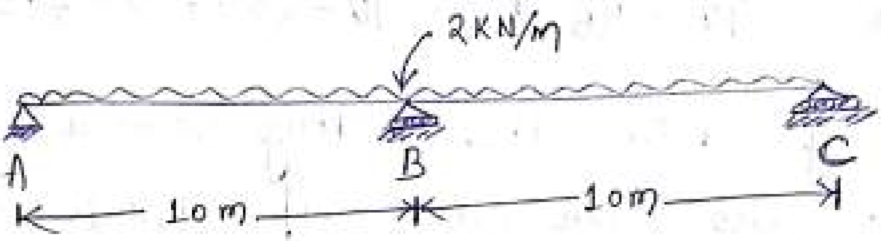
Continuous Beam

A continuous beam is a statically indeterminate multi-span beam on hinged support.

→ The end spans may be cantilever, may be freely supported or fixed supported.

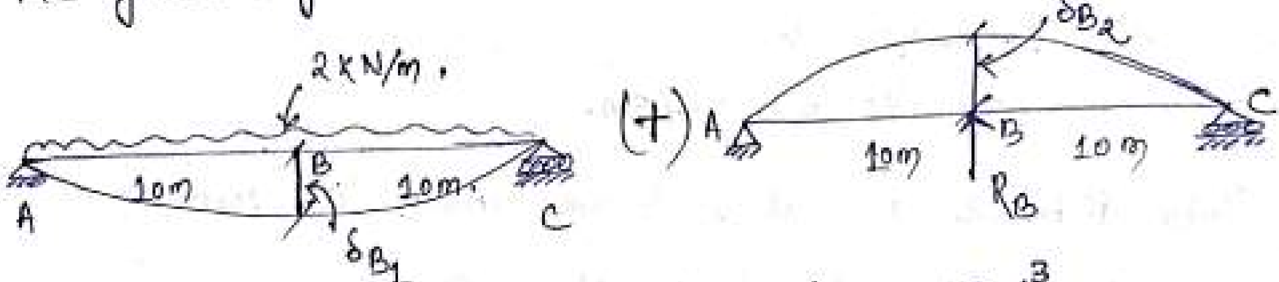
Problem-1

Analyse the continuous beam shown in figure and draw S.F.D., B.M.D.



Sol^n:— for this beam $D_s = R - 2 = 3 - 2 = 1$

The given figure can be simplified in to



$$\delta_{B_1} = -\frac{5wl^4}{384EI} \qquad \delta_{B_2} = +\frac{R_B l^3}{48EI}$$

Using Compatibility condition

$$\delta_B = 0$$

$$\Rightarrow \delta_{B_1} + \delta_{B_2} = 0$$

$$\Rightarrow \frac{-5wl^4}{384EI} + \frac{R_B l^3}{48EI} = 0$$

$$\Rightarrow \frac{R_B l^3}{48EI} = \frac{5wl^4}{384EI} \Rightarrow R_B = \frac{5wl}{8} = 25KN$$

Using equilibrium conditions for other Reactions

$\Sigma M_A = 0$

$\Rightarrow (R_C \times 20) + (R_B \times 10) - 2 \times 20 \times 10 = 0$

$\Rightarrow R_C = \frac{400 - (25 \times 10)}{20} = \frac{150}{20} = 7.5 KN.$

$\Sigma F_y = 0$

$\Rightarrow R_A + R_B + R_C = 2 \times 20$

$\Rightarrow R_A = 40 - R_B - R_C = 40 - 25 - 7.5 = 7.5 KN.$

After drawing S.F.D., the bending moment will be maximum at zero shear point and let, that point is at 'x' distance from 'A'.

The shear force is zero at this point.

$\therefore 7.5 - 2x = 0$

$\Rightarrow x = 7.5/2 = 3.75 m.$

This distance is also same from 'C' point.

The Bending moment at $A = M_A = 0$

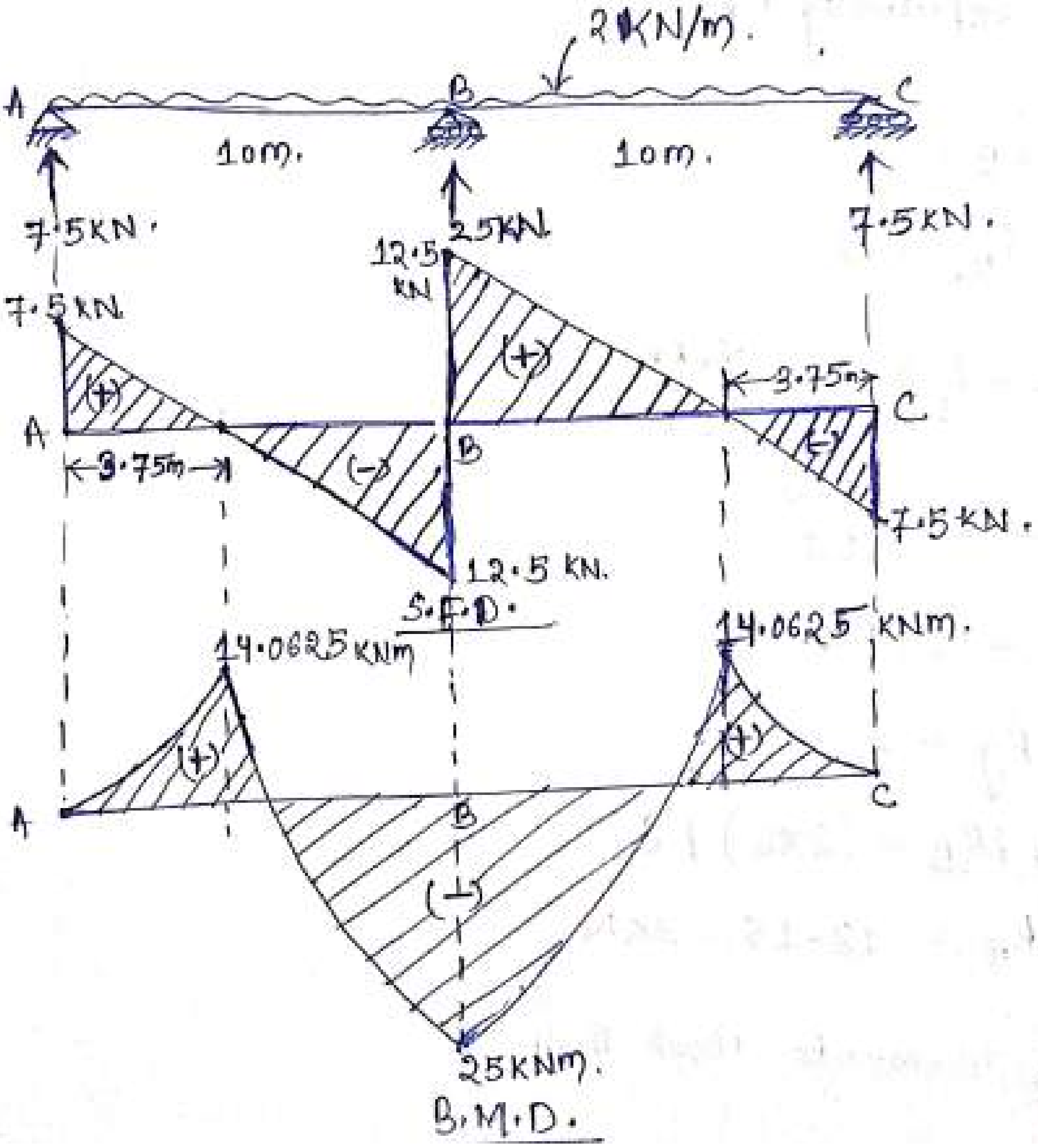
$M_B = (7.5 \times 10) - 2 \times 10 \times 5$

$= -25 KNm.$

$M_C = 0$

$M_x = (7.5 \times 3.75) - 2 \times 3.75 \times \frac{3.75}{2}$

$= 14.0625 KNm$


2KN/m.
A
B
C
1om.
1om.
7.5KN.
25KN
7.5KN.
7.5KN
12.5 KN
(+)
(+)
3.75m
A
B
C
3.75m
(-)
(-)
12.5 KN.
7.5KN.
S.F.D.
14.0625KNm
14.0625 KNm.
(+)
(+)
A
B
C
(-)
25KNm.
B.M.D.

Chapter-8

Trusses

Truss

When a structure formed by members in triangular form, the resulting figure is called a truss.

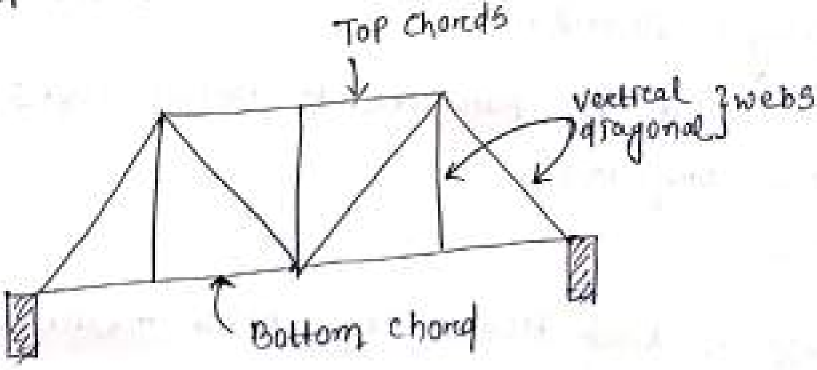
→ In truss, joints are pin connected and loads are applied at joints or end points.

