

* Basic Trigonometric Formulae:

1) $\sin^2 \theta + \cos^2 \theta = 1$
 $\sec^2 \theta - \tan^2 \theta = 1$
 $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

2) $\sin(A+B) = \sin A \cos B + \cos A \sin B$
 $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

3) $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

$\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$

$\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$

$\left(\frac{\sin \theta}{\cos \theta}\right) \cos \theta = \sin \theta$

$$4) \sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B \quad \& \quad \cos^2 B - \cos^2 A$$

$$\cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B \quad \& \quad \cos^2 B - \sin^2 A$$

$$5) \sin 2A = 2 \sin A \cdot \cos A \quad \& \quad \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A \quad \& \quad 1 - 2 \sin^2 A \quad \& \quad 2 \cos^2 A - 1 \quad \& \quad \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$10) \cot \frac{A}{2} = \frac{1 + \cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$$

$$6) 1 + \cos 2A = 2 \cos^2 A$$

$$1 - \cos 2A = 2 \sin^2 A$$

$$1 + \sin 2A = (\cos A + \sin A)^2$$

$$1 + \sin 2A = (\cos A - \sin A)^2$$

$$11) \frac{\cos A + \sin A}{\cos A - \sin A} = \tan\left(\frac{\pi}{4} + A\right)$$

$$7) \cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

$$\frac{\cos A - \sin A}{\cos A + \sin A} = \tan\left(\frac{\pi}{4} - A\right)$$

$$\tan^2 A = \frac{\sin^2 A}{\cos^2 A} \quad \& \quad \frac{1 - \cos 2A}{1 + \cos 2A}$$

$$12) \sin C + \sin D$$

$$8) \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$= 2 \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D$$

$$9) \sin A = 2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}$$

$$\cos A = 2 \cos^2 \frac{A}{2} \cdot \sin^2 \frac{A}{2}$$

$$= 2 \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D$$

$$= 2 \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2 \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

* Derivatives:-

$$1. \frac{d}{dx} (x^n) = nx^{n-1}$$

$$13. \frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$2. \frac{d}{dx} (\sin x) = \cos x$$

$$14. \frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$3. \frac{d}{dx} (\cos x) = -\sin x$$

$$15. \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$$

$$4. \frac{d}{dx} (\tan x) = \sec^2 x$$

$$16. \frac{d}{dx} (\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$5. \frac{d}{dx} (\sec x) = \sec x \cdot \tan x$$

$$17. \frac{d}{dx} (\sec^{-1} x) = \frac{1}{(x)\sqrt{x^2-1}}$$

$$6. \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$$

$$7. \frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$$

$$18. \frac{d}{dx} (\operatorname{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$8. \frac{d}{dx} (a^x) = a^x \ln a$$

$$19. \frac{d}{dx} (k) = 0, k \in \mathbb{R}$$

$$9. \frac{d}{dx} (e^x) = e^x$$

$$20. \frac{d}{dx} (\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$10. \frac{d}{dx} (e^{-x}) = -e^{-x}$$

$$21. \frac{d}{dx} \left(\frac{1}{x}\right) = \frac{-1}{x^2}$$

$$11. \frac{d}{dx} (\log x) = \frac{1}{x}$$

$$12. \frac{d}{dx} (\log_a x) = \frac{1}{x \ln a}$$

* Algebra of Derivatives:-

$$1. \frac{d}{dx} (f(x) \pm g(x)) = \frac{d}{dx} (f(x)) \pm \frac{d}{dx} (g(x))$$

$$2. \frac{d}{dx} (k f(x)) = k \frac{d}{dx} (f(x))$$

#

$$\frac{d}{dx} (x^n) = n x^{n-1}$$

$$\frac{d}{dx} (x^{-n}) = -n x^{-n-1}$$

$$\frac{d}{dx} (x^a) = a x^{a-1}$$

$$\frac{d}{dx} (x^k) = k x^{k-1}$$

$$\frac{d}{dx} (k) = 0$$

$$\frac{d}{dx} (\tan^2 A) = 2 \tan A \sec^2 A$$

$$\frac{d}{dx} (\sin 3A) = 3 \cos 3A$$

$$\frac{d}{dx} (\cos 3A) = -3 \sin 3A$$

$$\frac{d}{dx} (\sin A) = \cos A$$

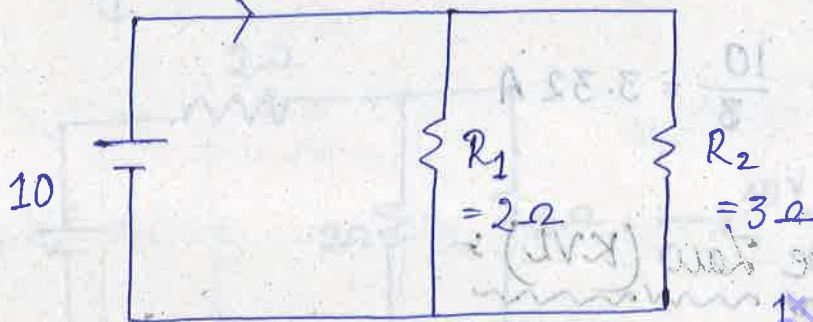
$$\frac{d}{dx} (\cos A) = -\sin A$$

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* Resistance - The property by virtue of which an element opposes the flow of current in an electric circuit is called resistance.

Unit = ohm (Ω).

$I = ?$



$$I = \frac{E}{R}$$

$E = \text{EMF}$

↓
Electromotive force

$$R = \frac{R_1 \times R_2}{R_1 + R_2}$$

→ parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{2} + \frac{1}{3}$$

$$= \frac{3+2}{6}$$

$$= \frac{5}{6}$$

$$R = \frac{6}{5}$$

$$= \frac{3 \times 2}{3 + 2}$$

$$= \frac{6}{5}$$

COR

$$I = \frac{E}{R}$$

$$= 10 \times \frac{5}{6}$$

$$= \frac{50}{6}$$

$$= 8.3 \text{ A}$$

Series Connection

$$R = R_1 + R_2$$

$$I_1 = \frac{I \times R_2}{R_1 + R_2} = \frac{8.3 \times 3}{5}$$

$$= \frac{24.9}{5} = 4.98 \text{ A}$$

$$= 5 \text{ A}$$

Parallel Connection

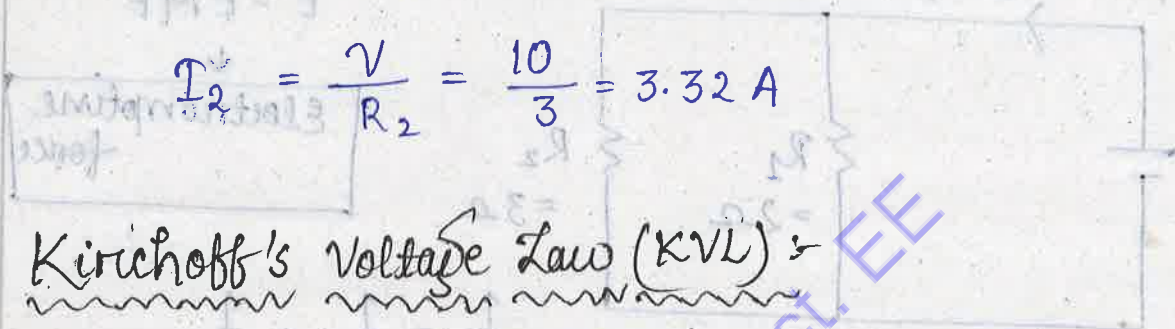
$$R = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$I_2 = \frac{I \times R_1}{R_1 + R_2} = \frac{8.3 \times 2}{5} = \frac{16.6}{5}$$

$$= 3.32 \text{ A}$$

$$I_1 = \frac{V}{R_1} = \frac{10}{2} = 5 \text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{10}{3} = 3.32 \text{ A}$$



* Kirchoff's Voltage Law (KVL) :

The summation of voltage drop across each element in a closed loop is always equal to zero is known as Kirchoff's Voltage Law.

* Kirchoff's Current Law (KCL) :

The summation of all the currents meeting at a junction is equal to zero, this is called Kirchoff's Current Law.

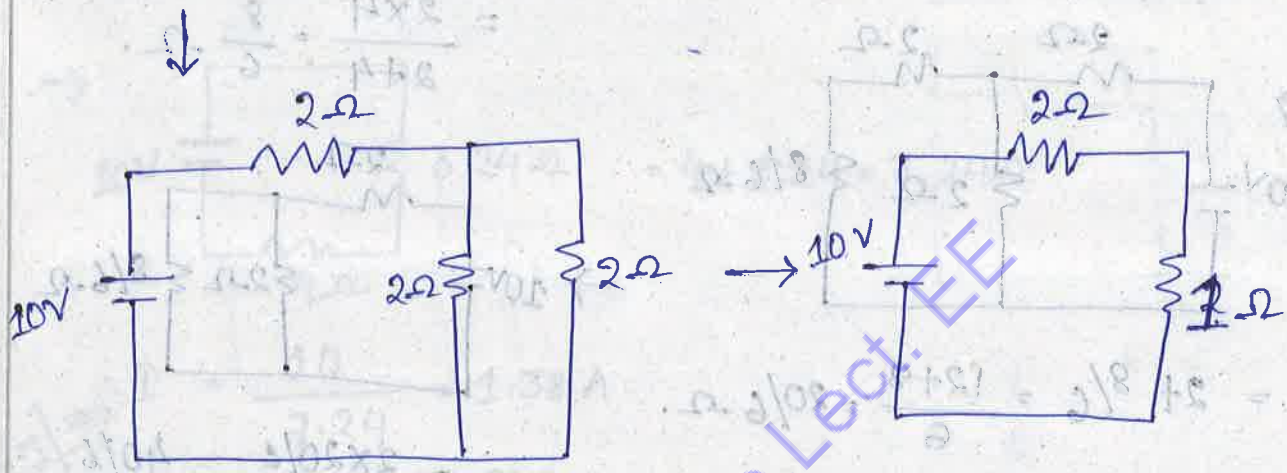
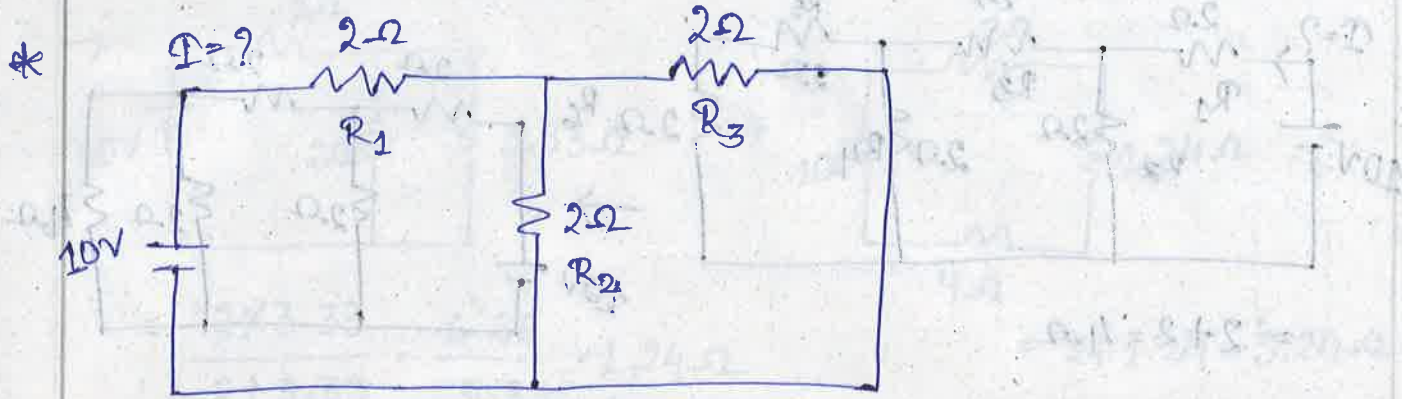
* Current Div. Formula :

Parallel

$$I_1 = \frac{I \times R_2}{R_1 + R_2}$$

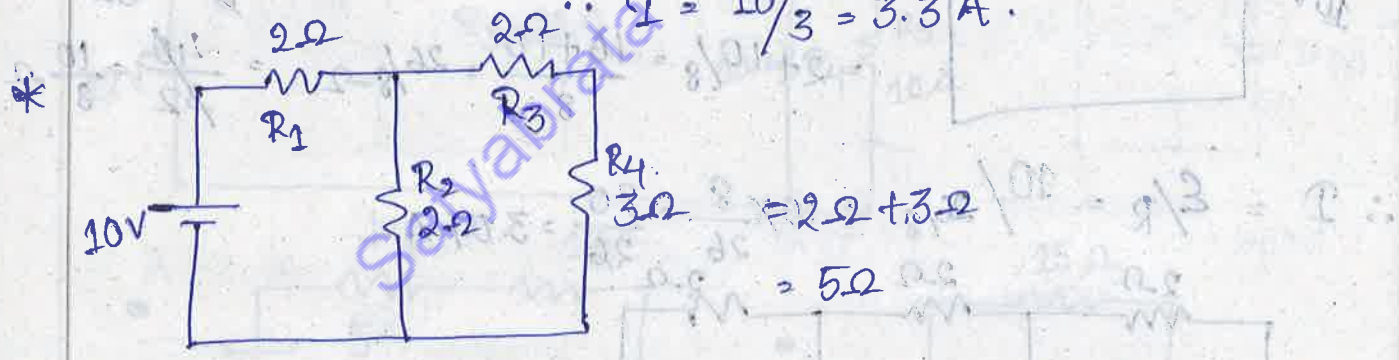
$$R = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$I_2 = \frac{I \times R_1}{R_1 + R_2}$$

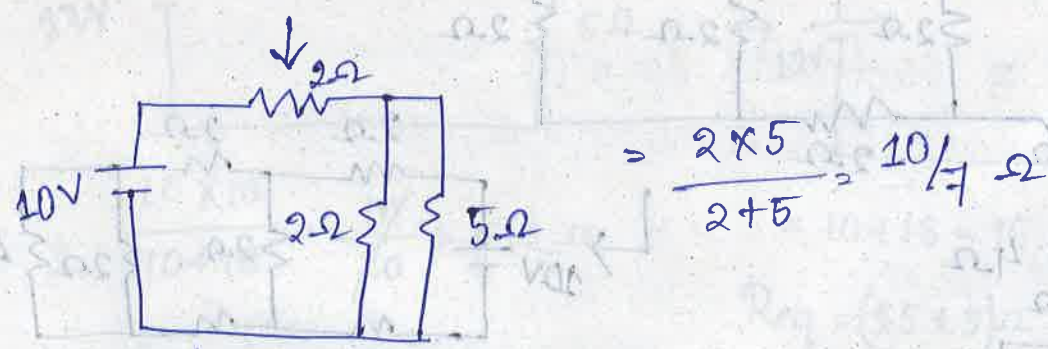


$\therefore R_{req} = 2 + 1 = 3\Omega$

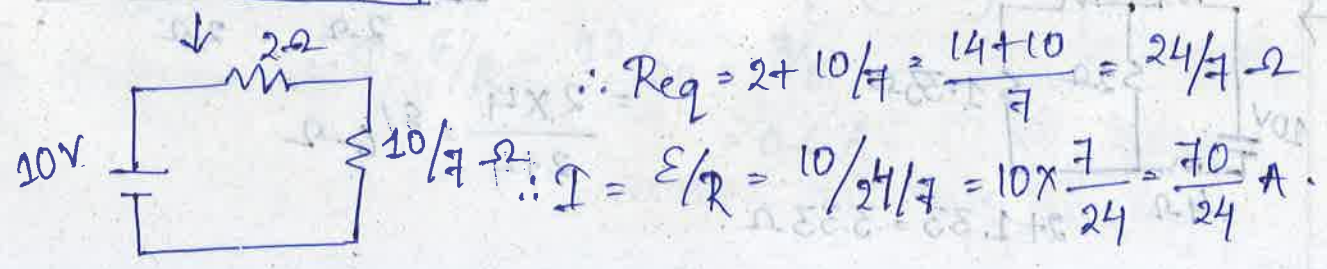
$\therefore I = \frac{10}{3} = 3.3\text{ A}$