→ No shear and bending moment are produced.

→ Only axial compression and axial tension are to be determined while analysing a truss.

Truss is composed of 3 basic parts.

- Top Chords
- Bottom Chords
- webs



Top chords

The beam at the top which is usually in compression is called top chord.

Bottom chord

The beam at the bottom which is usually in tension is called bottom chord.

Web

All the interior beams are called webs.
These can be (i) vertical or (ii) diagonal

Types of trusses

Truss

- As per work space
  - 1. Planar truss
  - 2. Space truss
- As per span length & loading
  - 1. Bridge truss
    - 1. Pratt Bridge truss
    - 2. Howe Bridge truss
    - 3. Warren Bridge truss
  - 2. Roof truss
    - 1. Pratt roof truss
    - 2. Fink roof truss
    - 3. Howe roof truss
    - 4. Warren roof truss
    - 5. King post roof truss

A truss can be of 2 types as per work space system:

1. planar truss
2. Space truss

1. Planar truss

Planar truss is that truss in which members lies in a 2D plane or a single plane.

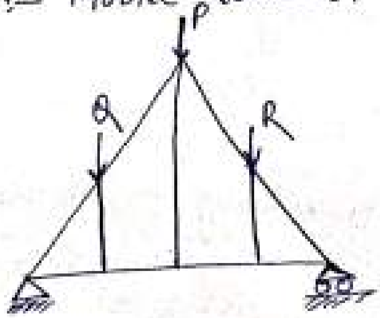
→ These are used in parallel to form roofs & bridges.

example:- roof truss, bridge truss

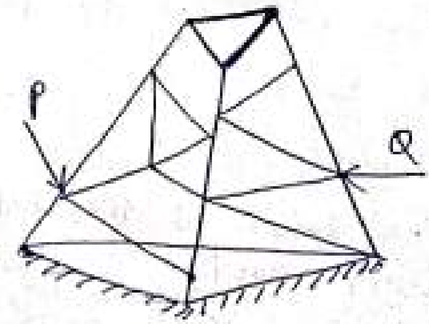
2. Space truss

Space truss is that truss in which members lie in a 3D plane or not in a single plane.

example:- Mobile towers, Transmission towers.



Roof - Truss



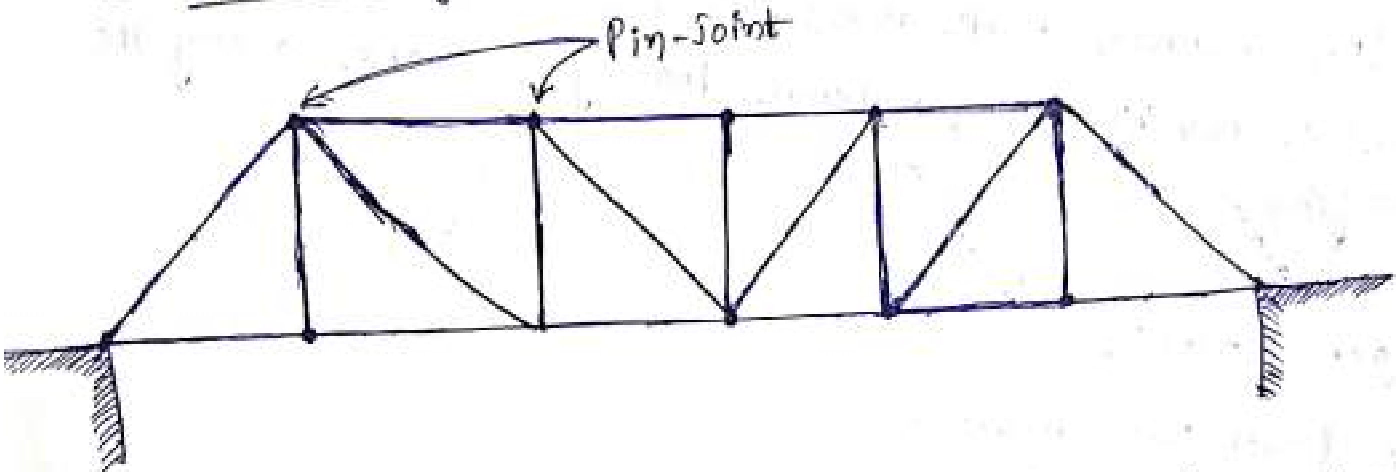
Towers

Trusses are also of two types according to loading & span length.

1) Bridge truss
2) Roof truss

(1) Bridge Truss :

1. Pratt bridge truss :-

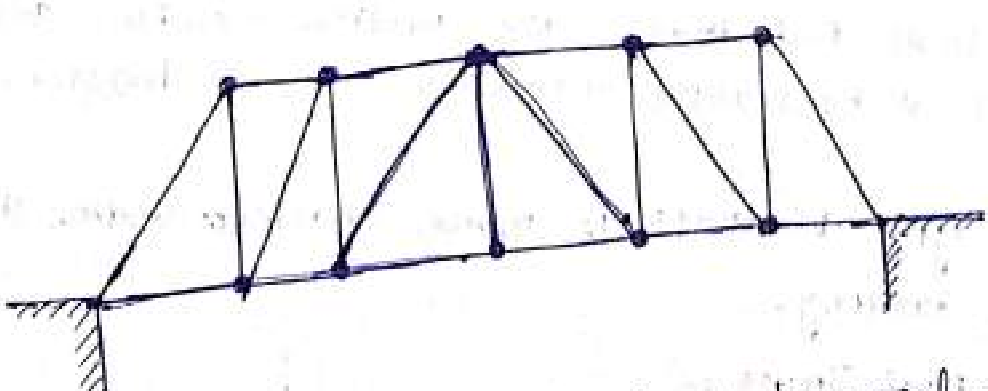


→ It includes vertical members & diagonal members.

→ Diagonal members are sloping downwards.

→ Diagonal members are subjected to tension & vertical members are subjected to compression.

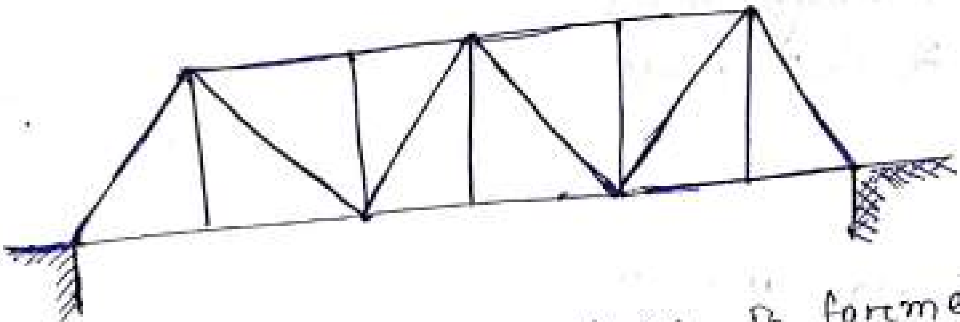
2. Howe Bridge Truss :



→ It includes vertical members & diagonal members.

→ Diagonal members are sloping upwards.

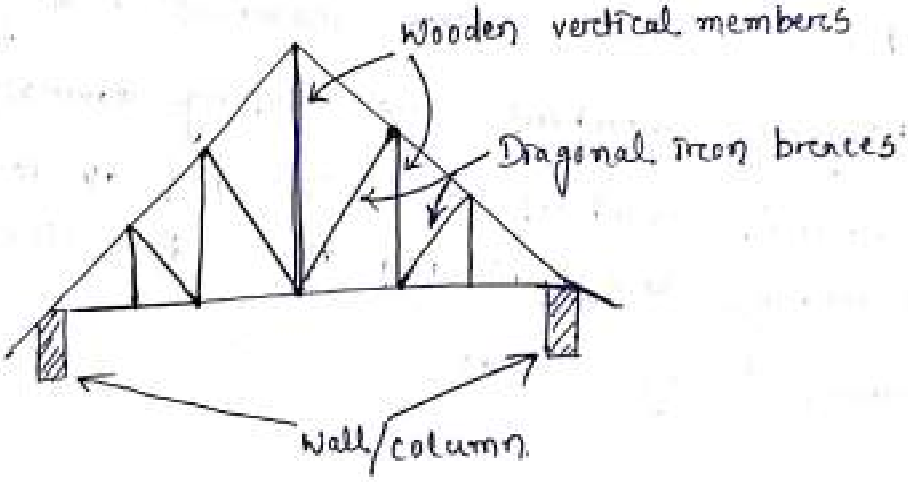
3. Warren Bridge Truss



→ In warren bridge truss, shape is formed by alternate inverted equilateral triangle shapes along its length.

2. Roof Truss

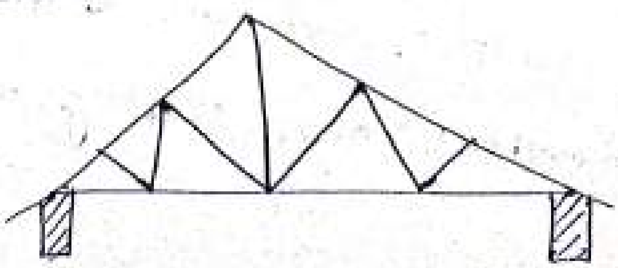
1. Pratt Roof truss :->



→ In Pratt Roof truss, the vertical members are in compression & horizontal members are in tension.

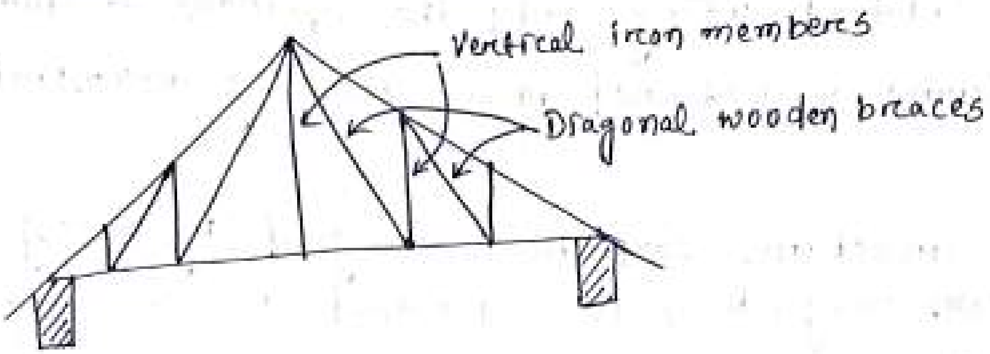
→ This type of truss is more efficient under static & vertical loading.

2. Fink Roof Truss :-



→ This types of truss is used for longer span.

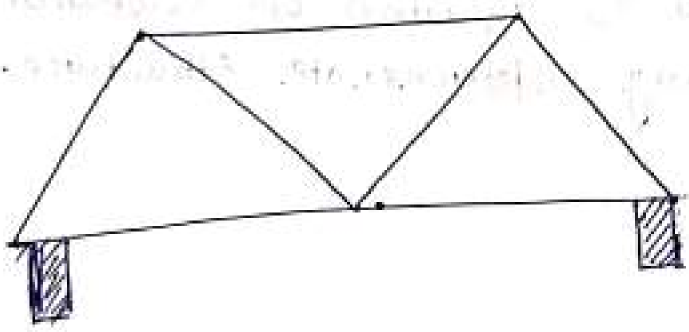
3. Howe Roof Truss:



→ In Howe Roof truss, the vertical members are in tension & horizontal members are in compression.

→ It is also more efficient.

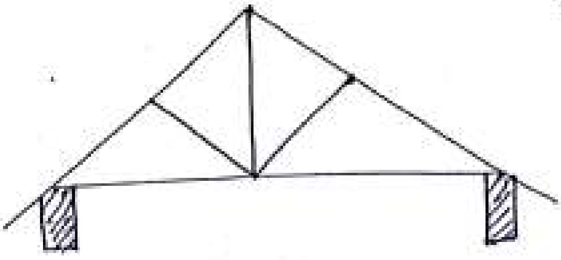
4. Warren Roof Truss:



→ In warren roof truss, diagonal members are in tension & compression alternatively.

→ It is used in building, normally of span length 20m-100m.

5. King Post Truss:



→ King post roof truss is used for simple roof truss.

→ It is the simplest form of truss in that it is constructed of the fewest number of truss members.

Static Determinacy

Statically Determinate truss:- When all the forces in a structure can be obtained using only the equations of static equilibrium, the structure is referred to as statically determinate.

or,

If a structure can be analysed by using equations of equilibrium only, then it is termed as statically determinate structure.

Statically Indeterminate truss:- If the unknown reactions cannot be determined or found simply by the equation equilibrium, the structure is referred to as statically indeterminate.

or,

If a structure is analysed by using equation of equilibrium as well as equation of compatibility, is termed as statically indeterminate structure.

If

$m = 2j-3$ → Statically determinate truss or Perfect truss or stable truss
$m > 2j-3$ → Statically Indeterminate truss or Redundant truss
$m < 2j-3$ → Deficient/Unstable truss

where,

m → no. of members

j → no. of joints

Degree of Indeterminacy (D_i)

The total no. of redundant forces in the structure/truss is called degree of Indeterminacy.

→ It is of two types

1. Degree of static Indeterminacy (D_s)
2. Degree of Kinetic Indeterminacy (D_K)

The degree of indeterminacy mainly static indeterminacy is calculated using the formula = $m-(2j-3)$.

Calculation of Indeterminacy and stability of truss

Problem-1

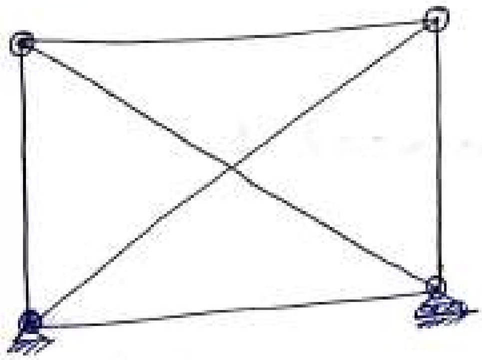
Calculate the degree of indeterminacy for the given truss.

$D_s = m-2j-3$, here, $m < 2j-3$

$m = 4$, $j = 4$ $\therefore D_s = 4-(2\times4-3) = -1$

$\therefore$ It is a defficient or unstable truss with degree of static indeterminacy -1.

2.

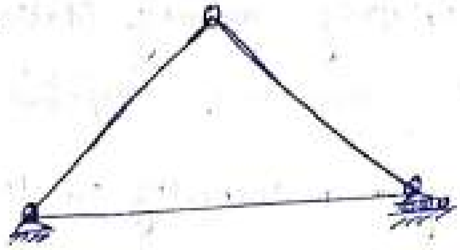


$m = 6$, $j = 4$ $\therefore D_s = 6-(2\times4-3) = 1$

here, $m > 2j-3$

$\therefore$ It is a Redundant / statically indeterminate truss having degree of static indeterminacy 1.

3.



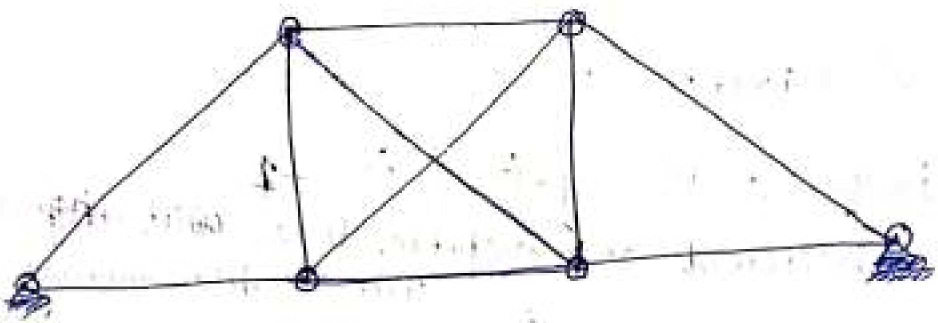
$m = 3, j = 3$

$D_s = 3 - (2\times3 - 3) = 0$

Here, $m = 2j-3$

$\therefore$ It is a Perfect/statically determinate/stable truss.

4.



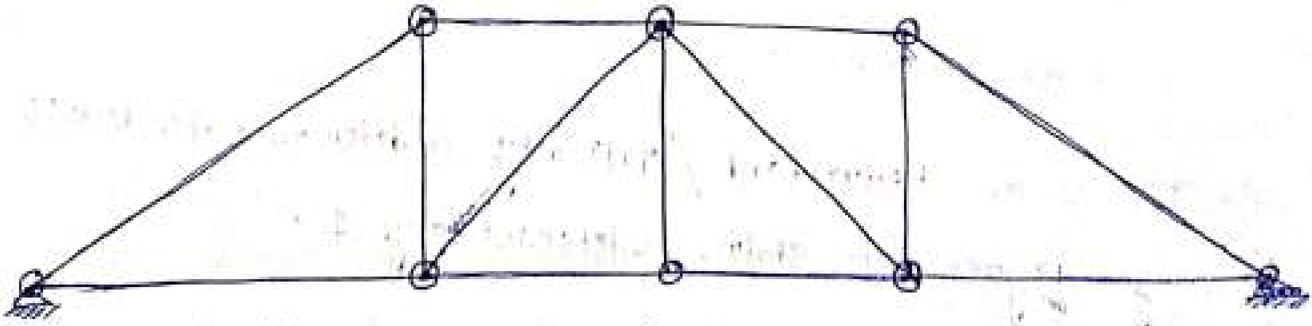
$m = 10$
$j = 6$

$D_s = 10 - (2\times6-3) = 1$

Here, $m > 2j-3$

$\therefore$ It is a Redundant or statically indeterminate beam.