$= 2\Omega + 3\Omega$
 $= 5\Omega$

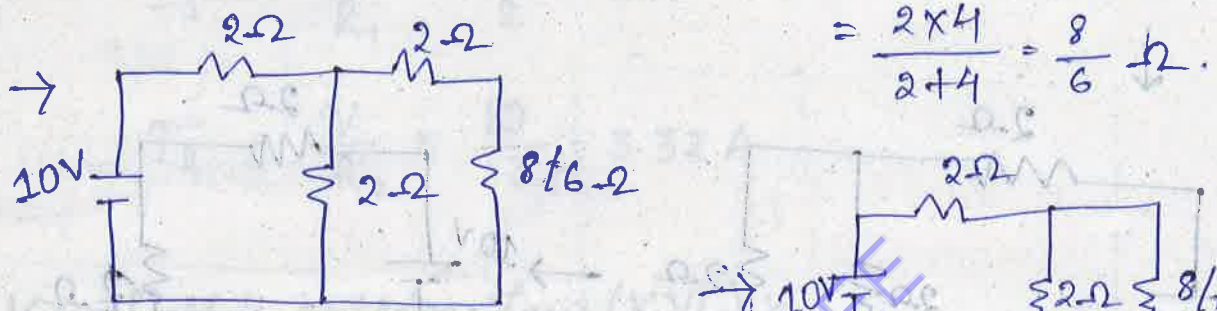
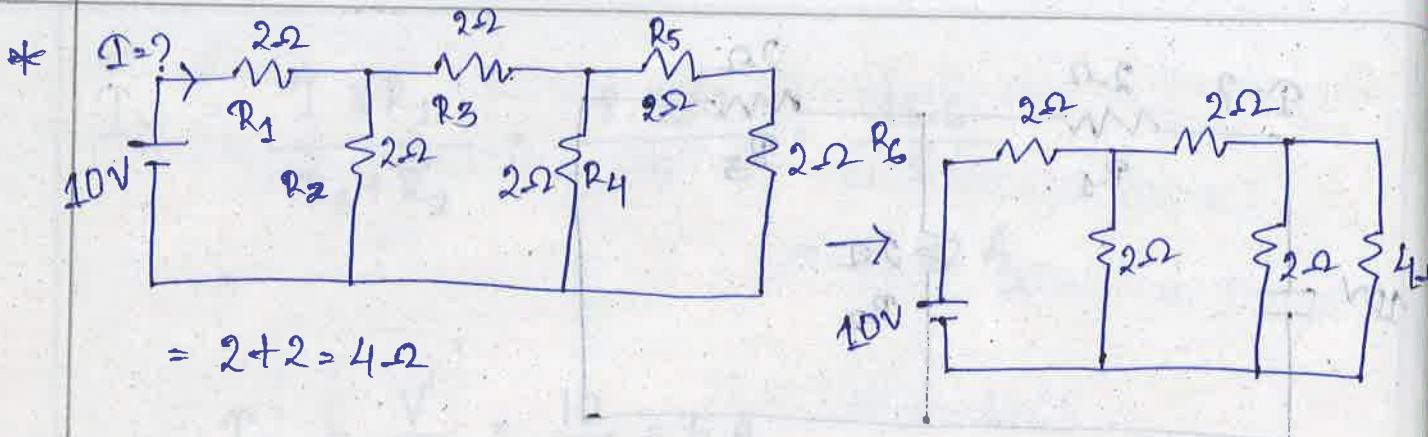


$= \frac{2 \times 5}{2 + 5} = \frac{10}{7} \Omega$

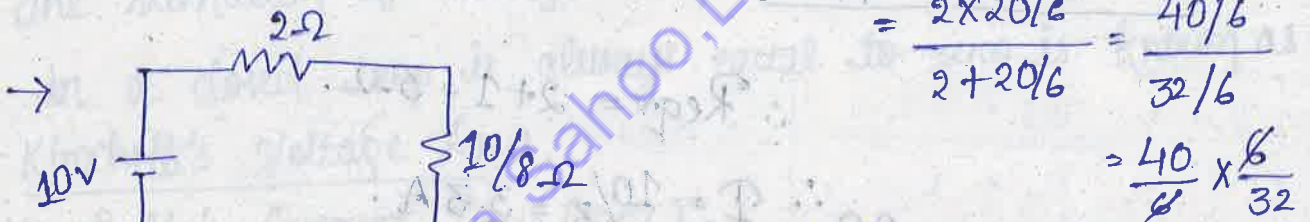


$\therefore R_{eq} = 2 + \frac{10}{7} = \frac{14 + 10}{7} = \frac{24}{7} \Omega$

$\therefore I = \frac{\epsilon}{R} = \frac{10}{\frac{24}{7}} = 10 \times \frac{7}{24} = \frac{70}{24} \text{ A}$

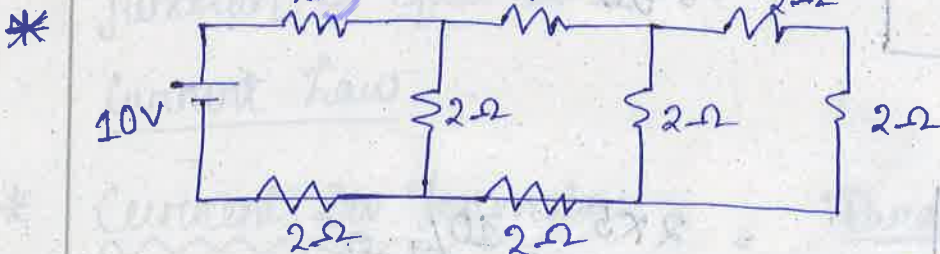


$= 2 + \frac{8}{6} = \frac{12 + 8}{6} = \frac{20}{6}\Omega$

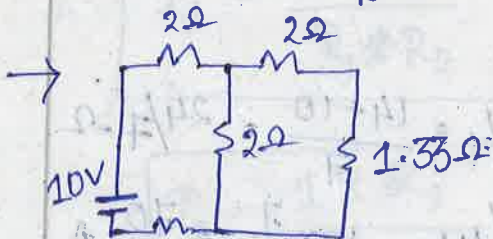


$= \frac{40}{32} \times \frac{6}{32} = \frac{40}{32} = \frac{10}{8}$

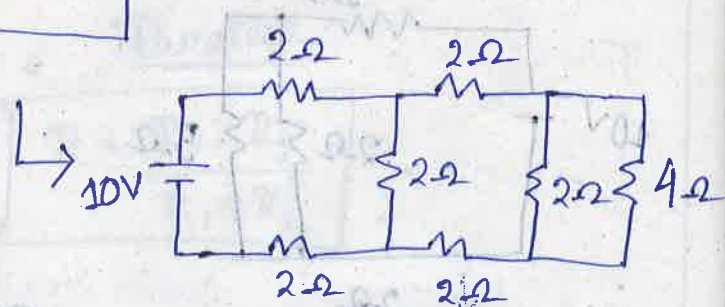
$\therefore I = \frac{E}{R} = \frac{10}{26/8} = 10 \times \frac{8}{26} = \frac{80}{26} = 3.07\text{ A}$



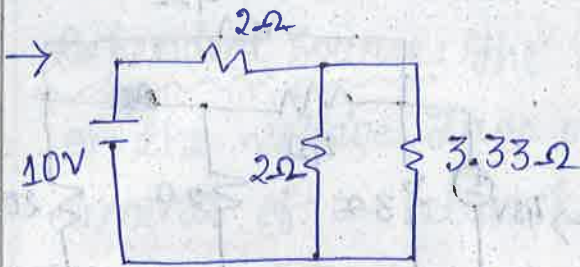
$= 2 + 2 = 4\Omega$



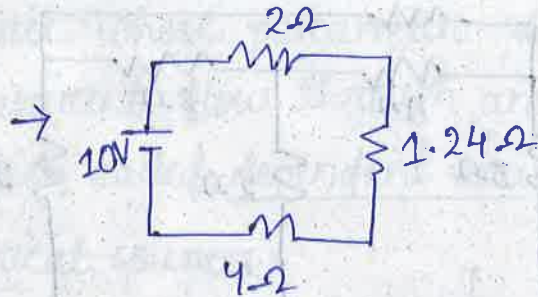
$2 + 1.33 = 3.33\Omega$



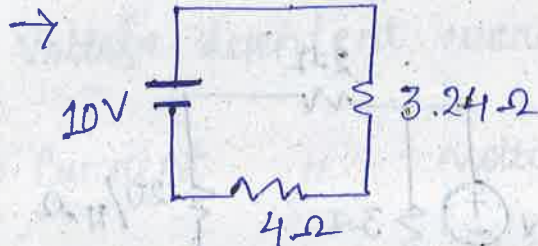
$= \frac{2 \times 4}{2 + 4} = \frac{8}{6}\Omega$



$$= \frac{2 \times 3.33}{2 + 3.33} = \frac{6.66}{5.33} = 1.24 \Omega$$



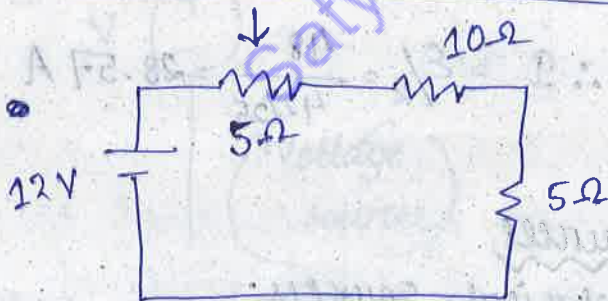
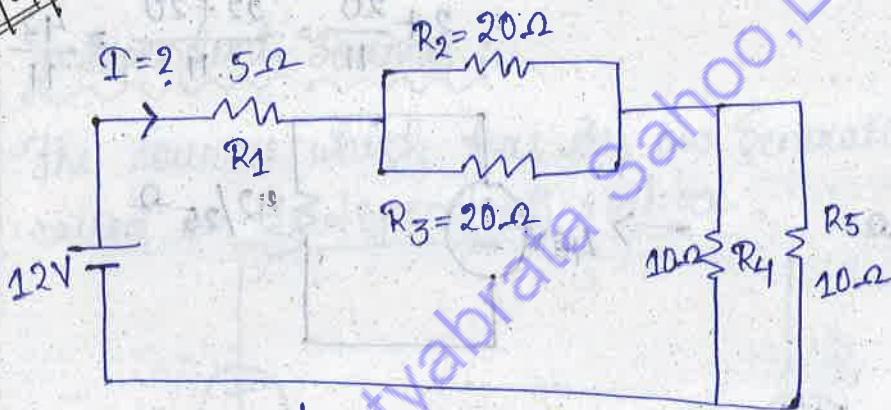
$$= 2 + 1.24 = 3.24 \Omega$$



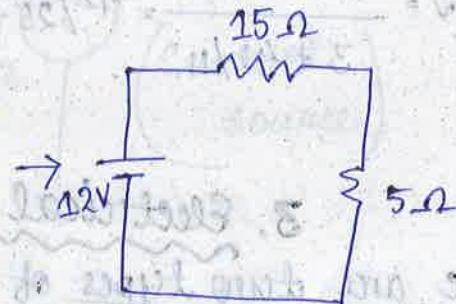
$$= 4 + 3.24 = 7.24 \Omega$$

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$$I = \frac{10}{7.24} = 1.38 \text{ A}$$



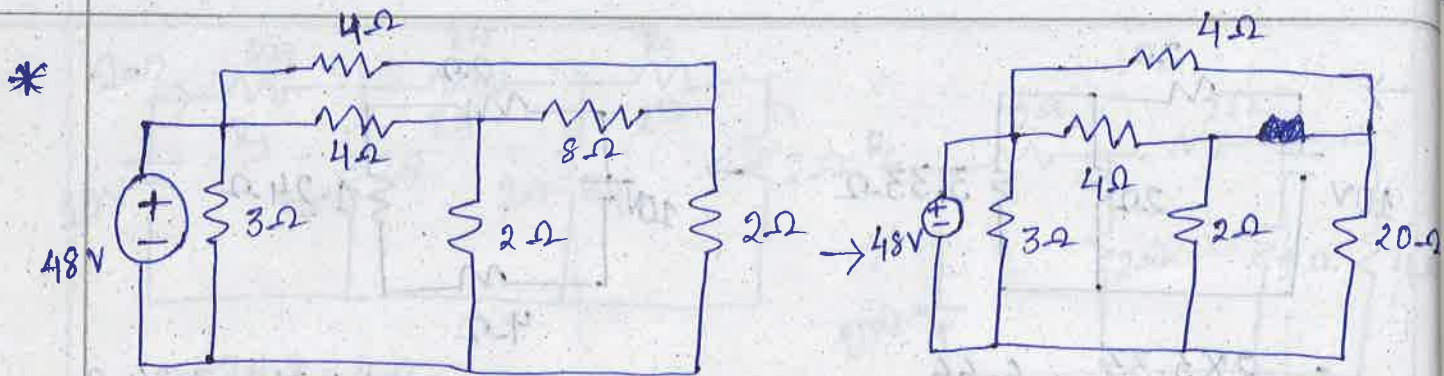
$$= \frac{10 \times 10}{10 + 10} = \frac{10}{2} = 5.2 \Omega$$



$$= 10 + 15 = 15 \Omega$$

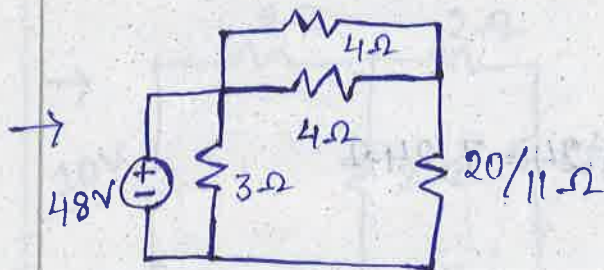
$$R_{eq} = (15 + 5) \Omega = 20 \Omega$$

$$\therefore I = E/R = 12/20 = 3/5 = 0.6 \text{ A}$$

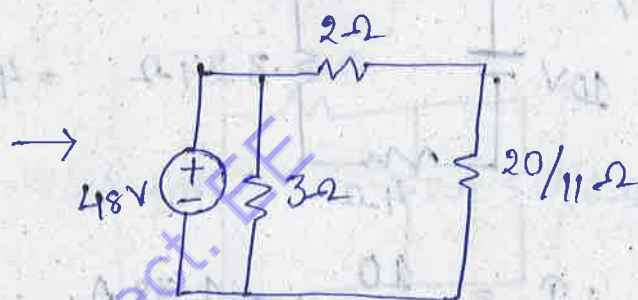


$$= 12 + 8 \Omega = 20 \Omega$$

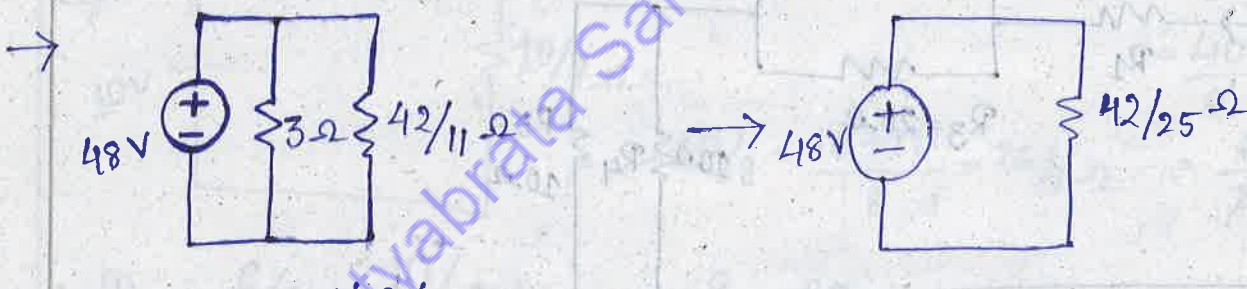
$$= \frac{20 \times 2}{20 + 2} = \frac{40}{22} = \frac{20}{11} \Omega$$



$$= \frac{4 \times 4}{4 + 4} = \frac{16}{8} = 2 \Omega$$



$$= 2 + \frac{20}{11} = \frac{22 + 20}{11} = \frac{42}{11} \Omega$$



$$R_{eq} = \frac{3 \times 42/11}{3 + 42/11} = 42/25 \Omega$$

$$\therefore I = \frac{E}{R} = \frac{48}{42/25} = 28.57 \text{ A}$$

3. Electrical Sources

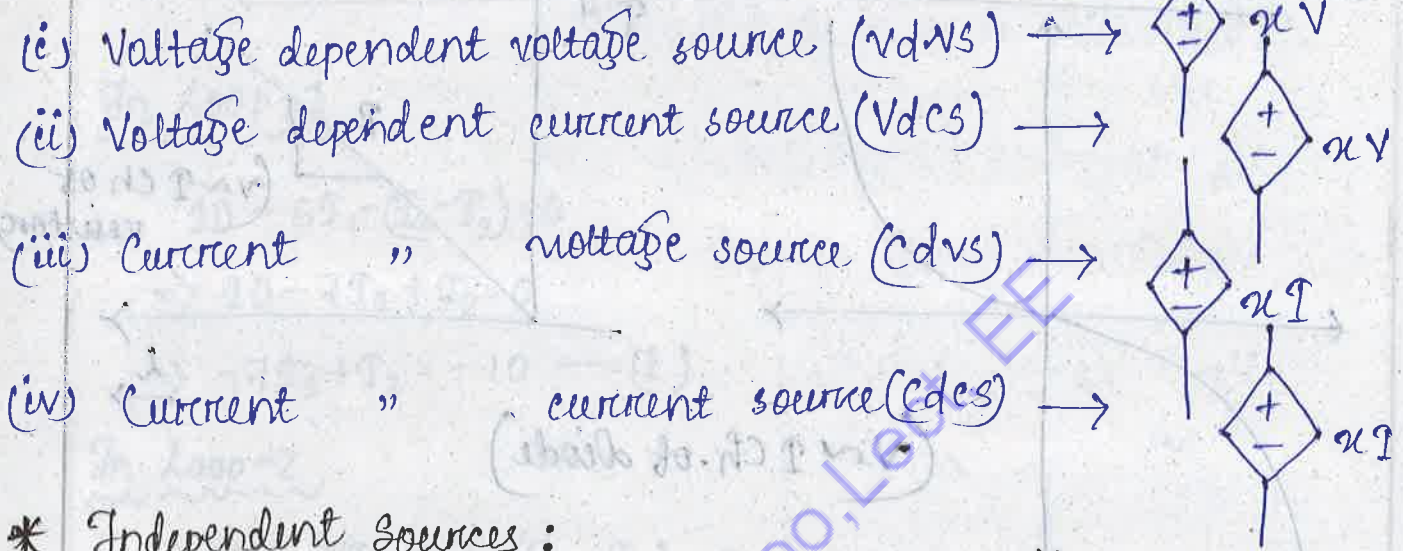
→ There are two types of electrical sources:
 * dependent source
 * Independent source

* Electrical source: The element in an electrical circuit which provides electricity to the network is called an electrical source.