5.



$m = 13$
$j = 8$

$D_s = 13 - (2\times8-3)$
$= 0$

Here $m = 2j-3$

$\therefore$ It is a Perfect/statically determinate/stable truss.

Advantages of trusses:-

→ Light weight, hence cost effective.

→ Suitable for long and high spans

→ Easy to assemble and less time taking.

→ Flexibility to assembling, dismantling & Re-use.

→ Suitable for dynamic loading & thermal effect.

→ Reduced deflection as compared to single member equivalent structure.

Disadvantages of trusses:-

→ Require more space.

→ Higher maintenance cost

→ Require engineered fabrication & skilled manpower for assembling.

→ Geometrically less stable.

→ Not suitable for multistorey building.

Uses of Trusses

1. Bridges
2. Roofs
   - → Airport Terminals
   - → Aircraft Hangers
   - → Sport Stadiums
   - → Auditoriums
   - → Industrial Buildings
   - → Garrage & ware house
3. Power Pylons & Electric Transmission
4. Cranes and weight lifting equipments

Analysis of Trusses: Truss analysis means determining the reactions and member forces.

→ Trusses can be analysed by two methods.

1. Analytical Method 2. Graphical Method.

1. **Analytical Method**: The method in which the parts of the trusses are analysed by drawing it's free body diagram and nature of forces.

In this method, the truss can be analysed by two methods.

1. Method of joints
2. Method of sections

2. **Graphical Method**: The method in which truss is analysed rapidly and in a simple way, that is called graphical method.

→ This method is used for getting rapid solution.

→ In this method, the space diagrams, vector diagrams and force tables are made to get the solution.

→ Comparison to analytical method, this method is less accurate because precision in diagrams is required and they should be drawn to proper scale.

Assumptions for analysing a perfect frame:

The following assumptions are made while computing the forces in the members of a perfect frame:

1. All the members are pin jointed.
2. The truss is loaded only at the joints.
3. The truss is a perfect truss.
4. The self-weight of the members is neglected.
5. The members of a truss are straight two-force members with the forces acting co-linear with the centre line of the members.

Method of Joints

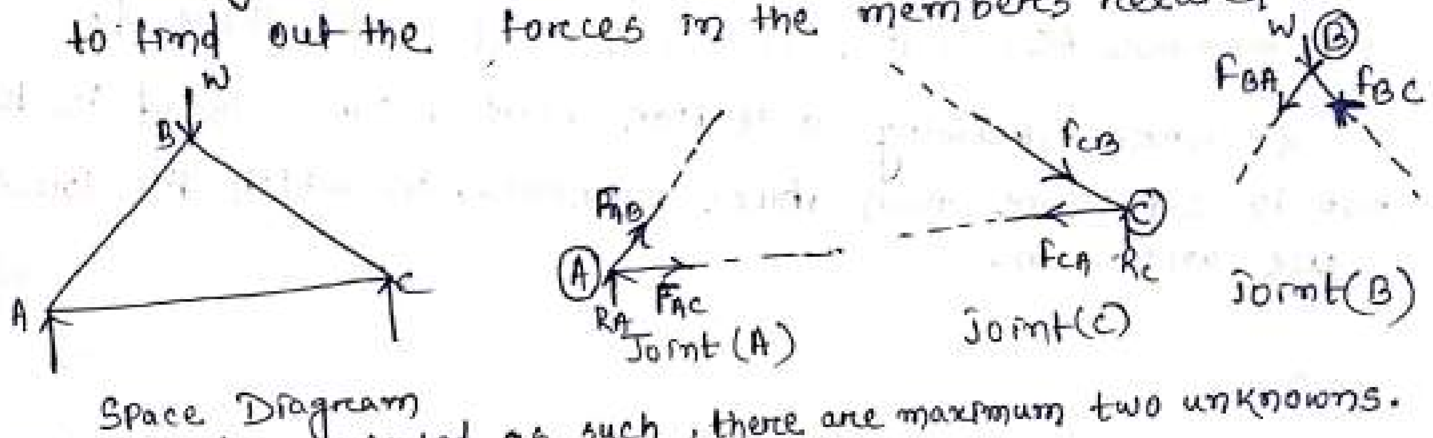
This type of method is used when we have to find out the forces in all members of the truss.

Steps :-

→ In this method, each and every joint is treated as a free body in equilibrium as shown in figure.

→ The unknown forces are then determined by equilibrium equations i.e. $\Sigma V = 0$ and $\Sigma H = 0$ i.e., sum of all vertical forces and horizontal forces is equated to zero.

→ According to requirement, various joints are analysed to find out the forces in the members related to it.



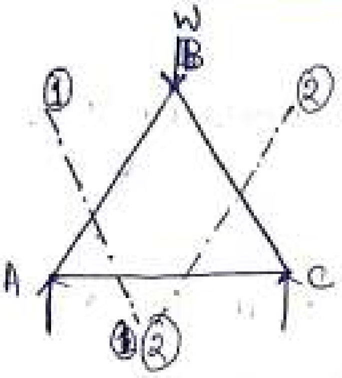
Note:- Joint should be selected as such, there are maximum two unknowns.

Method of Sections or Method of moments

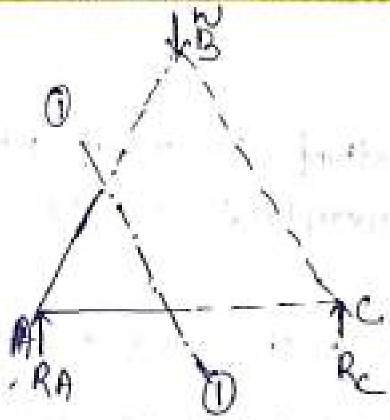
This method is used, when the forces in a few members of a truss are required to be found out.

→ In this method, a section line is passed through the member or members, in which the forces are required to be found out as shown in figure.

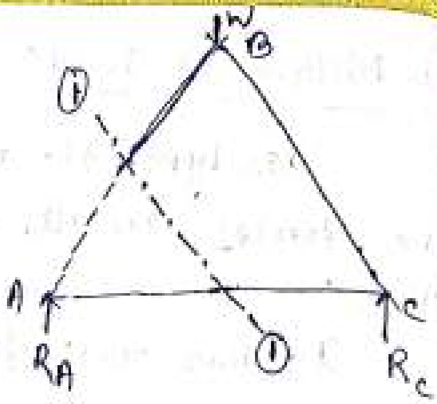
→ A part of the structure, on any one side of the section line, is then treated as a free body in equilibrium under the action of external forces as shown in figure.



Space diagram



Left Part



Right part

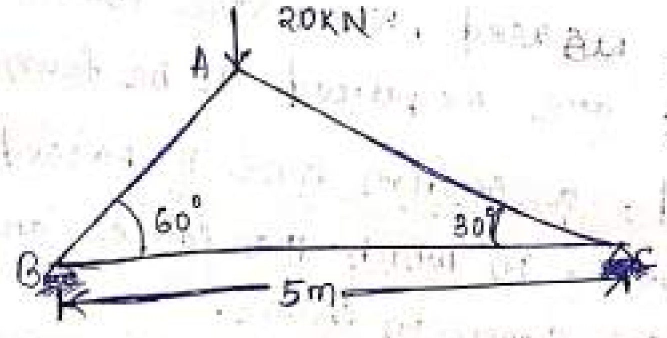
→ The unknown forces are then found out by the application of equilibrium, or the principle of statics i.e. $\Sigma M = 0$.

Note:- 1. To start with, we have taken section line 1-1, cutting the members AB and BC. Now in order to find out the forces in the member AC, section line 2-2 may be considered.

2. While drawing a section line, care should be taken not to cut more than three members, in which the forces are unknown.

Method of Joints (Problems)

1. Find the forces in members AB, AC and BC of the given truss.

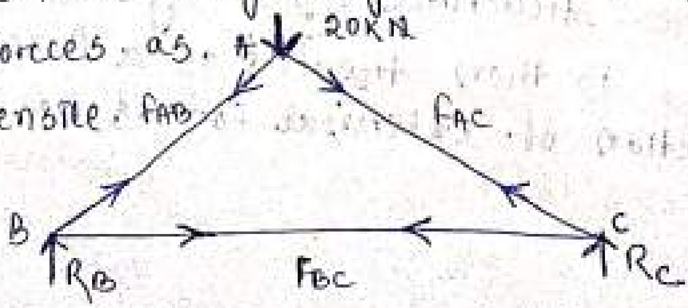


Assumption

Joint • ⟶ Tension

Joint • ⟵ Compression

Drawing the free body diagram of the given truss assuming the nature of forces as Compressive/Tensile.



R_B, R_C

Taking moment about 'B'

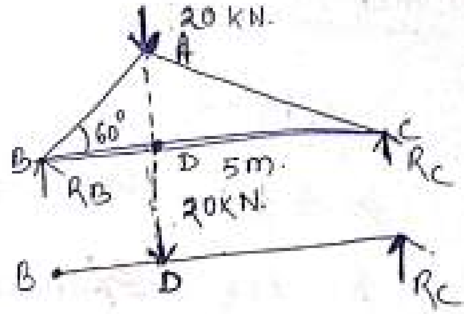
$R_c \times 5 = 20 \times BD$

$\Rightarrow R_c \times 5 = 25$

$\Rightarrow R_c = 5\,KN.$

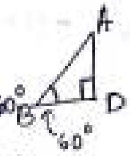
$R_B + R_C = 20\,KN.$

$\Rightarrow R_B = 20 - 5 = 15\,KN.$



In ΔABC

$\cos 60^\circ = \frac{AB}{BC}$



$\Rightarrow AB = BC \cos 60^\circ$

$\Rightarrow AB = 5/2 = 2.5m$

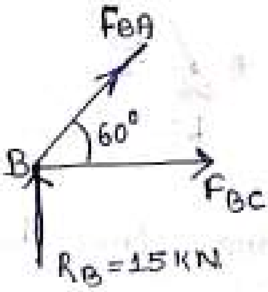
$\cos 60^\circ = \frac{BD}{AB}$

$\Rightarrow BD = AB \cos 60^\circ$

$\Rightarrow BD = 1.25m.$

Member forces

Joint 'B'



$\Sigma F_H = 0$ or $\Sigma F_x = 0$

$\Rightarrow F_{BC} + F_{BA} \cos 60^\circ = 0$

$\Rightarrow F_{BC} = -F_{BA} \cos 60^\circ$

$\Rightarrow F_{BC} = -F_{BA}/2$ ---- (1)

$\Sigma F_y = 0$ or $\Sigma F_V = 0$

$\Rightarrow R_B - F_{BA} \sin 60^\circ = 0$

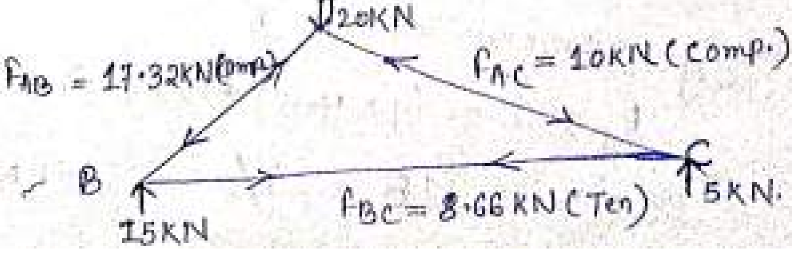
$\Rightarrow 15 = F_{BA} \sin 60^\circ$

$\Rightarrow F_{BA} = -\frac{15 \times 2}{\sqrt{3}}\,KN = 17.32\,KN$ (comp.)

Putting value of F_{BA} in eqn (1)

$F_{BC} = -F_{BA}/2 = +\frac{15 \times \cancel{2}}{\sqrt{3}} \times \frac{1}{\cancel{2}} = +\frac{15}{\sqrt{3}}\,KN = 8.66\,KN$ (Tens)

Final Drawing
Showing the nature of forces with their values,



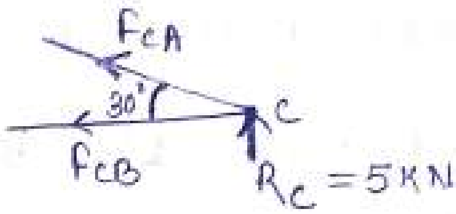
Joint 'c'

$\Sigma f_y = 0$

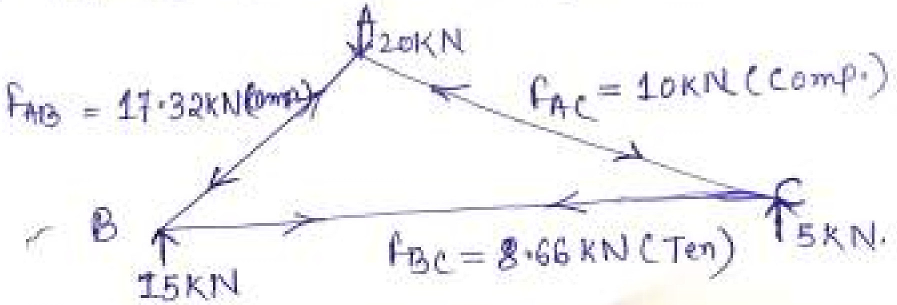
$\Rightarrow 5 + f_{CA} \sin 30^\circ = 0$

$\Rightarrow f_{CA} = -5 \times \sin 30^\circ$

$\Rightarrow f_{CA} = -5 \times 2 = 10\, KN$ (Comp.)

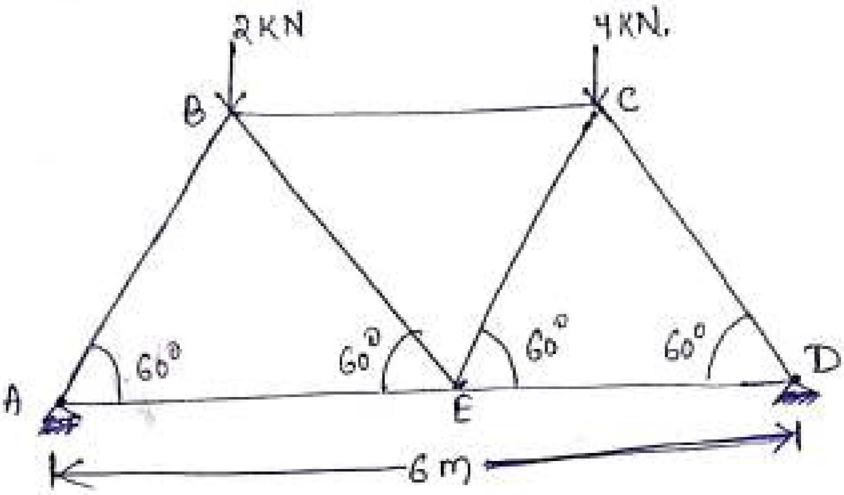


Final Drawing showing the nature of forces with their values,

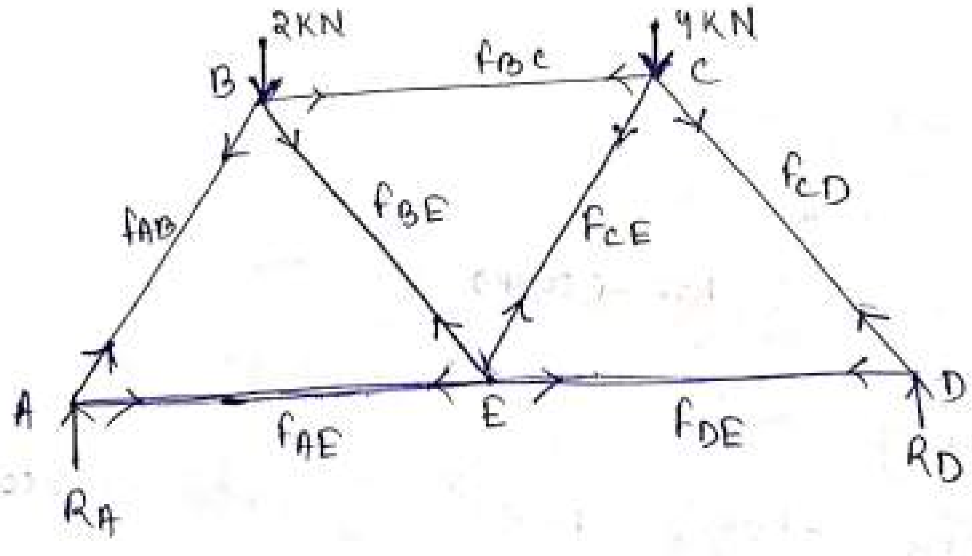


Problem-2

Calculate the forces in all the members of the given truss.



Drawing the free member body diagram of the given truss assuming the nature of forces as tensile.



Calculation of R_A & R_D

Taking moment about 'A'

$$R_D \times 6 = (4 \times 4.5) + (2 \times 1.5)$$

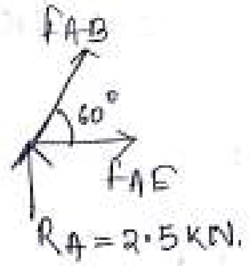
$$\Rightarrow R_D = (18 + 3)/6 = 3.5\,KN.$$

$R_A + R_D = 2+4$

$\Rightarrow R_A = 6 - 3.5 = 2.5\,KN.$

Member forces

Joint 'A'



$\Sigma H = 0$

$\Rightarrow F_{AB}\cos 60^\circ + F_{AE} = 0$

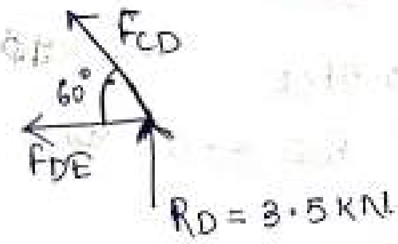
$\Sigma V = 0$

$\Rightarrow F_{AB}\sin 60^\circ + 2.5 = 0$

$\Rightarrow F_{AB} = -2.5/\sin 60^\circ = -2.887KN = 2.887KN$ (Comp.)