* Dependent source: The source whose parameter depends on the voltage drop or current flow through any other elements of a network which called dependent source.

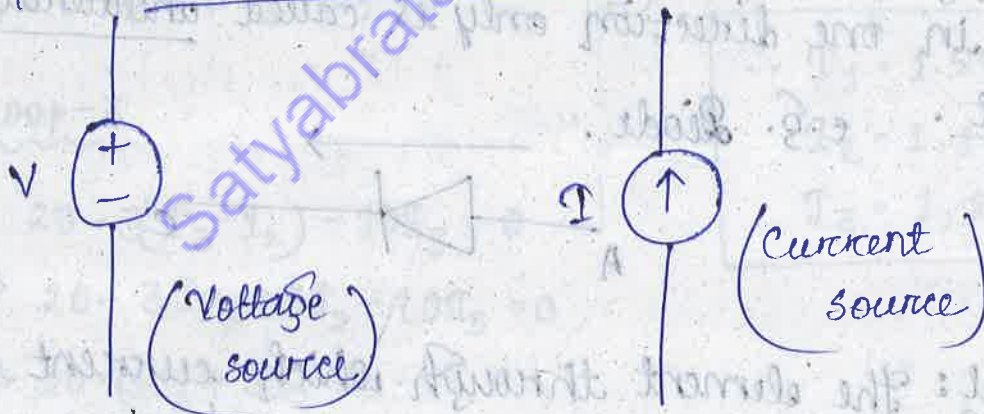
They are two types of electrical sources:

→ dependent source which are 4 types:



* Independent sources:

The sources which has its own generating capability is called an independent source.



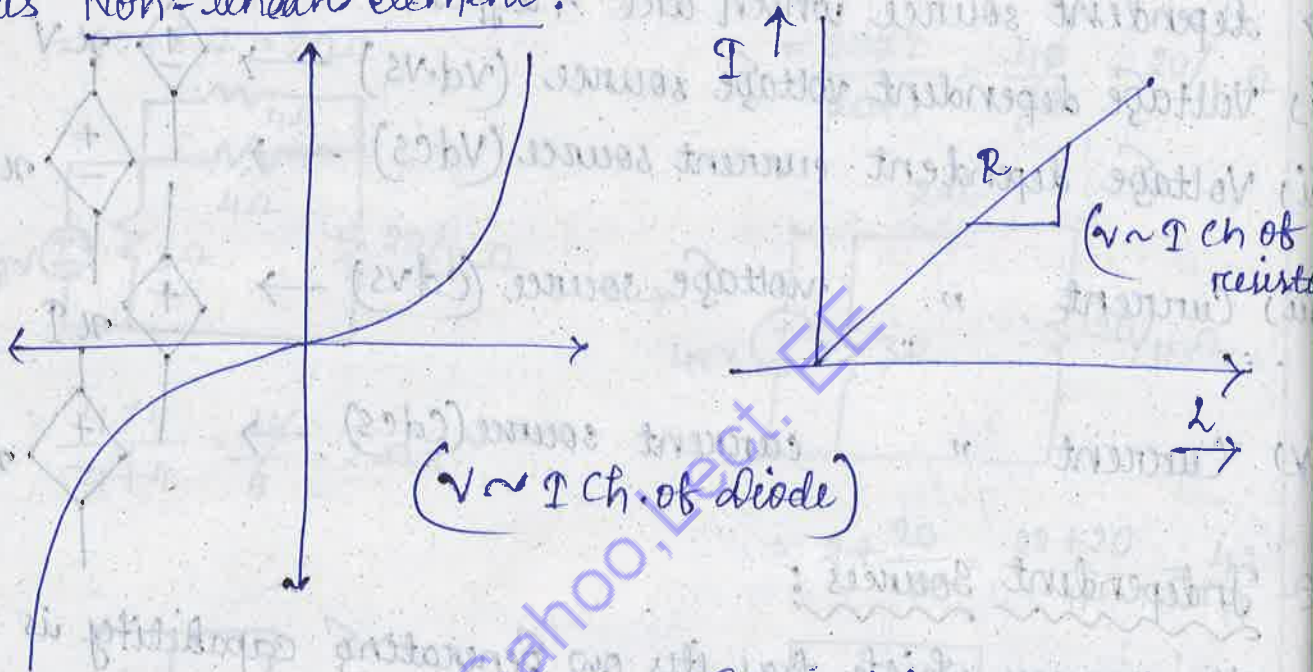
* Electrical Elements:

(1) Active elements: The element which can produce electricity in a circuit is called an active element.

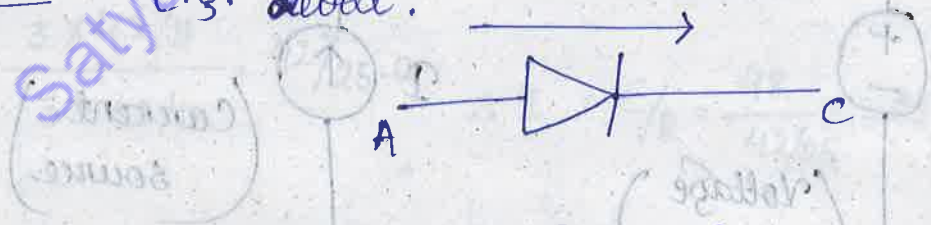
(2) Passive elements: The element which consumes electricity in a network is called passive element.

* Linear element: The element which has linear voltage and current relationship is called linear element.

* Non-linear element: The element which has ~~non-linear~~ non-linear voltage and current relationship is called as Non-linear element.

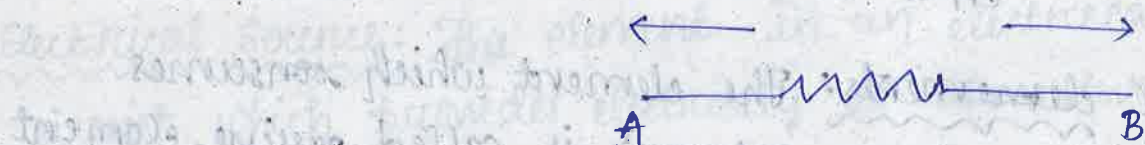


* Unilateral: The element through which current can flow in one direction only is called unilateral element. e.g. Diode.

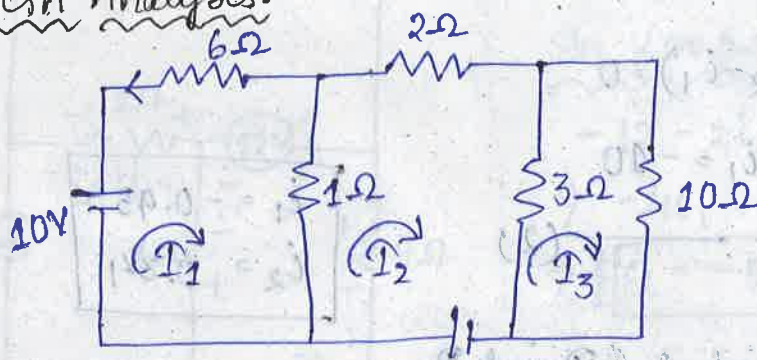


* Bilateral: The element through which current can flow in both the ~~two~~ directions is called bilateral element.

e.g. Resistor



* Mesh Analysis:



In Loop-1

$$10 - 6I_1 - (I_1 - I_2) = 0$$

$$\Rightarrow 10 - 7I_1 + I_2 = 0$$

$$\Rightarrow -7I_1 + I_2 = -10 \quad \text{--- (1)}$$

In Loop-2

$$2I_2 - 3(I_2 - I_3) - (I_2 - I_1) = 0$$

$$\Rightarrow 2I_2 - 3I_2 + 3I_3 - I_2 + I_1 = 0$$

$$\Rightarrow -6I_2 + 3I_3 + I_1 = 0 \quad \text{--- (2)}$$

In Loop-3

$$20 - 3(I_3 - I_2) - 10I_3 = 0$$

$$\Rightarrow 20 - 3I_3 + 3I_2 - 10I_3 = 0$$

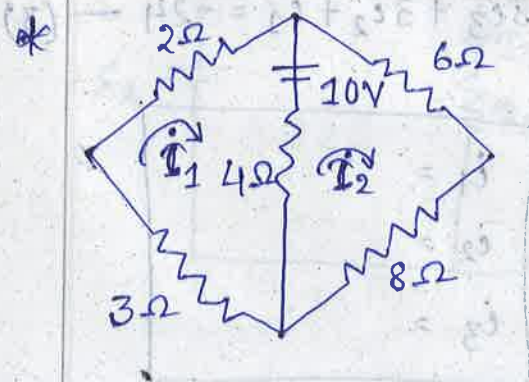
$$\Rightarrow 20 - 13I_3 + 3I_2 = 0$$

$$\Rightarrow -13I_3 + 3I_2 = -20 \quad \text{--- (3)}$$

$$\therefore I_1 = 1.59 \text{ A}$$

$$I_2 = 1.17 \text{ A}$$

$$I_3 = 1.8 \text{ A}$$



In Loop-1

$$10 - 3i_1 - 2i_1 - 4(i_1 + i_2) = 0$$

$$\Rightarrow -3i_1 - 2i_1 - 4i_1 + 4i_2 = 10$$

$$\Rightarrow -9i_1 + 4i_2 = 10 \quad \text{--- (1)}$$

In Loop-2

$$10 - 6i_2 - 8i_2 - 4(i_2 - i_1) = 0$$

$$\Rightarrow -6i_2 - 8i_2 - 4i_2 + 4i_1 = -10$$

$$\Rightarrow -18i_2 + 4i_1 = -10 \quad \text{--- (2)}$$

$$i_1 = -0.95$$

$$i_2 = 0.34$$

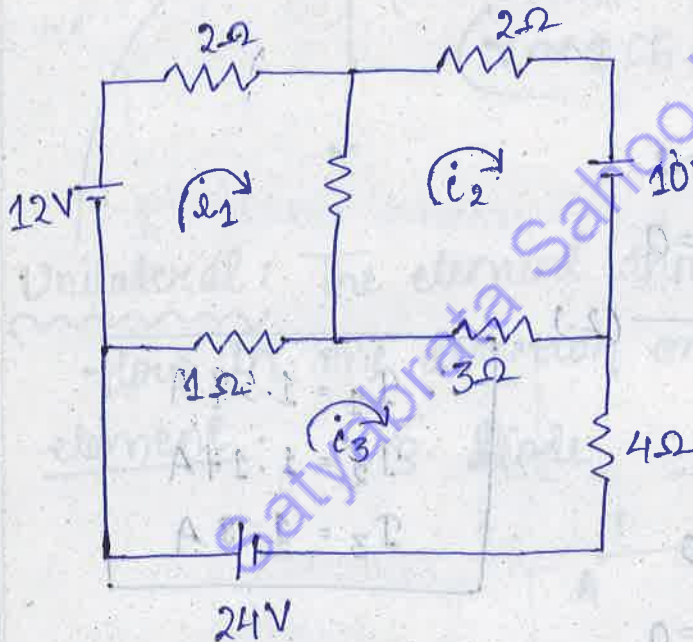
\therefore Potential at point A & Point B,

$$V_A = 10 - 0.95 \times 2 = 8.1 \text{ V}$$

$$V_B = 10 - 0.34 \times 6 = 7.96 \text{ V}$$

$$V_{AB} = V_A - V_B = 0.14 \text{ V}$$

*



In Loop-1

$$12 - 2i_1 = 10(i_2 - i_1) - 1(i_1 - i_3)$$

$$\Rightarrow 12 - 2i_1 - 10i_1 + 10i_2 - i_1 + i_3 = 0$$

$$\Rightarrow -13i_1 + 10i_2 + i_3 = -12 \quad \text{--- (1)}$$

In Loop-3

$$24 - 4i_3 - 18(i_3 + i_1) - 3(i_3 - i_2) = 0$$

$$\Rightarrow 24 - 4i_3 - 18i_3 + 18i_1 - 3i_3 + 3i_2 = 0$$

$$\Rightarrow -8i_3 + 3i_2 + 18i_1 = -24 \quad \text{--- (3)}$$

In Loop-2

$$-10 + 2i_2 + 3(i_2 - i_3) + 10(i_1 - i_2) = 0$$

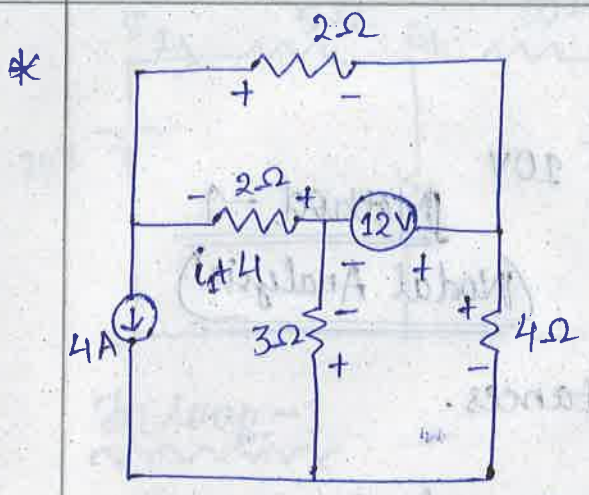
$$\Rightarrow -10 + 2i_2 + 3i_2 - 3i_3 + 10i_1 - 10i_2 = 0$$

$$\Rightarrow -5i_2 + 3i_3 + 10i_1 = 10 \quad \text{--- (2)}$$

$$i_1 = 0.95$$

$$i_2 = 0.34$$

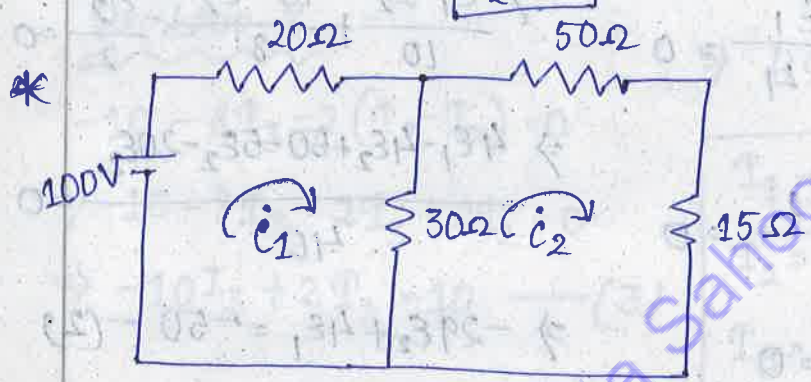
$$i_3 = 0.34$$



In Loop-1
 $-12 - 2i_1 - 2(i_1 + 4) = 0$
 $\Rightarrow -4i_1 = 20$
 $\Rightarrow i_1 = -5A$

In Loop-2
 $12 - 4i_2 - 3(i_2 + 4) = 0$
 $\Rightarrow 7i_2 = 0$
 $\Rightarrow i_2 = 0$

~~$i_3 = -4A$~~
 $i_3 = -4A$

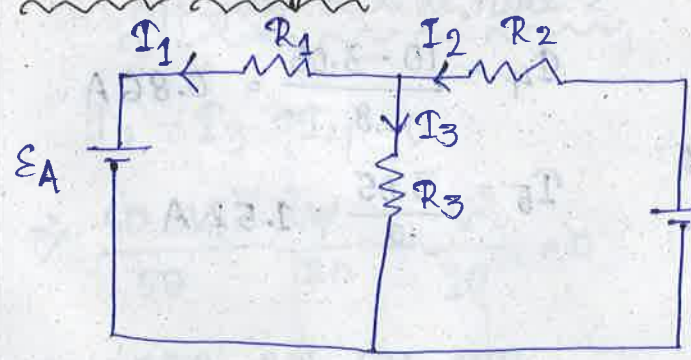


$i_1 = 2.46A$
 $i_2 = 0.77A$

In Loop-1
 $100 - 20i_1 - 30(i_1 - i_2) = 0$
 $\Rightarrow 100 - 20i_1 - 30i_1 + 30i_2 = 0$
 $\Rightarrow -50i_1 + 30i_2 = -100 \quad (1)$

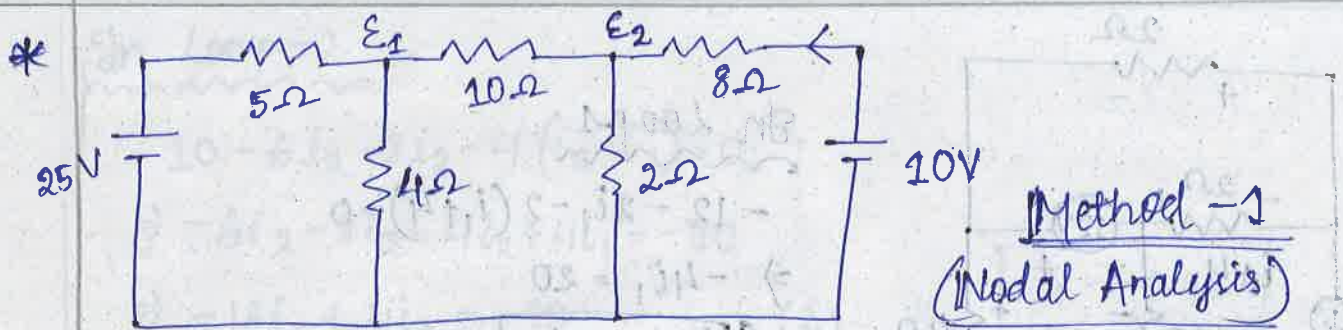
In Loop-2
 $-50i_2 - 15i_2 - 30(i_2 - i_1) = 0$
 $\Rightarrow -50i_2 - 15i_2 - 30i_2 + 30i_1 = 0$
 $\Rightarrow -95i_2 + 30i_1 = 0 \quad (2)$

* Nodal Analysis:



Applying KCL at node is
 $I_1 + I_2 - I_3 = 0$

$\Rightarrow \frac{\epsilon_A - \epsilon_B}{R_1} + \frac{\epsilon_B - \epsilon_m}{R_2} - \frac{\epsilon_m}{R_3} = 0$



Find current through resistances.

Soln:

At node - 1

$$I_1 - I_2 - I_3 = 0$$

$$\Rightarrow \frac{25 - \epsilon_1}{5} - \frac{\epsilon_1 - \epsilon_2}{10} - \frac{\epsilon_1}{4} = 0$$

$$\Rightarrow \frac{100 - 4\epsilon_1 - 2\epsilon_1 + 2\epsilon_2 - 5\epsilon_1}{20} = 0$$

$$\Rightarrow 100 - 4\epsilon_1 - 2\epsilon_1 + 2\epsilon_2 - 5\epsilon_1 = 0$$

$$\Rightarrow -11\epsilon_1 + 2\epsilon_2 = -100 \quad (1)$$

Solving (1) & (2),

$$-11\epsilon_1 + 2\epsilon_2 = -100 \quad (1)$$

$$-29\epsilon_2 + 4\epsilon_1 = -50 \quad (2)$$

$$\therefore I_1 = \frac{25 - 9.64}{5} = 3.07 \text{ A}$$

$$I_2 = \frac{9.64 - 3.05}{10} = 0.65 \text{ A}$$

$$I_3 = \frac{9.64}{4} = 2.41 \text{ A}$$

At node - 2

$$I_2 + I_4 - I_5 = 0$$

$$\Rightarrow \frac{\epsilon_1 - \epsilon_2}{10} + \frac{10 - \epsilon_2}{8} - \frac{\epsilon_2}{2} = 0$$

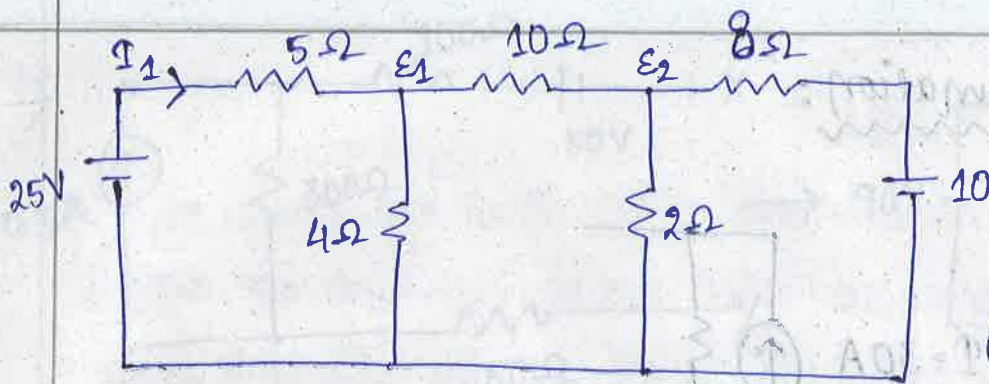
$$\Rightarrow \frac{4\epsilon_1 - 4\epsilon_2 + 50 - 5\epsilon_2 - 20\epsilon_2}{40} = 0$$

$$\Rightarrow -29\epsilon_2 + 4\epsilon_1 = -50 \quad (2)$$

$$\begin{aligned} \epsilon_1 &= 9.64 \text{ V} \\ \epsilon_2 &= 3.05 \text{ V} \end{aligned}$$

$$I_4 = \frac{10 - 3.05}{8} = 0.86 \text{ A}$$

$$I_5 = \frac{3.05}{2} = 1.52 \text{ A}$$



Method - 2
(Mesh Analysis)

In Loop - 1

$$25 - 5I_1 - 4(I_1 - I_2) = 0$$

$$\Rightarrow 25 - I_1 - 4I_1 + 4I_2 = 0$$

$$\Rightarrow -9I_1 + 4I_2 = -25 \quad (1)$$

In Loop - 2

$$-10I_2 - 2(I_2 - I_3) - 4(I_2 - I_1) = 0$$

$$\Rightarrow -10I_2 - 2I_2 + 2I_3 - 4I_2 + 4I_1 = 0$$

$$\Rightarrow -16I_2 + 2I_3 + 4I_1 = 0$$

In Loop - 3

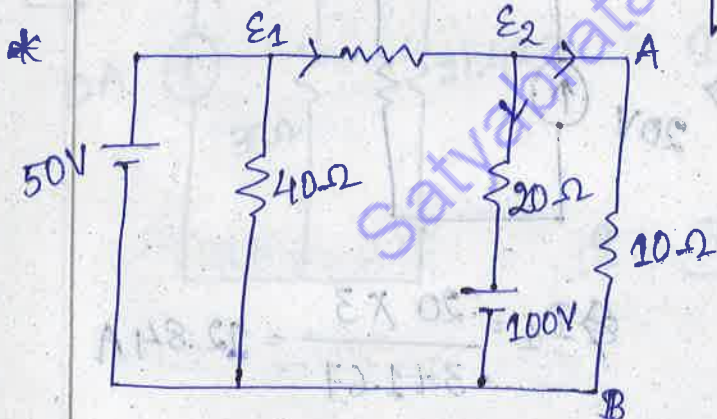
$$-10 - 8I_3 - 2(I_3 - I_2) = 0$$

$$\Rightarrow -10 - 8I_3 - 2I_3 + 2I_2 = 0$$

$$\Rightarrow -10I_3 + 2I_2 = 10 \quad (3)$$

$$\Rightarrow 4I_1 - 16I_2 + 2I_3 = 0 \quad (2)$$

$I_1 = 3.07A$	$I_4 = 2.42A$
$I_2 = 0.65A$	$I_5 = 1.252A$
$I_3 = -0.86A$	



Applying KCL at node - 2

$$I_1 - I_3 - I_4 = 0$$

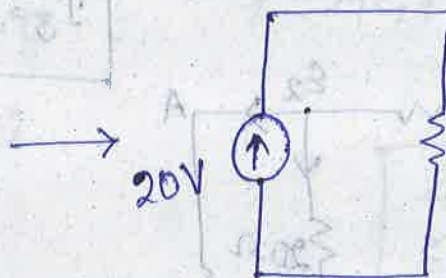
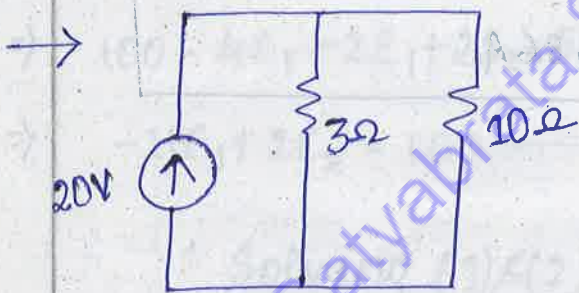
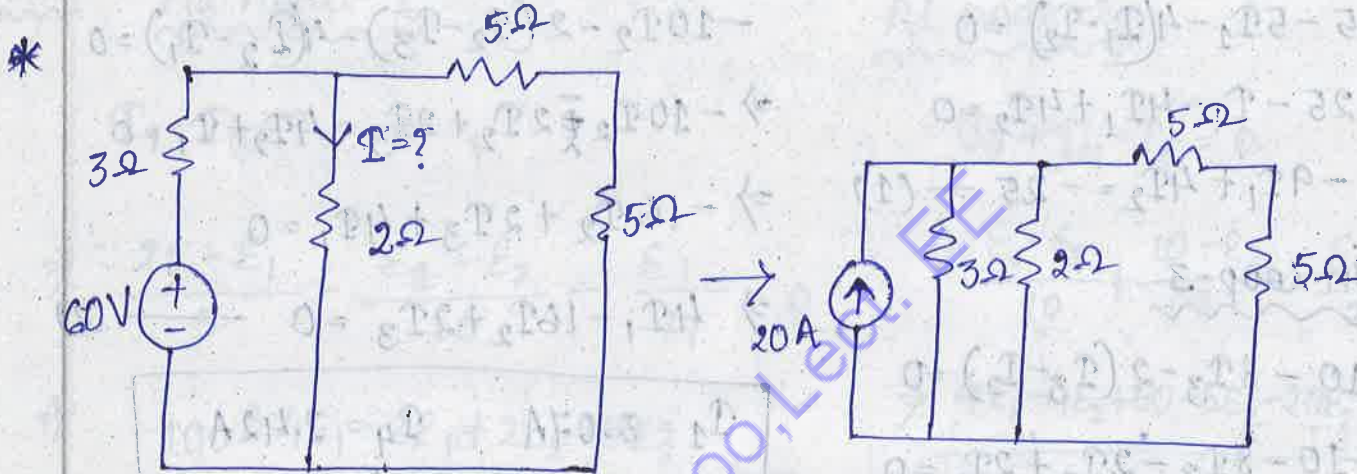
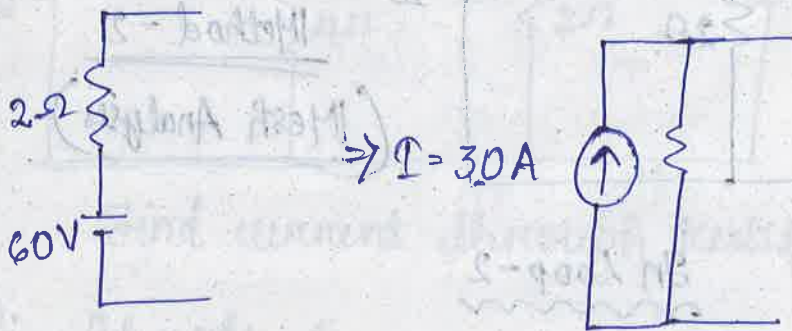
$$\Rightarrow \frac{50 - V}{50} - \frac{V}{20} - \frac{V}{10} = 0$$

$$\Rightarrow \frac{100 - 2V - 5V - 10V}{100} = 0$$

$$\Rightarrow -17V + 100 = 0$$

$$\Rightarrow \boxed{V = 5.88V}$$

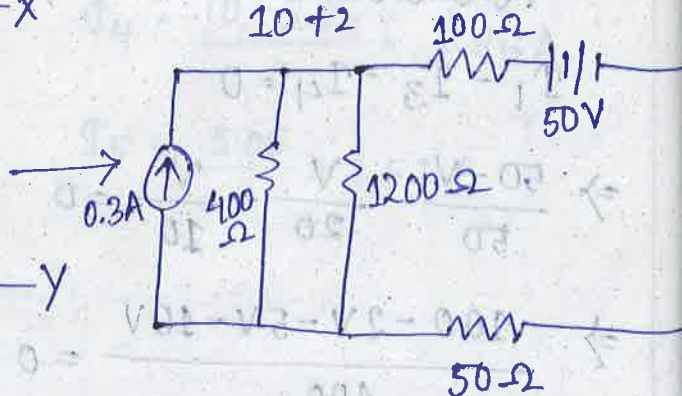
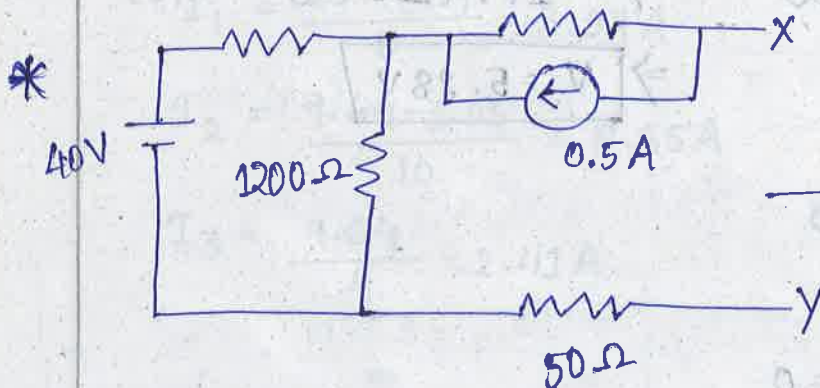
* Source Transformation:

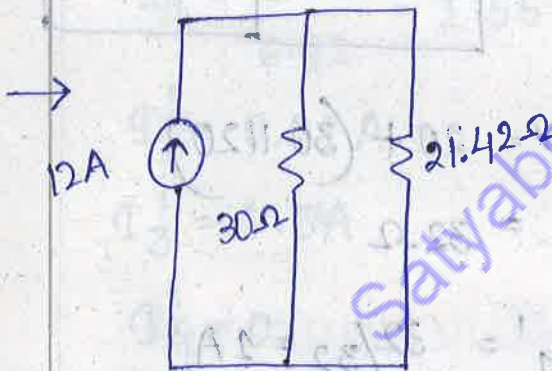
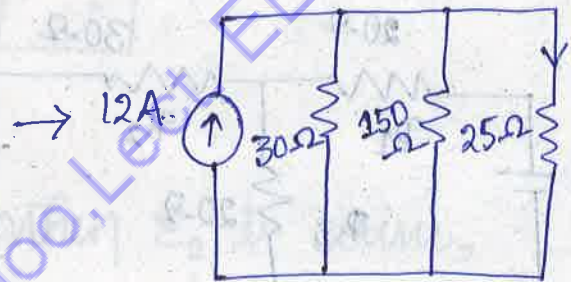
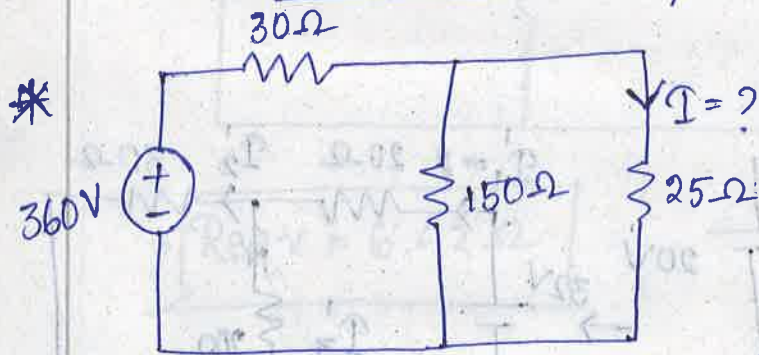
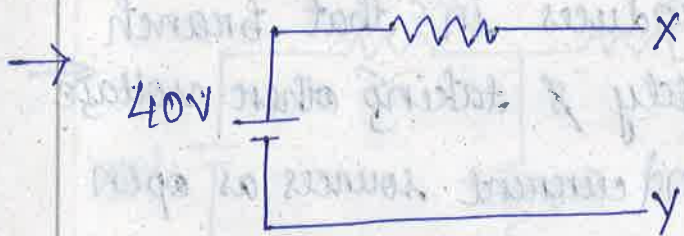
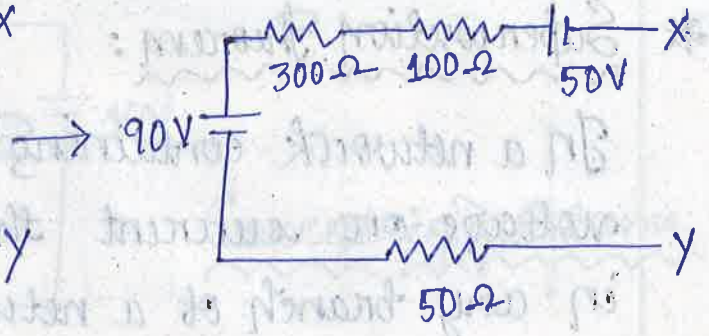
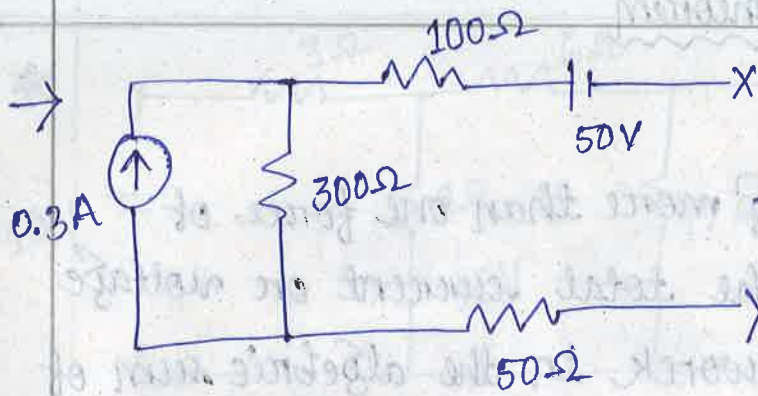