$\therefore F_{AE} = -F_{AB}\cos 60^\circ = +2.88\cos 60^\circ = 1.444KN$ (Tension)

Joint 'D'



$\Sigma H = 0$

$\Rightarrow F_{CD}\cos 60^\circ + F_{DE} = 0$

$\Rightarrow F_{DE} = -F_{CD}/2$

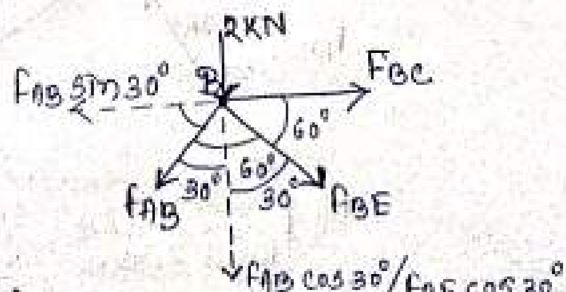
$\Sigma V = 0$

$\Rightarrow F_{CD}\sin 60^\circ + 3.5 = 0$

$\Rightarrow F_{CD} = -3.5/\sin 60^\circ = -4.042KN = 4.042KN$ (comp)

$\therefore F_{DE} = -F_{CD}/2 = +4.042/2 = 2.021KN$ (Tension)

Joint 'B'



$\Sigma H = 0$

$\Rightarrow F_{AB}\sin 30^\circ - F_{BE}\cos 60^\circ - F_{BC} = 0$

$\Rightarrow -2.887\sin 30^\circ - F_{BE}/2 - F_{BC} = 0$

$\Rightarrow 0.5F_{BE} + F_{BC} = -1.444$ ---- (i)

$\Sigma V = 0$

$\Rightarrow F_{AB}\cos 30^\circ + F_{BE}\cos 30^\circ + 2 = 0$

$\Rightarrow -2.887 \times \frac{\sqrt{3}}{2} + F_{BE} \times \frac{\sqrt{3}}{2} + 2 = 0$

$\Rightarrow F_{BE} \times \frac{\sqrt{3}}{2} = 0.5$

$\Rightarrow F_{BE} = 0.577 KN$ (Tension)

Putting F_{BE} value in eq^n (1)

$0.5 F_{BE} + F_{BC} = -1.444$

$\Rightarrow 0.5 \times 0.577 + F_{BC} = -1.44$

$\Rightarrow F_{BC} = -1.732$ KN $= 1.732$ KN (comp.)

joint 'C'

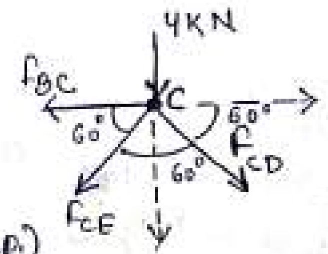
$\Sigma H = 0$

$F_{BC} + F_{CE}\cos 60^\circ - F_{CD}\cos 60^\circ = 0$

$\Rightarrow -1.732 + 0.5 F_{CE} - (-4.042 \times 0.5) = 0$

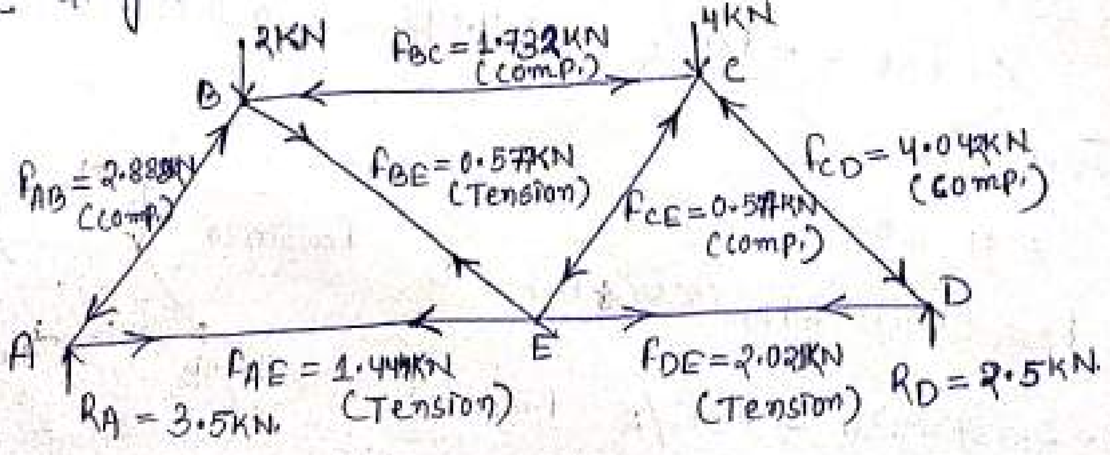
$\Rightarrow 0.5 F_{CE} = -0.289$

$\Rightarrow F_{CE} = -0.578 KN = 0.578 KN$ (comp.)



final Diagram

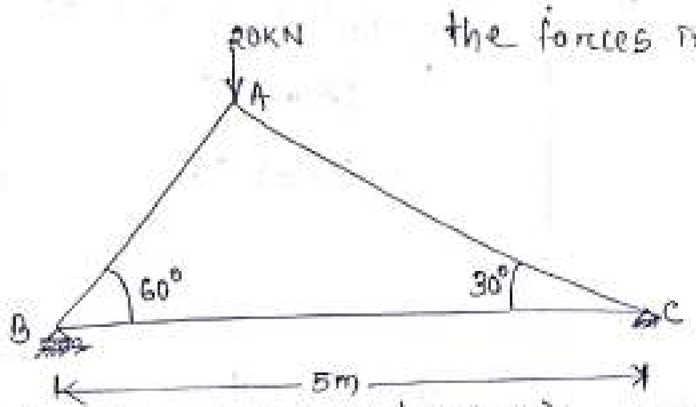
Showing the nature of the forces with their values, the final diagram of truss will be.



Method of sections (Problems)

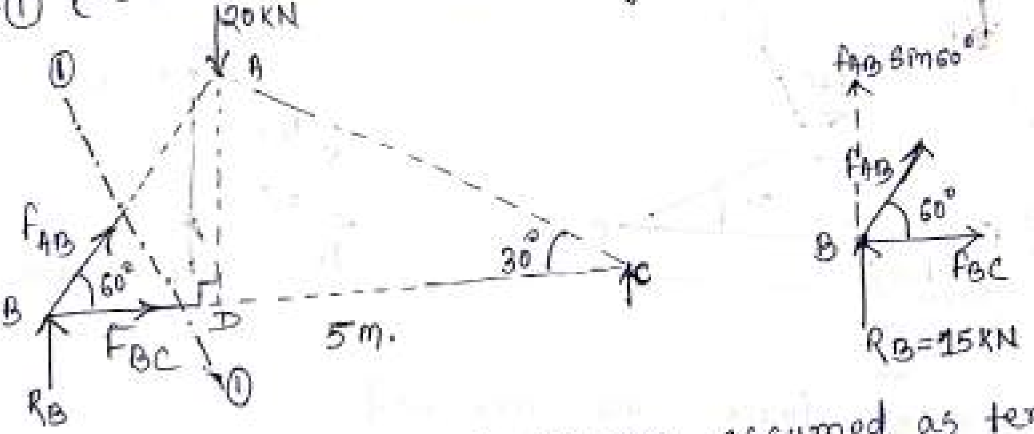
Problem-1

Find the forces in the members AB & BC for the given truss / find the forces in the members AC & BC.



Soln – (find out R_B & R_C as discussed before)

Drawing a section line through the members AB & BC and making the free body diagram of the left part of section line ①-① (as it is smaller than the right part)



Here, the f_{AB} & f_{BC} forces are assumed as tensile.

Taking moments of the forces acting in the left part of the truss only about the joint 'c' and equating the same,

$(f_{AB}\sin 60° \times 5) + (R_B \times 5) = 0$

$\Rightarrow f_{AB}\sin 60° \times \cancel{5} = -R_B \times \cancel{5}$

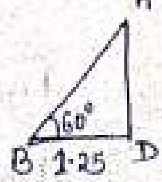
$\Rightarrow f_{AB} = -15/\sin 60° = -17.32\,KN$ (comp.)

Then, taking moments of the forces acting in the left part of the truss only about the joint 'A' and equating the same,

$F_{BC} \times AD - R_B \times 1.25 = 0$ → (f_{AB} component is not considered here, as it is connected to 'A')

$\Rightarrow f_{BC} \times 1.25\tan 60° - R_B \times 1.25 = 0$

$\Rightarrow F_{BC} = \dfrac{R_B \times 1.25}{1.25\tan 60°} = \dfrac{15 \times 1.25}{1.25 \times \tan 60°} = \dfrac{15}{\tan 60°}$



$\tan 60° = \dfrac{AD}{BD}$

$\Rightarrow AD = BD\tan 60°$

$= 1.25\tan 60°$

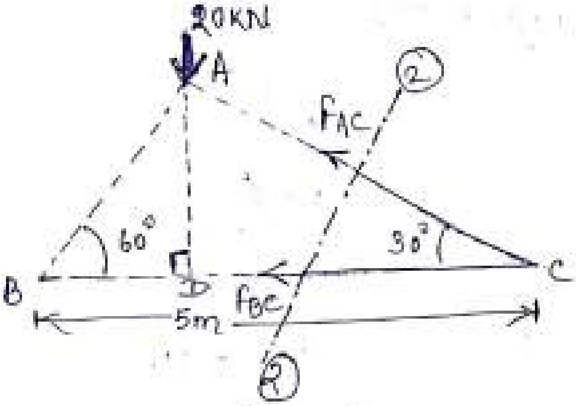
$\Rightarrow f_{BC} = 8.66\,KN$ (Ten.)

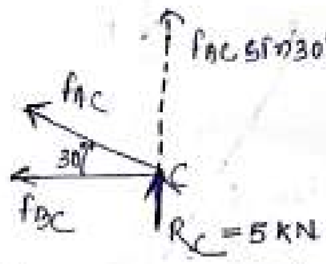
Now tabulating the results

Sl. No.	Member	Magnitude of force in KN.	Nature of force.
1	AB	17.32 KN	Compression
2	BC	8.66 KN	Tension

or,

Drawing a section line through the members AC & BC and making the free body diagram of the right part of the section line 2-2 (as it is smaller than left part).





Here, the F_{AC} & F_{BC} forces are assumed as tensile.

Taking moments of the forces acting in the right part of the truss only about the joint 'B' and equating the same,

$$F_{AC}\sin 30° \times 5 + R_C \times 5 = 0$$

$$\Rightarrow F_{AC} = -R_C/\sin 30° = -10\text{ KN} = 10\text{ KN (comp.)}$$

Then, taking moments of the forces acting in the right part of the truss only about the joint 'A' and equating the same,

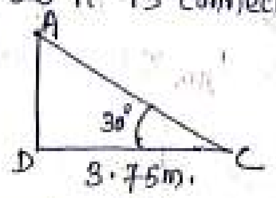
$$-F_{BC} \times AD + R_C \times CD = 0$$ → (F_{AC} component is not considered here, as it is connected to 'A')

$$\Rightarrow -F_{BC} \times 3.75\tan 30° + R_C \times 3.75 = 0$$

$$\Rightarrow F_{BC} \times 3.75\tan 30° = R_C \times 3.75$$

$$\Rightarrow F_{BC} = R_C/\tan 30° = 5/\tan 30°$$

$$= 8.66\text{KN (Ten)}$$



$$\tan 30° = \frac{AD}{CD}$$

$$\Rightarrow AD = CD \tan 30°$$

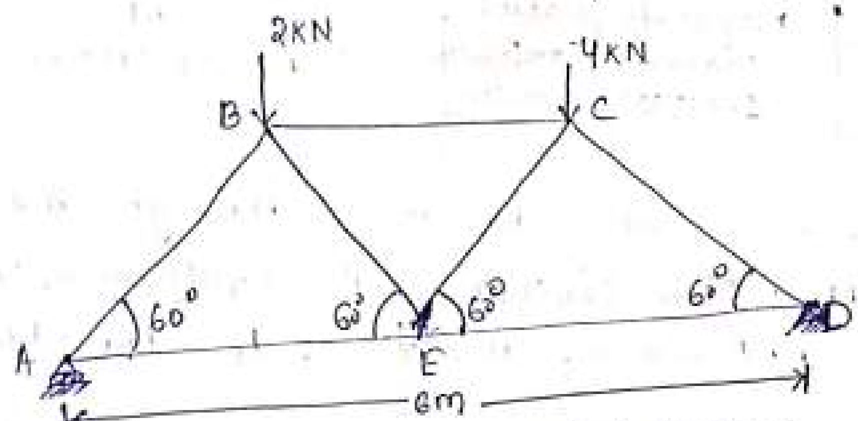
$$\Rightarrow AD = 3.75 \tan 30°$$

Now tabulating the results.

Sl. No.	Member	Magnitude of force in KN.	Nature of force.
1.	AC	10 KN.	compression
2	BC	8.66KN.	Tension

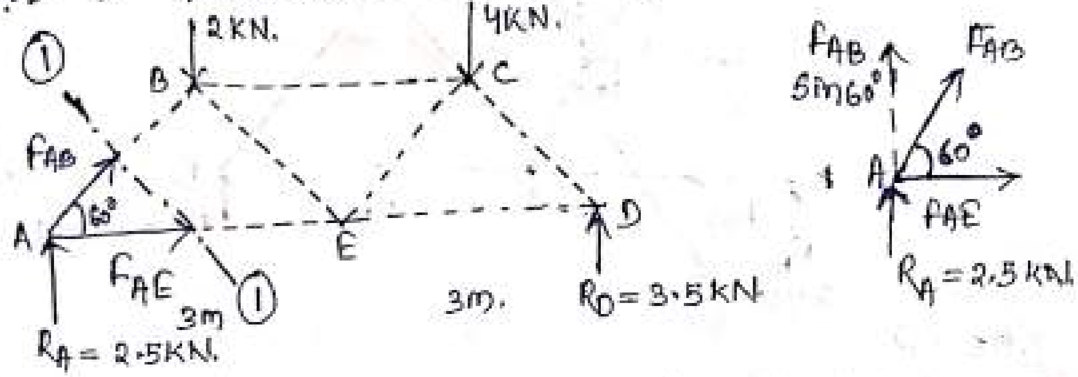
Problem-2

Find the forces in the members AB & AE/ BC, BE & AE/BC, CE & ED/CD & DE



Soln:- find R_A & R_D as calculated before.

Passing a section (1-1) cutting the truss through members AB & AE. Now considering the equilibrium of the left part of the truss & directions of F_{AB} & F_{AE} as tensile.



Taking moments of the forces acting in the left part of the truss only, about the joint 'E' and equating the same,

$$F_{AB} \sin 60^\circ \times 3 + R_A \times 3 = 0$$ ('F_{AE} component is not considered as it is connected to joint 'E')

$$\Rightarrow F_{AB} = -R_A/\sin 60^\circ = -2.5/\sin 60^\circ = -2.887 KN$$
$$= 2.887 KN \text{ (comp.)}$$

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Then, taking the moments of the forces in the left part of the truss only, about joint 'B' and equating the same,

$$F_{AE} \times BF = R_A \times 1.5$$ (F_{AB} component is not considered as it is connected to joint 'B')

$$\Rightarrow F_{AE} \times \cancel{1.5}\tan 60^\circ = 2.5 \times \cancel{1.5}$$

$$\Rightarrow F_{AE} = 2.5/\tan 60^\circ = 1.443 KN \text{ (Ten)}$$

$$\tan 60^\circ = \frac{BF}{AF}$$

$$\Rightarrow BF = 1.5 \tan 60^\circ$$

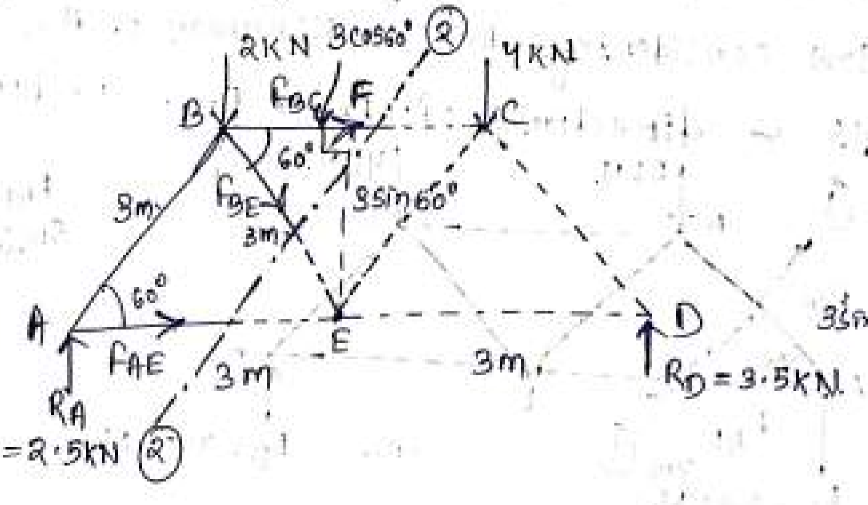
Tabulating the results;

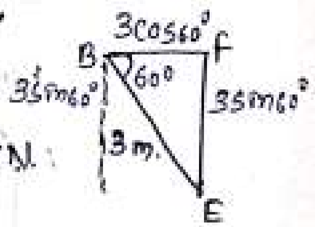
Sl. no.	members	Magnitude	Nature
1	AB	2.887 KN	Compression
2	AE	1.443 KN	Tension

or,

Passing a section (2-2) cutting the truss through the members BC, BE & AE. Now considering the equilibrium of the left part of the truss. Assuming the forces P_{BC}, P_{BE} & P_{AE} as tensile.