$$= \frac{10 \times 2}{10 + 2} = \frac{20}{12} = 1.67\Omega$$

$$I_1 = \frac{20 \times 3}{3 + 1.67} = 12.84A$$

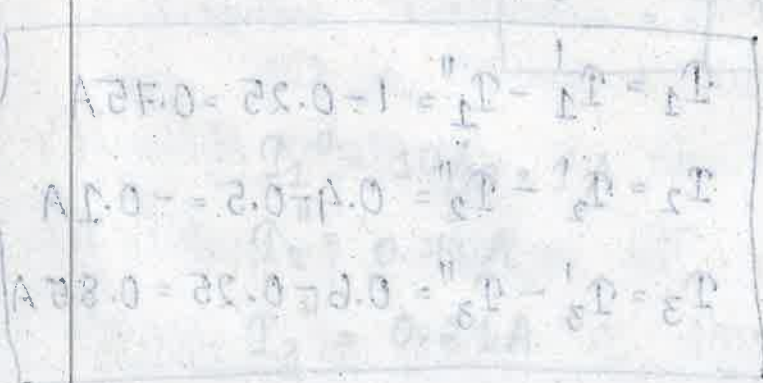
$$I = \frac{12.84 \times 10}{10 + 2} = 10.7A$$





$$I_1 = \frac{12 \times 30}{30 + 21.42} = \frac{360}{51.42} = 7.001 \text{ A}$$

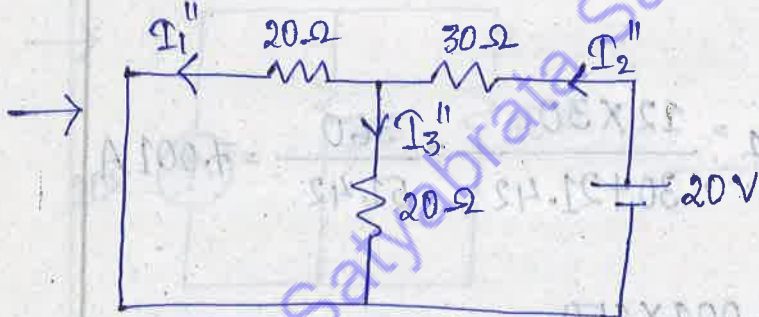
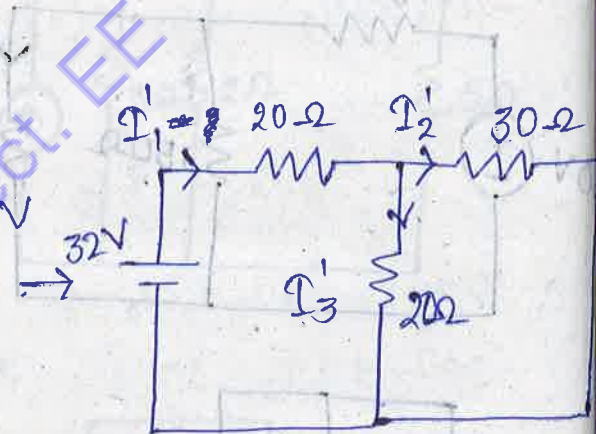
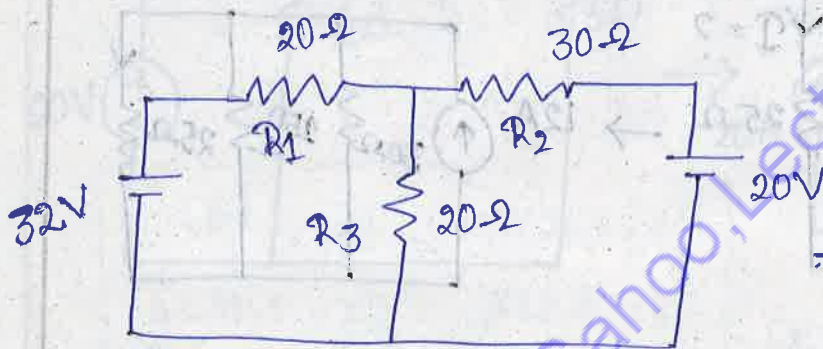
$$I = \frac{7.001 \times 150}{175} = 6 \text{ A}$$



4. Network Theorem

* Superposition Theorem:

In a network containing more than one force of voltage or current the total current or voltage in any branch of a network is the algebraic sum of the current or voltage produced in that branch by his source acting separately & taking other voltage sources as short circuit and current sources as open circuit.



$$R_{eqv} = 30 + (20 \parallel 20)$$

$$= 40 \Omega$$

$$I_1'' = 1 \times 20 / 40 = 0.25 \text{ A}$$

$$I_2'' = 0.5 \text{ A}$$

$$I_3'' = 1 \times 20 / 40 = 0.25 \text{ A}$$

$$R_{eqv} = 20 + (30 \parallel 20)$$

$$= 32 \Omega$$

$$I_1' = 32 / 32 = 1 \text{ A}$$

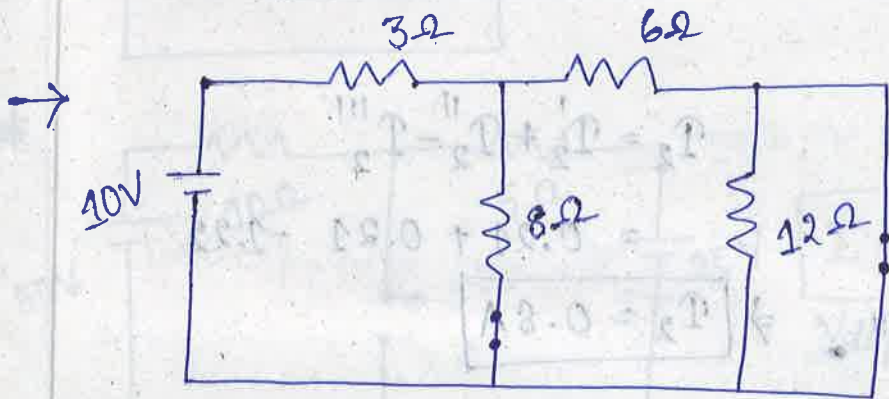
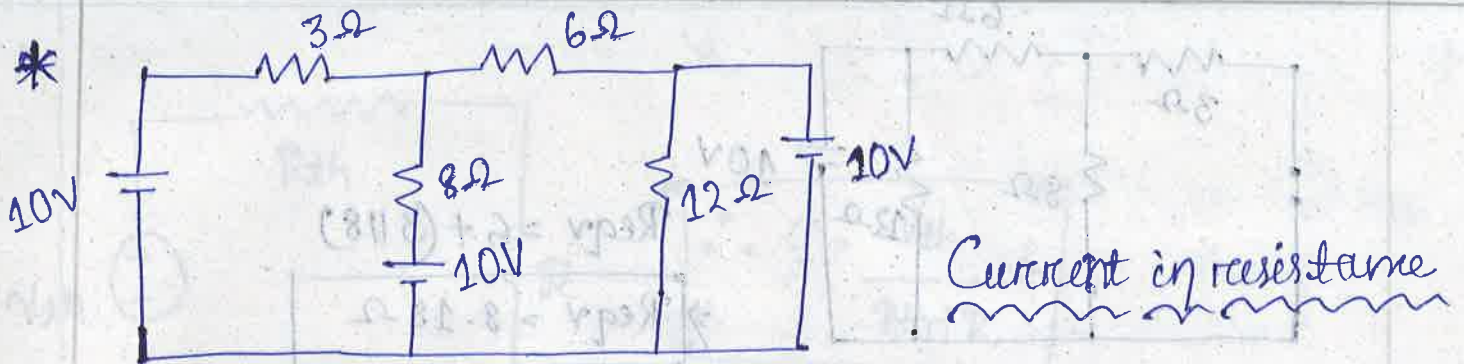
$$I_2' = 1 \times 20 / 50 = 0.4 \text{ A}$$

$$I_3' = 1 \times 30 / 50 = 0.6 \text{ A}$$

$$I_1 = I_1' - I_1'' = 1 - 0.25 = 0.75 \text{ A}$$

$$I_2 = I_2' - I_2'' = 0.4 - 0.5 = -0.1 \text{ A}$$

$$I_3 = I_3' - I_3'' = 0.6 - 0.25 = 0.35 \text{ A}$$



$$\Rightarrow \boxed{R_{eqv} = 6.42 \Omega}$$

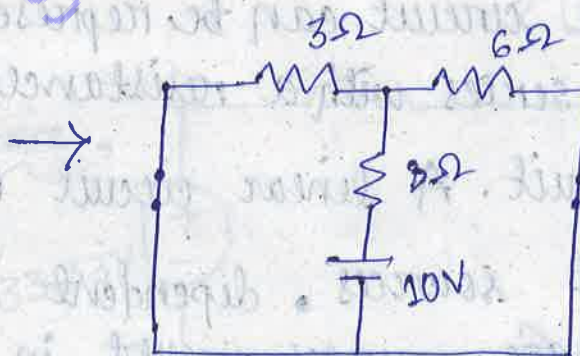
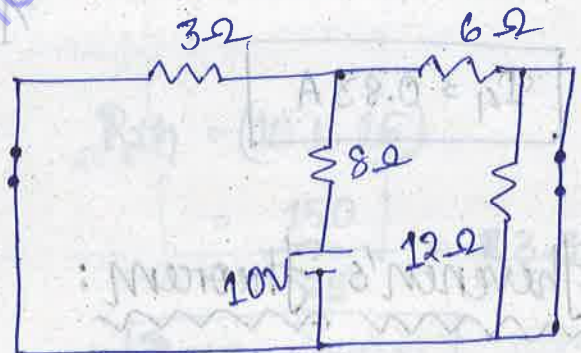
$$I_1' = \frac{10}{6.42} = 1.55 \text{ A}$$

$$I_2' = 0.21 \text{ A}$$

$$I_3' = 1.34 \text{ A}$$

$$I_4' = 0$$

When \mathcal{E}_2 is active,



$$\Rightarrow R_{eqv} = 6 + (3 \parallel 8)$$

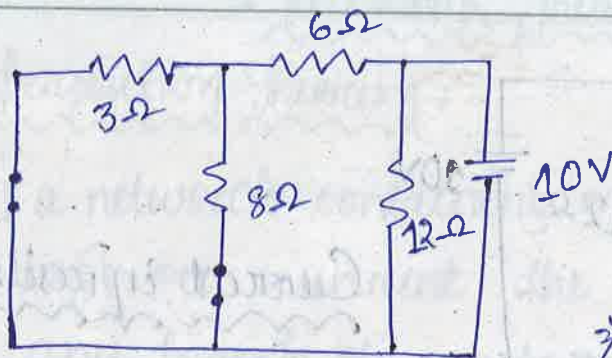
$$= 8.98 \Omega$$

$$\Rightarrow \boxed{R_{eqv} = 10 \Omega}$$

$$I_1'' = 10/10 = 1 \text{ A}$$

$$I_2'' = 0.79 \text{ A}$$

$$I_3'' = 0.21 \text{ A}$$



$$R_{eqv} = 6 + (3 \parallel 8)$$

$$\Rightarrow R_{eqv} = 8.18 \Omega$$

$$I_1''' = 0.32 \text{ A}$$

$$I_2''' = 10/8 = 1.22 \text{ A}$$

$$I_3''' = 0.88 \text{ A}$$

$$I_2 = I_2' + I_2'' - I_2'''$$

$$= 0.21 + 0.21 - 1.22$$

$$I_1 = I_1' - I_2'' - I_1''' \Rightarrow I_2 = 0.8 \text{ A}$$

$$= 1.55 - 1 - 0.32$$

$$I_3 = I_3' - I_3'' + I_3'''$$

$$= 0.32 - 0.79 + 0.88$$

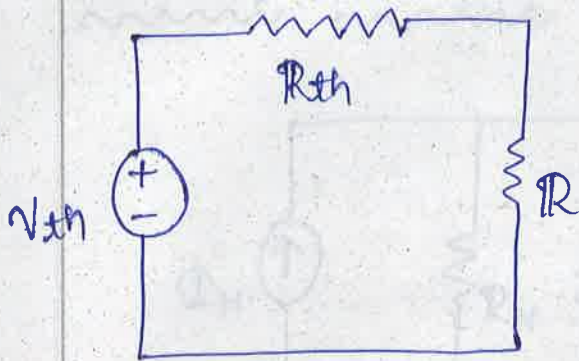
$$\Rightarrow I_1 = 0.23 \text{ A}$$

$$\Rightarrow I_3 = 1.31 \text{ A}$$

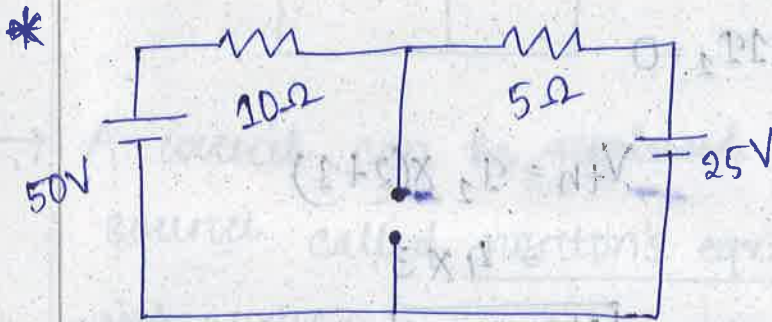
$$I_4 = 0.83 \text{ A}$$

* Thevenin's Theorem:

It states that, the circuit can be represented as voltage source in series with a resistance. It is known as thevenin's circuit. A linear circuit may contain independent sources, dependent sources & resistors. The voltage source present in the thevenin's equivalent circuit is covered as thevenin's equivalent voltage (V_{th}). The resistance present in the thevenin's equivalent circuit is called as thevenin's equivalent resistance (R_{th}).



$$I = \frac{V_{th}}{R_{th} + R}$$

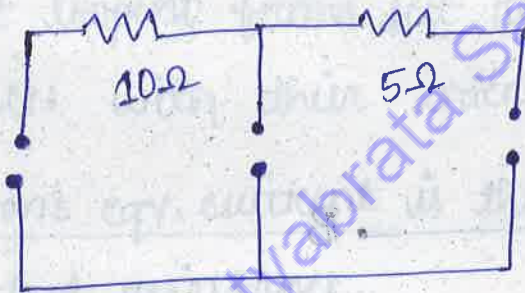


$$I = 1.66 \text{ A}$$

$$V_{th} = 50 - I \times 10 = 50 - 1.67 \times 10 = 50 - 16.67$$

$$\Rightarrow V_{th} = 33.33 \text{ V}$$

Forc Rth,

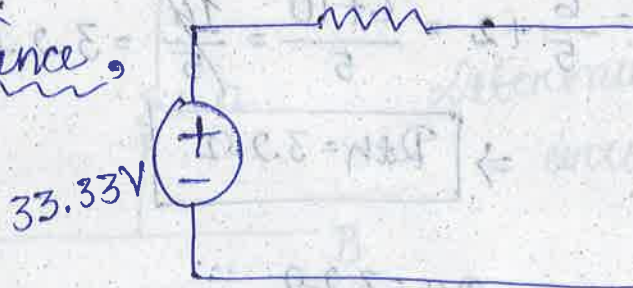


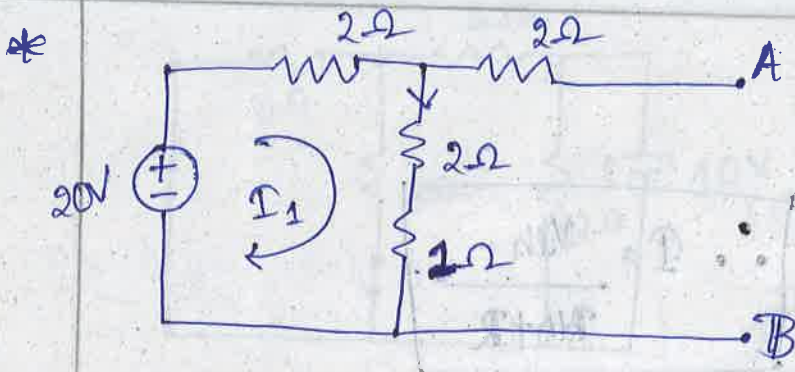
$$R_{th} = (10 \parallel 15)$$

$$= \frac{150}{25} = 3.33 \Omega$$

$$\Rightarrow R_{th} = 3.33 \Omega$$

Thevenin eqv. resistance,





Applying KVL,

$$-20 + 2I_1 + 2I_1 + 1I_1 = 0$$

$$\Rightarrow -20 + 5I_1 = 0$$

$$\Rightarrow 5I_1 = 20$$

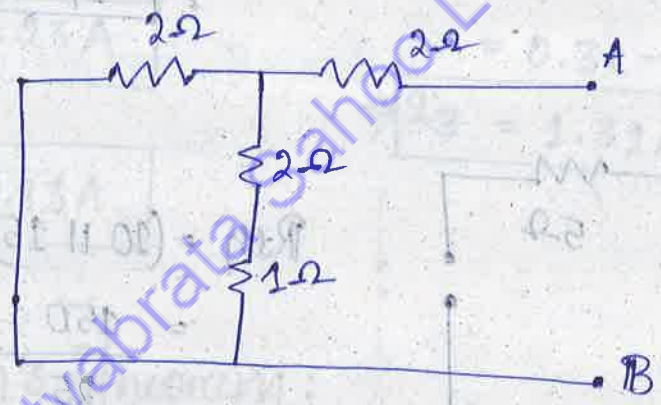
$$I_1 = 4A$$

$$V_{th} = I_1 \times (2+1)$$

$$= 4 \times 3$$

$$V_{th} = 12V$$

For R_{th} ,

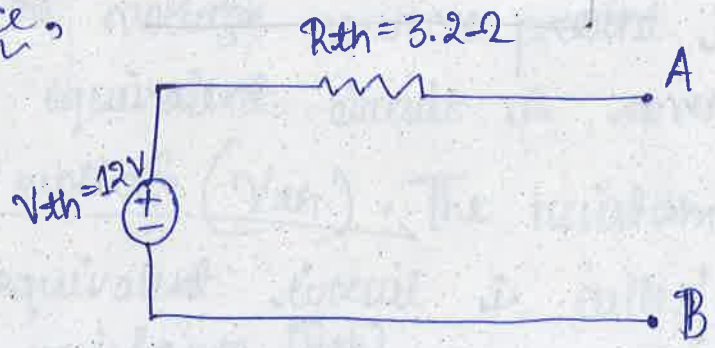


$$R_{th} = (2 || 3) + 2$$

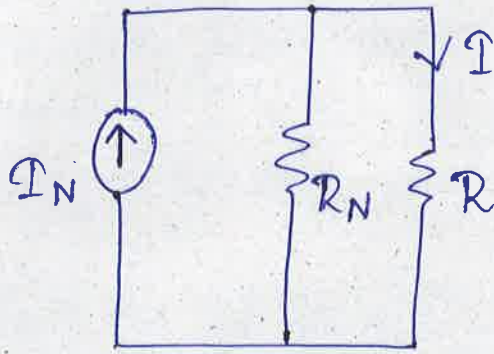
$$= \frac{2 \times 3}{2+3} + 2 = \frac{6}{5} + 2 = \frac{6+10}{5} = \frac{16}{5} = 3.2 \Omega$$

$$\Rightarrow R_{th} = 3.2 \Omega$$

Thenerin' eq.
resistance,



* Norton's Theorem:

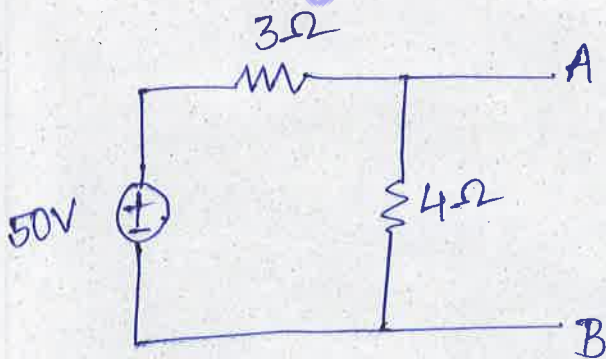


$$I = \frac{I_N \times R_N}{R + R_N}$$

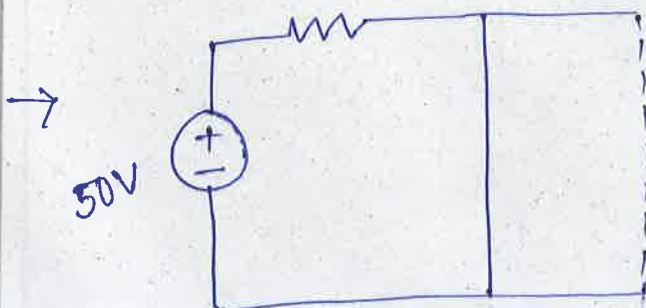
→ A circuit can be ~~replaced~~ represented as a current source called Norton's eqv. current source in parallel with resistance called Norton's eqv. resistance.

→ Norton's eqv. resistance is the eqv. resistance of the circuit looking from test resistance by removing all the sources with their internal resistance.

→ Norton's eqv. current is the short circuit current through the test resistance.

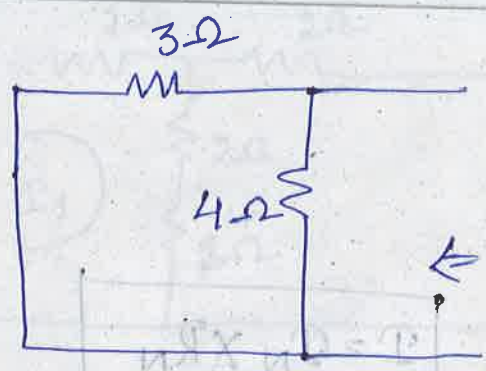


determine norton's eqv. circuit across A & B.

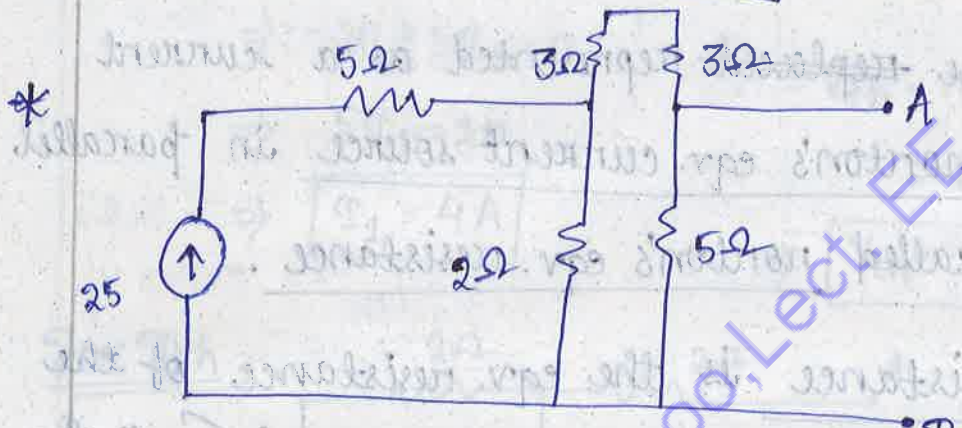
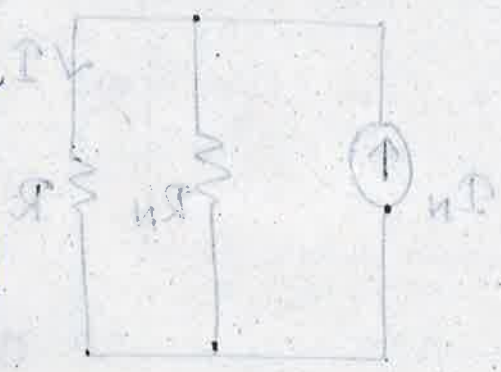


$$I_{sc} = \frac{50}{3} = 16.67$$

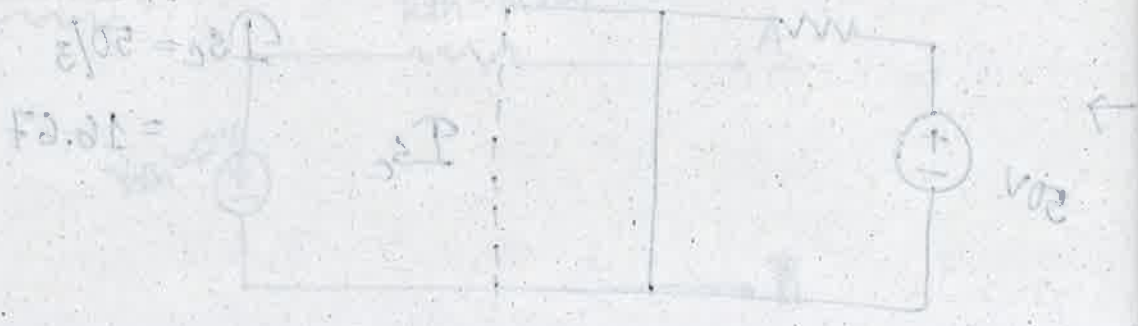
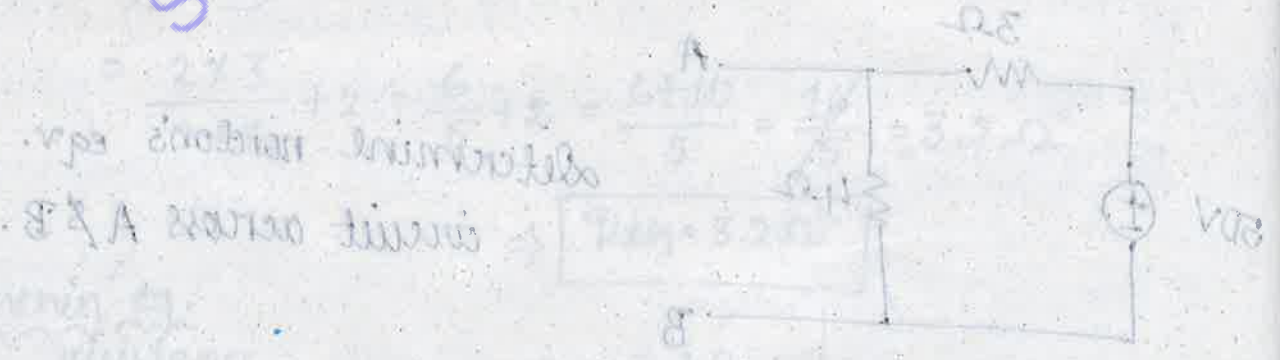
Norton's theorem:



$R_{eqv.} = 1.7 \Omega$



Norton's eq. resistance is the eq. resistance of the circuit looking from the terminals A & B by removing all the sources with their internal resistances.



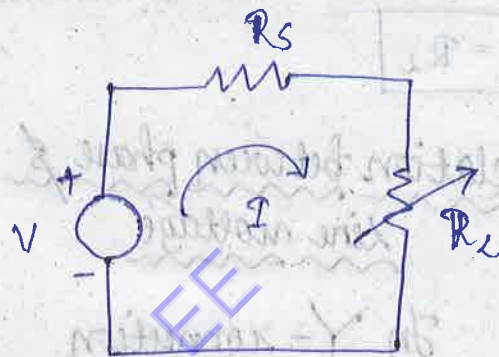
* Maximum Power Transfer Theorem:

→ According to this theorem, maximum power will be transferred to the load when load resistance is equal to the source resistance.

Assuming the load resistance is a variable resistance.

→ Current in the circuit

$$I = \frac{V}{R_s + R_L}$$



Power delivered to load,

$$P = I^2 \times R_L$$

$$= \left(\frac{V}{R_s + R_L} \right)^2 \times R_L$$

$$= \frac{V^2 R_L}{(R_s + R_L)^2}$$

$$\frac{dP}{dR_L} = 0$$

$$\Rightarrow \frac{d}{dR_L} \left[\frac{V^2 R_L}{(R_s + R_L)^2} \right] = 0$$

$$\frac{d}{dt} \left(\frac{A}{B} \right) = \frac{B \frac{d}{dt} A - A \frac{d}{dt} B}{B^2}$$

$$\Rightarrow \frac{(R_s + R_L)^2 \frac{d}{dR_L} (V^2 R_L) - (V^2 R_L) \frac{d}{dR_L} (R_s + R_L)^2}{(R_s + R_L)^4} = 0$$

$$\Rightarrow \frac{V^2 (R_s + R_L)^2 - (V^2 R_L) 2 (R_s + R_L)}{(R_s + R_L)^4} = 0$$

$$\Rightarrow V^2(R_s + R_L)^2 - 2V^2R_L(R_s + R_L) = 0$$

$$\Rightarrow (R_s + R_L) - 2R_L = 0$$

$$\Rightarrow R_s + R_L - 2R_L = 0$$

$$\Rightarrow R_s - R_L = 0$$

$$\Rightarrow \boxed{R_s = R_L}$$

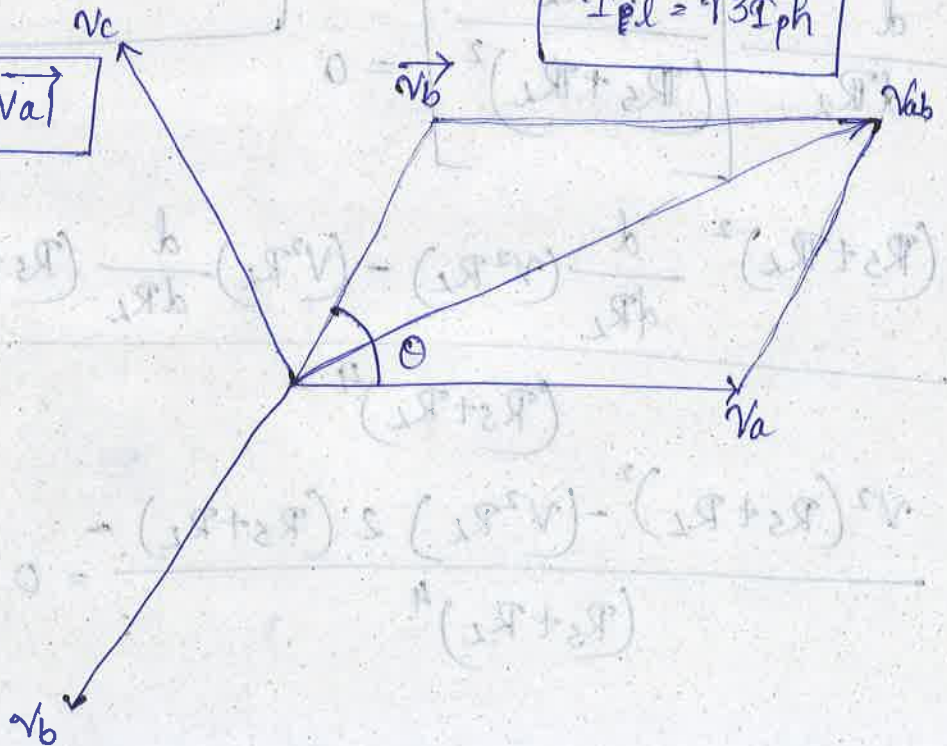
* Relation between phase & line voltage

In Y-connection

$$\begin{aligned} \vec{V}_{ab} &= \vec{V}_a - \vec{V}_b \\ &= \vec{V}_a + (-\vec{V}_b) \end{aligned}$$

$$\begin{aligned} |V_{ab}|^2 &= |V_a|^2 + |V_b|^2 + 2|V_a||V_b|\cos\theta \\ &= |V_a|^2 + |V_a|^2 + 2|V_a||V_a|\cos 60^\circ \\ &= 2V_a^2 + 2V_a^2 \cdot \frac{1}{2} \\ &= 3V_a^2 \end{aligned}$$

$$\Rightarrow \boxed{|V_{ab}| = \sqrt{3}|V_a|}$$



In Y-connection

$$\begin{aligned} V_{line} &= \sqrt{3} \times V_{ph} \\ I_L &= I_{ph} \end{aligned}$$

In Δ-connection

$$\begin{aligned} V_{line} &= V_{ph} \\ I_L &= \sqrt{3} I_{ph} \end{aligned}$$

"All the three phase values are generally mentioned as line values, except (resistance/impedence) which is a per phase quantity."

Hence, all the calculations should be made in per phase quantity & then it is to be converted into line values/quantities.

$$P_{\text{total}} = 3 \times V_{\text{ph}} \times I_{\text{ph}} \times \cos \phi$$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos \phi$$

$$\Rightarrow P_{\text{total}} = \sqrt{3} V_L I_L \cos \phi$$

Y-connection

$$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos \phi$$

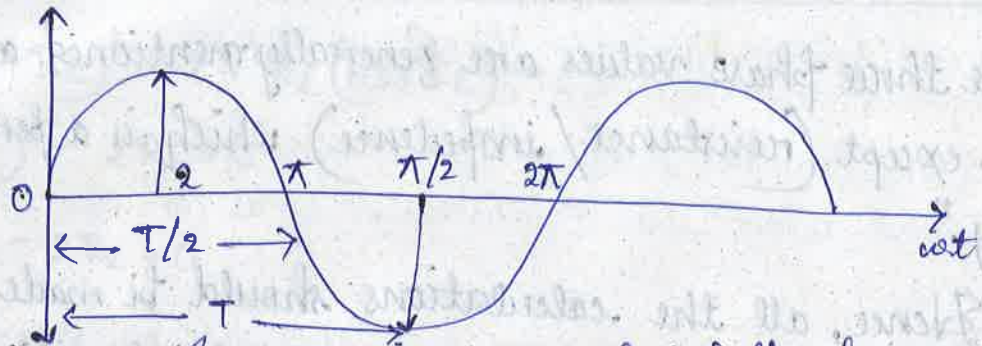
$$\Rightarrow P_{\text{total}} = \sqrt{3} V_L I_L \cos \phi$$

Δ-connection

* Introduction to AC Voltage:

- Alternating Voltage:
- In general the sin wave is more useful for alternating voltages.
- For any wave the time taken to complete one cycle is called time period.
- It can be measured from 0-crossing of one cycle to 0-crossing of another cycle.
- The frequency is defined as the no. of cycles that a wave completes in one sec.

$$f = \frac{1}{T} \quad [\text{Hertz (Hz)}]$$



* The period a sign wave is 20 ms. what is the frequency?

$$1 \text{ ms} = 10^{-3} \text{ sec}$$

$$\Rightarrow 20 \text{ ms} = 20 \times 10^{-3} \text{ sec}$$

$$f = \frac{1}{20 \times 10^{-3}}$$

$$= \frac{1000}{20} = 50 \text{ Hz.}$$

Thus, $\boxed{\text{frequency} = 50 \text{ Hz}}$

A sign wave completes a half cycle in 180° or π radian.