Now taking moments of the forces acting in left part of the truss only, about joint 'E' and equating the same,

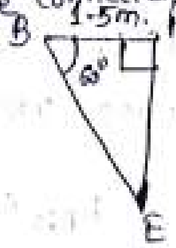




$\Sigma M_E = 0$

or,

$$-F_{BC} \times 1.5 \tan 60^\circ - 2.5 \times 3 + 2 \times 1.5 = 0$$ (F_{AE} & F_{BE} is not considered here as they are connected to joint 'E')



$$\Rightarrow F_{BC} \times 1.5 \tan 60^\circ = -7.5 + 3$$

$$\Rightarrow F_{BC} \times \cancel{1.5} \tan 60^\circ = -\cancel{4.5}\ 3$$

$$\Rightarrow F_{BC} = -3/\tan 60^\circ = -1.732 KN$$

$$= 1.732 KN \text{ (comp.)}$$

$$\tan 60^\circ = \frac{EF}{BF}$$

$$\Rightarrow EF = 1.5 \tan 60^\circ$$

$$= 2.6 m.$$

$\Sigma F_x = 0$

$\Rightarrow F_{AE} + F_{BC} + F_{BE} \cos 60^\circ = 0$ ...... (1)

$\Sigma F_y = 0$

$\Rightarrow R_A - 2 - F_{BE} \sin 60^\circ = 0$

$\Rightarrow F_{BE} = (2.5 - 2)/\sin 60^\circ$

$\Rightarrow F_{BE} = 0.577 \text{ KN (Ten.)}$

Putting the value of F_{BE} & F_{BC} in eqn (1)

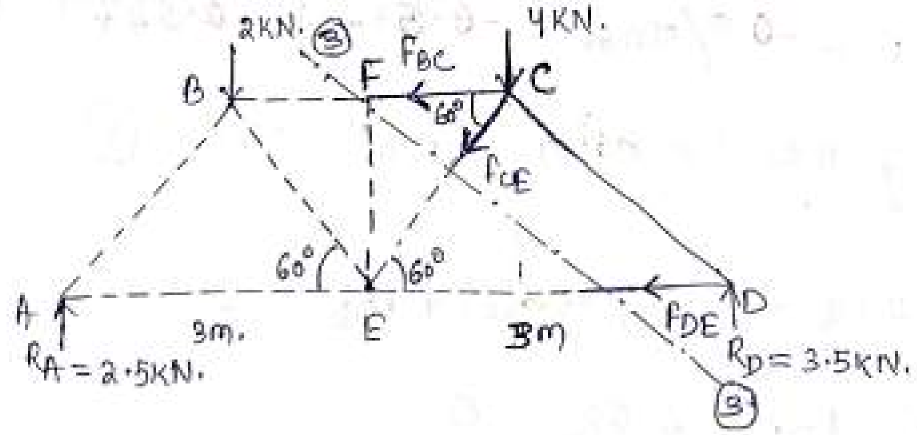
$F_{AE} - 1.732 + 0.577 \cos 60^\circ = 0$

$\Rightarrow F_{AE} = 1.443 \text{ KN (Ten.)}$

Tabulating the results,

Sl. No.	Members	Magnitude	Nature
01	AE	1.443 KN	Tension
02	BC	1.732 KN	Compression
03	BE	0.577 KN	Tension

OR,



Passing a section (3-3) cutting the truss through the members BC, CE, DE. Now considering the equilibrium of the right part of the truss. Let the directions of the forces F_{BC}, F_{CE} & F_{DE} be assumed as tensile.

Taking moments of the forces in the right part of the truss only, about the joint 'E' & equating the same,

$\Sigma M_E = 0$

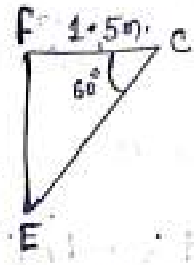
$\Rightarrow F_{BC} \times 2.598 - (4 \times 1.5) + R_D \times 3 = 0$

$\Rightarrow F_{BC} \times 2.598 - 6 + (3.5 \times 3) = 0$

$\Rightarrow F_{BC} \times 2.598 + 4.5 = 0$

$\Rightarrow F_{BC} = -4.5/2.598 = -1.732\ KN$

$= 1.732\ (\text{comp.})$



$\tan 60° = \frac{EF}{CF}$

$\Rightarrow EF = CF \tan 60° = 1.5 \tan 60°$

$= 2.598m.$

$\Sigma F_x = 0$

$-F_{BC} - F_{CE} \cos 60° - F_{DE} = 0$

$\Rightarrow F_{BC} + F_{CE} \cos 60° + F_{DE} = 0$ ------ (1)

$\Sigma F_y = 0$

$\Rightarrow R_D - 4 - F_{CE} \sin 60° = 0$

$\Rightarrow 3.5 - 4 - F_{CE} \sin 60° = 0$

$\Rightarrow F_{CE} \sin 60° = -0.5$

$\Rightarrow F_{CE} = -0.5/\sin 60° = -0.577\ KN = 0.577\ KN\ (\text{Comp.})$

Putting the value of F_{CE} & F_{BC} in eqn -- (1)

$-1.732 - 0.577 \cos 60° + F_{DE} = 0$

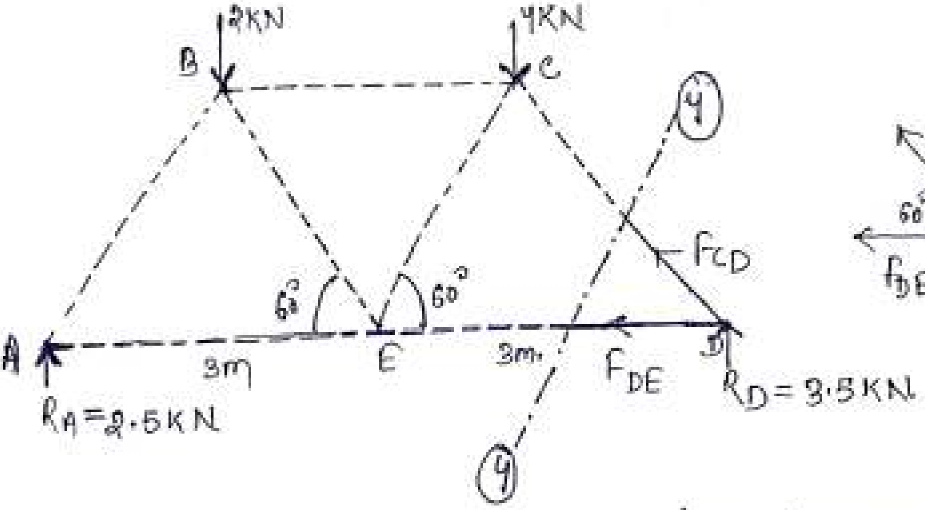
$\Rightarrow F_{DE} - 2.02 = 0$

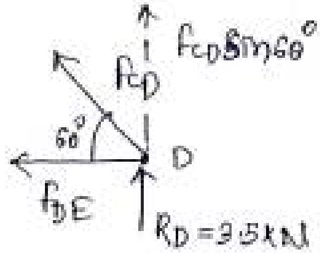
$\Rightarrow F_{DE} = 2.02\ KN\ (\text{Ten})$

Tabulating the results,

Sl. NO.	Members	Magnitude	Nature.
1	BC	1.732 KN	Compression
2.	CE	0.577 KN	Compression
3	DE	2.02 KN	Tension

or, Passing a section (4-4) cutting the truss through members CD & DE. Now considering the equilibrium of the right part of the truss & directions of F_{CD} & F_{DE} as tensile.





$\Sigma F_x = 0$

$\Rightarrow -F_{DE} - F_{CD}\cos 60° = 0$

$\Rightarrow F_{DE} + F_{CD}\cos 60° = 0$ ---- (1)

$\Sigma F_y = 0$

$\Rightarrow R_D + F_{CD}\sin 60° = 0$

$\Rightarrow F_{CD} = -R_D/\sin 60° = -3.5/\sin 60° = -4.041$ KN $= 4.041$ KN (comp)

or, taking moment about 'E', the forces in the CD & DE members can be calculated (as before)

Putting the value of F_{CD}, in eqn (1)

$F_{DE} + F_{CD}\cos 60° = 0$

$\Rightarrow F_{DE} + (-4.041) \times \frac{1}{2} = 0$

$\Rightarrow F_{DE} = 2.02$ KN (Ten.)

Tabulating the results,

Sl. No.	Members	Magnitude	Nature.
1	CD	4.041 KN	Compression
2	DE	2.02 KN	Tension