Or,

Quater cycle in 90° or $\pi/2$ radian.

Or,

Full cycle in 360° or 2π radian.

* Phase: \rightarrow The phase is an angular distance that specifies the position of sign wave relative to resistance.

$$1) v(t) = v_m \sin \omega t$$

$$2) v(t) = v_m \sin (\omega t - 90^\circ) \longrightarrow \text{lagging}$$

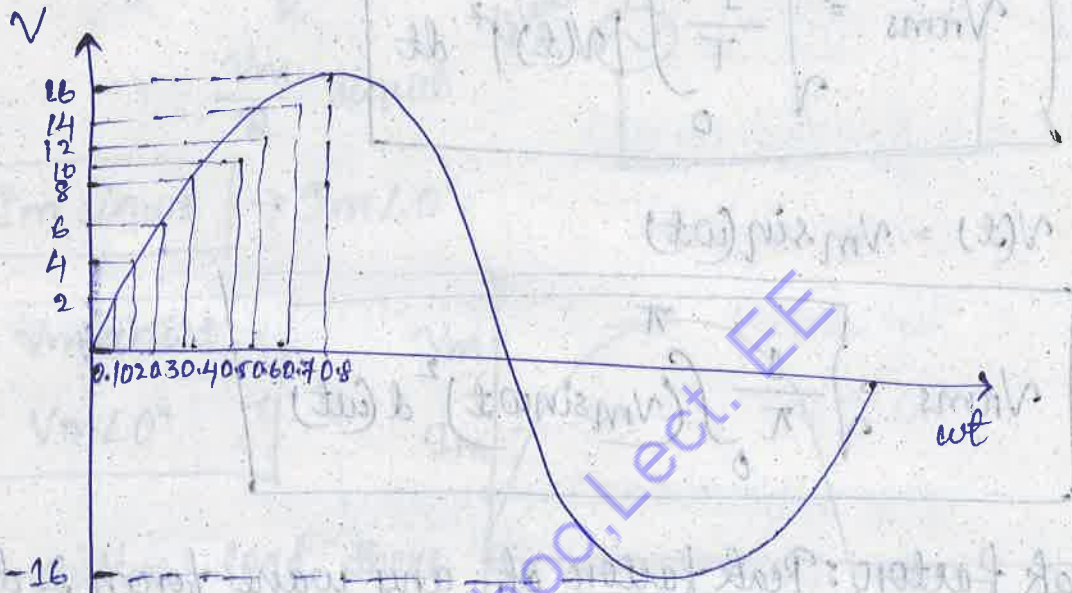
$$3) v(t) = v_m \sin (\omega t + 90^\circ) \longrightarrow \text{leading}$$

$$\boxed{v(t) = v_m \sin (\omega t)}$$

$$\boxed{f = \frac{1}{T}}$$

* Voltage and current values of a sine wave:

● Instantaneous value: The value of



● Peak Value: The peak value of sine wave is the max. value of the wave during +ve or -ve half cycle.

● Peak to peak value: Peak to peak value of a sine wave is the value from +ve to -ve peak.

● Average value: In general, average value of any funcⁿ with time period 'T' is given by

$$\text{Var} = \frac{1}{T} \int_0^T v(t) dt$$

→ The average value of a sine wave is the total area under the half cycle, divided by distance of the curve.

$$\text{Var} = \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t d(\omega t)$$

* RMS:

→ The root mean sq. (rms) value of a sign wave is a measure of heating effect, and is equal to the DC voltage produces same heating effect.

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T |V(t)|^2 dt}$$

$$V(t) = V_m \sin(\omega t)$$

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_0^{\pi} (V_m \sin \omega t)^2 d(\omega t)}$$

* Peak factor: Peak factor of any wave form is defined as the ratio of peak value & rms value.

$$\text{Peak factor} = \frac{V_m}{V_{rms}}$$

* Form factor: Form factor of any wave form is defined as the ratio between rms value & avg. value.

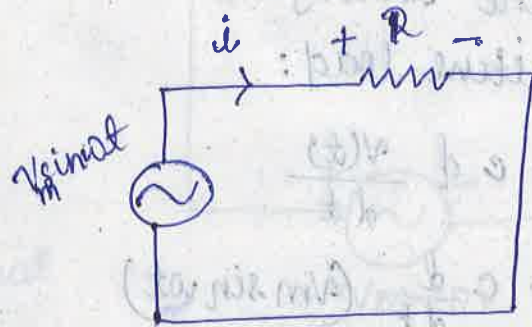
$$\text{Form factor} = \frac{V_{rms}}{V_{av}}$$

$$\frac{1}{\pi} \int_0^{\pi} \sin^2 \omega t d(\omega t)$$

* Phase relation in a resistor:

In this ckt,

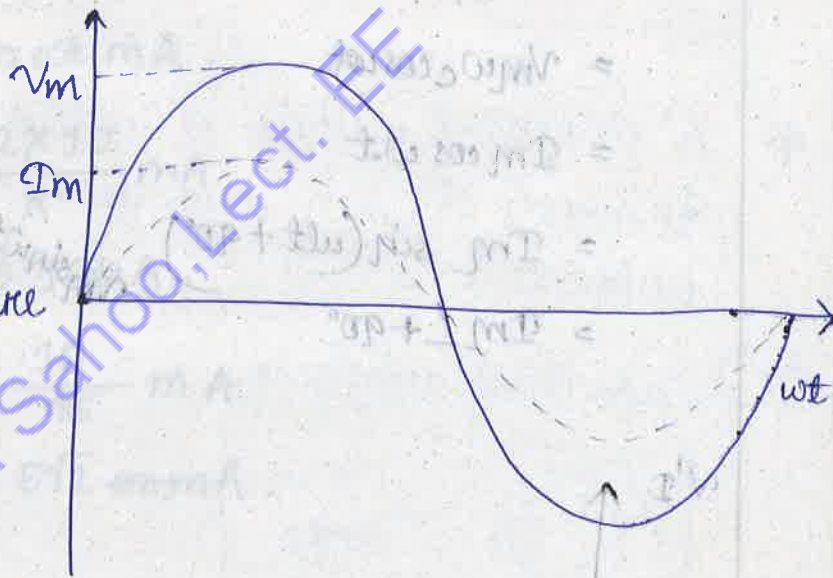
$$\begin{aligned}
 i(t) &= \frac{v(t)}{R} \\
 &= \frac{V_m \sin \omega t}{R} \\
 &= \frac{V_m}{R} \sin \omega t
 \end{aligned}$$



$$\Rightarrow i(t) = I_m \sin \omega t \rightarrow I_m \angle 0^\circ$$

$$\begin{aligned}
 v(t) &= V_m \sin \omega t \\
 &= V_m \angle 0^\circ
 \end{aligned}$$

→ For a resistive load there is no phase diff. between supply voltage & current.



* Phase relation in an Inductor:

voltage induced in an inductor.

$$v(t) = L \frac{di(t)}{dt}$$

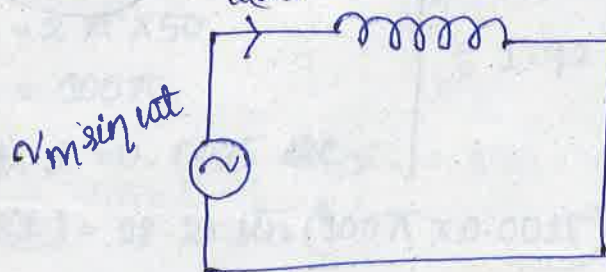
$$= \frac{V_m}{L} - \frac{\cos \omega t}{\omega}$$

$$\Rightarrow i(t) = \frac{1}{L} \int v(t) dt$$

$$= \frac{V_m}{\omega L} - \cos \omega t$$

$$= \frac{1}{L} \int V_m \sin(\omega t) d(\omega t)$$

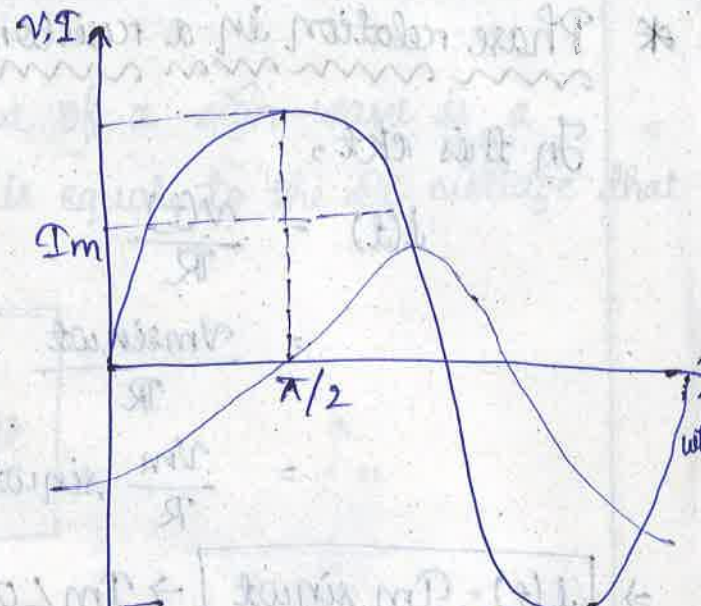
$$= I_m \sin(\omega t - 90^\circ)$$



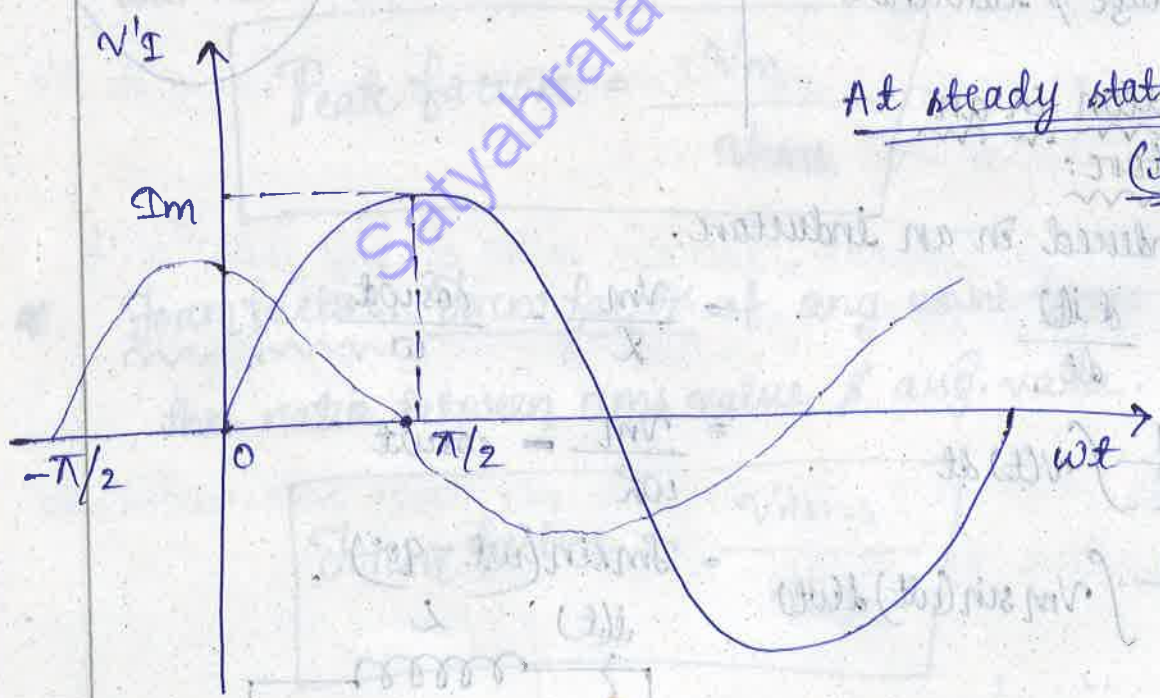
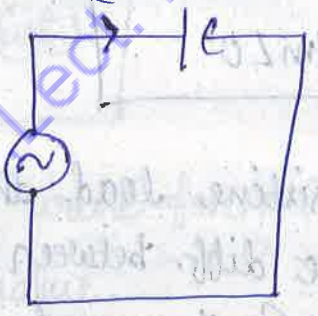
* Phase relation in a capacitor:

Current through a capacitive load:

$$\begin{aligned}
 i(t) &= c \frac{v(t)}{dt} \\
 &= c \frac{d}{dt} (V_m \sin \omega t) \\
 &= c \omega V_m \cos \omega t \\
 &= V_m \omega c \cos \omega t \\
 &= I_m \cos \omega t \\
 &= I_m \sin(\omega t + 90^\circ) \\
 &= I_m \angle +90^\circ
 \end{aligned}$$



∴ current is lagging by $i(t)$ 90° from voltage.



At steady state $(t \rightarrow \infty)$



* A sinusoidal voltage is ~~100~~ applied to the circuit. Determine rms current, average current, peak current & peak to peak current.

Solⁿ: Applied voltage,

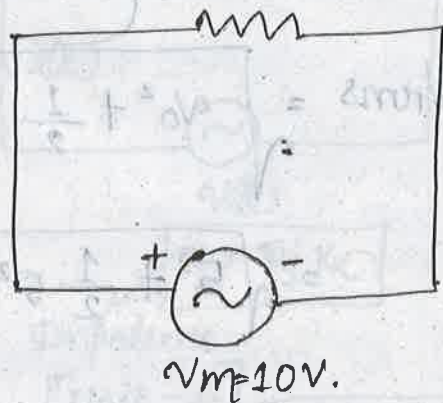
$$V(t) = V_m \sin \omega t$$

$$= 10 \sin \omega t$$

$$i(t) = \frac{V(t)}{R} = \frac{10}{1K} \sin \omega t$$

$$= 0.01 \sin \omega t \text{ A}$$

$$= 10 \sin \omega t \text{ mA}$$



$$\text{Avg. current} = \frac{2I_m}{\pi} = \frac{2 \times 10}{\pi} \text{ mA}$$

$$= 3.36 \text{ mA}$$

$$\text{RMS current} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ mA}$$

$$= 5\sqrt{2} \text{ mA}$$

$$\text{Peak current} = 10 \text{ mA}$$

$$\text{Peak to peak} = 20 \text{ mA}$$

* An alternating current varying with a frequency of 50 Hz & RMS value of 20 A. Write the equⁿ for instantaneous value & find its value at 0.0025 sec & 0.0125 sec.

Solⁿ: $f = 50 \text{ Hz}$

$$I_{\text{rms}} = 20 \text{ A}$$

$$i(t) = I_m \sin \omega t$$

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$$

$$\Rightarrow I_m = \sqrt{2} \times I_{\text{rms}}$$

$$= 28.28$$

$$i(t) = 28.28 \sin \omega t$$

$$\omega = 2\pi f$$

$$= 2\pi \times 50$$

$$= 100\pi$$

$$\text{At } t = 0.0025 \text{ sec,}$$

$$i(t) = 28.28 \sin(100\pi \times 0.0025)$$

$$= 0.367 \text{ A}$$

$$\text{At } t = 0.0125 \text{ sec,}$$

$$i(t) = 28.28 \sin(100\pi \times 0.0125)$$

$$= 1.92 \text{ A}$$

* Determine the rms value of voltage defined by $v(t) = 5 + 5 \sin\left(314t + \frac{\pi}{6}\right)$.

Ans:

$$V_{rms} = \sqrt{V_0^2 + \frac{1}{2} (V_{c1}^2 + V_{c2}^2 + V_{c3}^2 + \dots)}$$

$$= \sqrt{5^2 + \frac{1}{2} 5^2} = \sqrt{25 + \frac{25}{2}}$$

$$= \sqrt{\frac{50 + 25}{2}} = \sqrt{\frac{75}{2}}$$

$$= 6.12 \text{ V}$$

* A sinusoidal voltage is applied to a capacitor, the frequency of sign wave is 2 kHz. determine capacitive resistance?

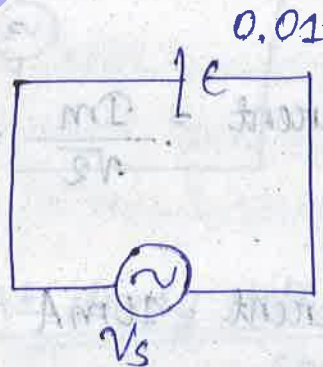
Soln:

Capacitive reactance,

$$X_c = \frac{1}{\omega c} = \frac{1}{2\pi f c}$$

$$= \frac{1}{2\pi \times 2 \times 10^3 \times 0.01 \times 10^{-6}}$$

$$= 7.96 \text{ K}\Omega$$



* Determine the inductive reactance & rms current in the circuit given.

Soln:

$$V_{rms} = 10 \text{ V}$$

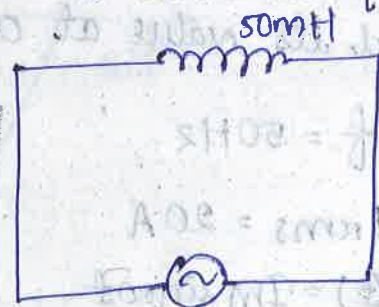
$$I_{rms} = \frac{V_{rms}}{X_L}$$

$$X_L = \omega L = 2\pi f L$$

$$X_L = 2\pi \times 10 \times 10^3 \times 50 \times 10^{-3} = 3.14 \text{ K}\Omega$$

$$I_{rms} = \frac{10}{3.14 \times 10^3}$$

$$= 3.18 \text{ mA}$$



$$V_s = 10 \text{ V}$$

$$10 \text{ KHz}$$

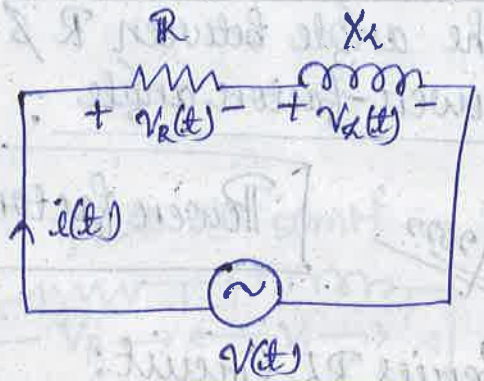
Impedance Diagram:

Applying KVL,

$$V(t) - V_R(t) - V_L(t) = 0$$

$$\Rightarrow V(t) = V_R(t) + V_L(t)$$

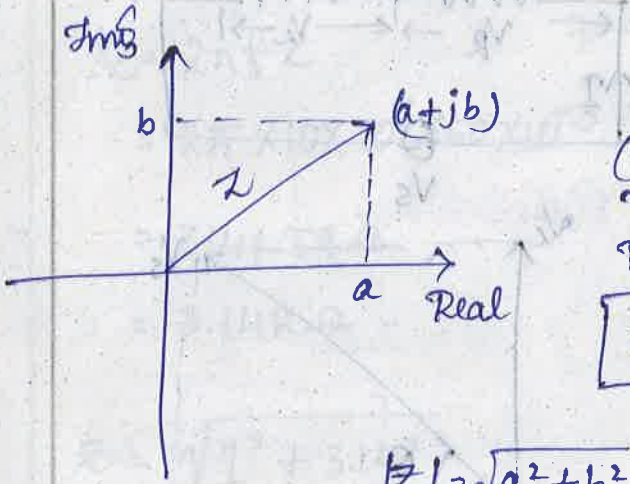
$$= i(t) \times R + L \frac{di(t)}{dt}$$



$$Z = R + jX_L$$

Impedance
[Unit - Ω]

$$i(t) = \frac{V(t)}{Z}$$



Complex No. Notation

$$Z = a + bj$$

Rectangular

Imaginary

$$|Z| = \sqrt{a^2 + b^2}$$

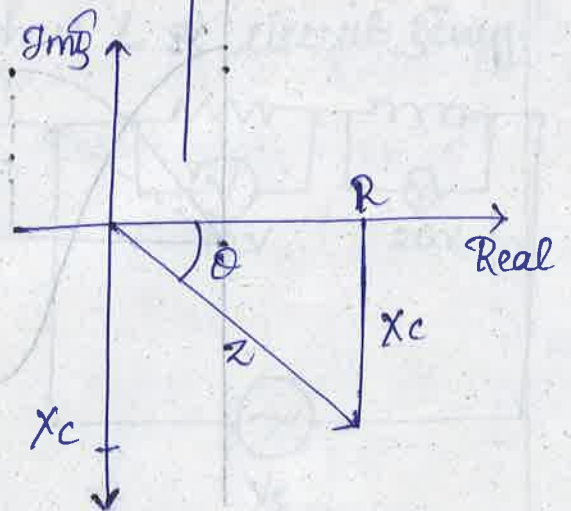
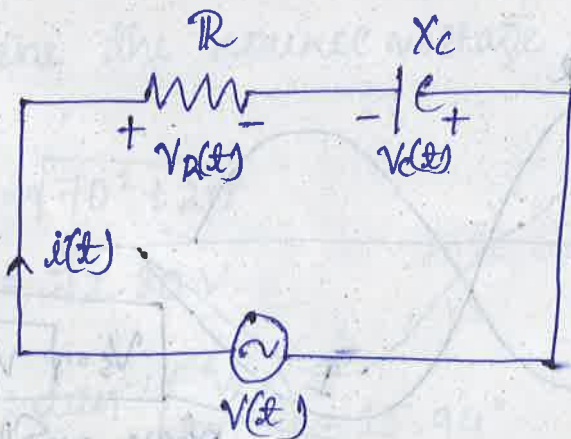
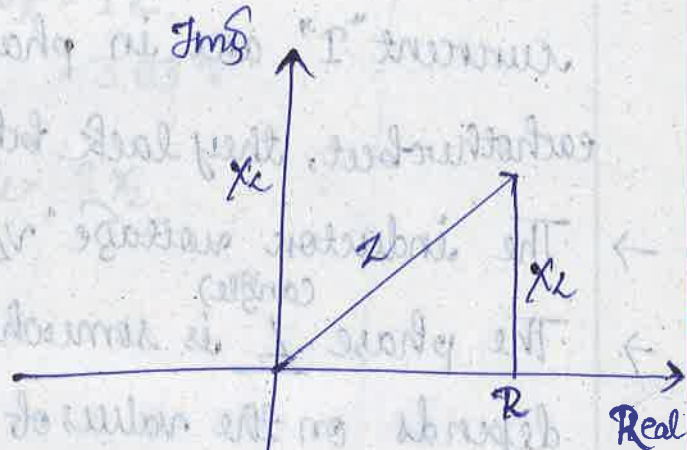
$$\angle \theta = \tan^{-1}(b/a)$$

For capacitor,

$$V(t) - V_R(t) - V_C(t) = 0$$

$$\Rightarrow V(t) = V_R(t) + V_C(t)$$

$$= i(t) \times R + \frac{1}{C} \int i(t) dt$$

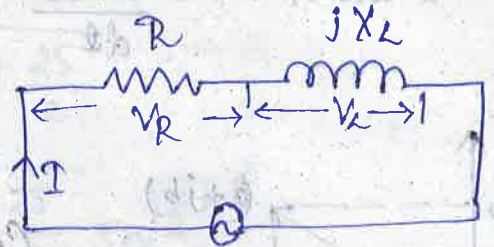


→ The angle between R & Z in the impedance Δ is called power-factor angle.

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Power factor = $\cos \theta$

* Series RL Circuit:

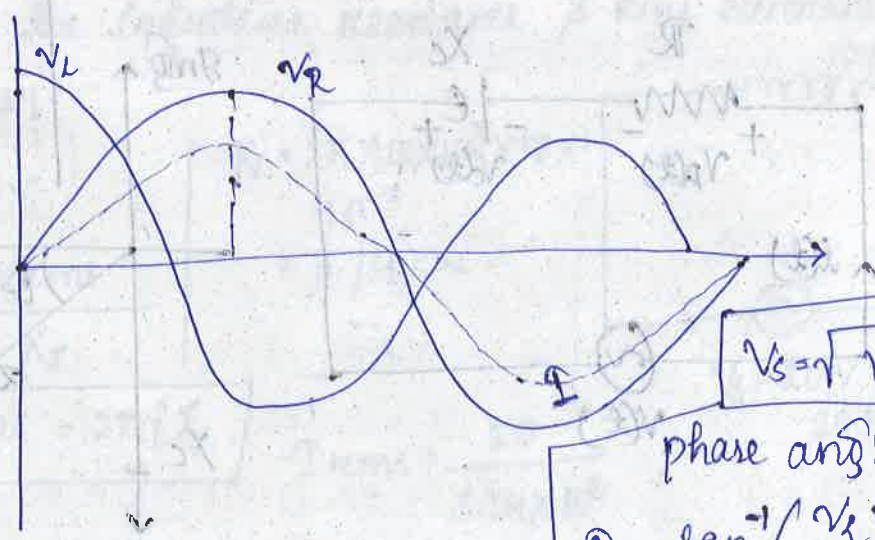
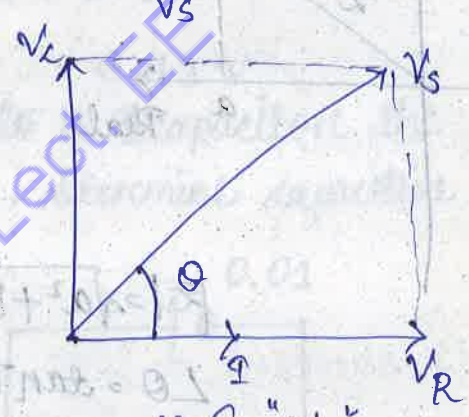


→ If AC supply is given to a series RL circuit, the current & voltage across all the elements are sinusoidal.

→ The resistor voltage " V_R " and current " I " are in phase with each other but, they lag behind supply voltage " V_s ".

→ The inductor voltage " V_L " leads the source voltage " V_s ".

→ The phase angle is somewhere between 0° & 90° which depends on the values of " R " & " X_L ".



$V_s = \sqrt{V_R^2 + V_L^2}$

phase angle,

$\theta = \tan^{-1} \left(\frac{V_L}{V_R} \right)$

1)

Soln:

$$Z = \sqrt{R^2 + X_L^2}$$

$$X_L = 2\pi fL$$

$$= 2\pi \times 10 \times 10^3 \times 50 \times 10^{-3}$$

$$= 3141.59 \Omega$$

$$= 3.14 \text{ k}\Omega$$

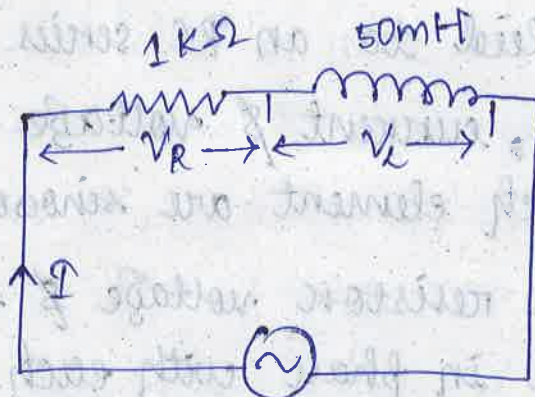
$$Z = \sqrt{1^2 + 3.14^2}$$

$$= \sqrt{10.8596}$$

$$= 3.29 \text{ k}\Omega$$

$$|I| = \frac{V_s}{|Z|} = \frac{10}{3.29}$$

$$= 3.03 \text{ mA}$$



$$V_s = 10 \text{ V}$$

$$\text{phase angle } (\theta) = \tan^{-1}\left(\frac{X_L}{R}\right) = 72.33^\circ$$

$$V_R = IR$$

$$= 3.03 \text{ V}$$

$$V_L = IX_L$$

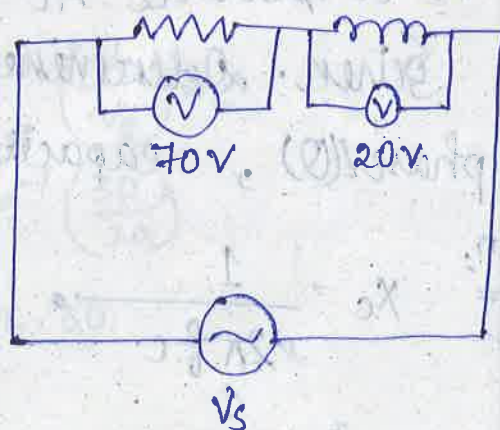
2) Determine the source voltage & phase \angle of circuit given.

Soln:

$$V_s = \sqrt{70^2 + 20^2}$$

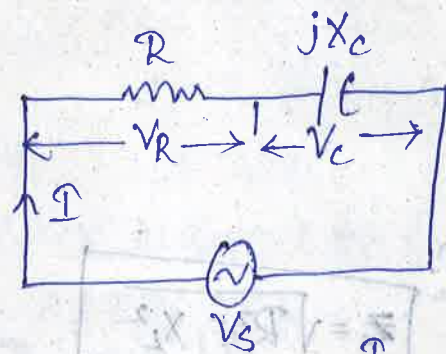
$$= 72.80 \text{ V}$$

$$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right) = 15.94^\circ$$

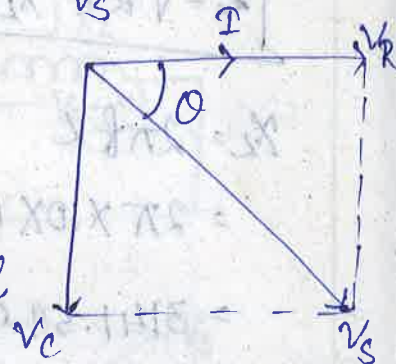


* Series RC circuit:

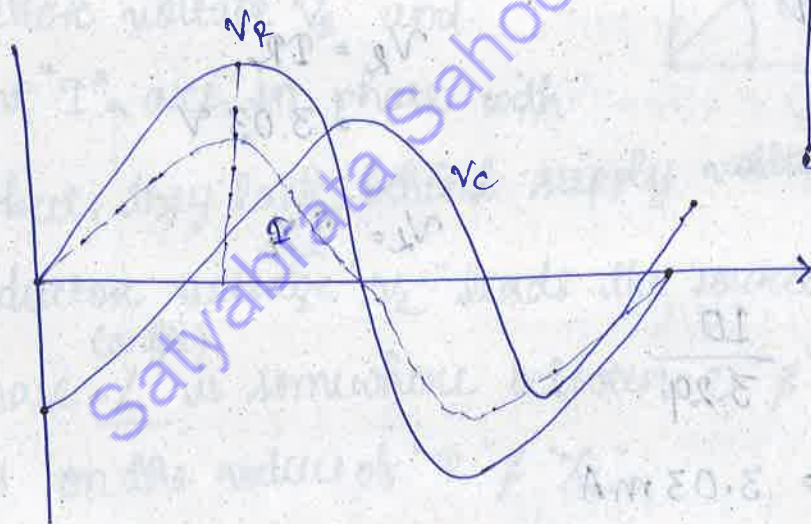
→ When a sinusoidal voltage is applied to an RC series circuit the current & voltage across each element are sinusoidal.



→ The resistor voltage & current are in phase with each other, the capacitor voltage lags behind source voltage.



→ The phase \angle between current & capacitor voltage is 0° to 90° .



$$V_s = \sqrt{V_R^2 + V_C^2}$$

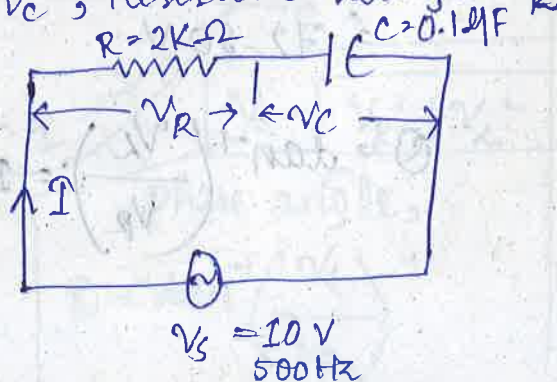
$$\theta = \tan^{-1} \left(\frac{V_C}{V_R} \right)$$

1) A 10V, 500 Hz AC voltage is applied to the circuit given. Determine total impedance 'Z', current 'I', phase $\angle(\theta)$, capacitive voltage 'V_C', resistive voltage 'V_R'.

Solⁿ:

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 500 \times 0.1 \times 10^{-6}} = 3183.09 \Omega$$



$$Z = (2000 - j3183.09) \Omega$$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$$= \sqrt{(2000)^2 + (3183.09)^2}$$

$$= 3759.26 \Omega$$

$$I = \frac{V_s}{|Z|} = \frac{10}{3759.26}$$

$$= 2.66 \text{ mA}$$

$$\theta = \tan^{-1} \left(\frac{X_c}{R} \right) = \tan^{-1} \left(\frac{3183.09}{2000} \right)$$

$$= 57.85^\circ$$

$$V_R = IR$$

$$= 2000 \times 2.66 \times 10^{-3}$$

$$= 5.32 \text{ V}$$

$$V_c = IX_c$$

$$= 2.66 \times 10^{-3} \times 3183.09$$

$$= 8.46 \text{ V}$$

2)

sol?

$$V_R = 20 \text{ V}, V_c = 30 \text{ V}$$

$$V_s = \sqrt{V_R^2 + V_c^2}$$

$$= \sqrt{20^2 + 30^2}$$

$$= \sqrt{400 + 900}$$

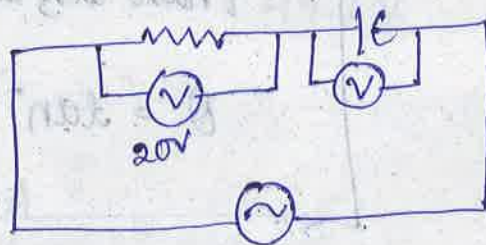
$$= \sqrt{1300}$$

$$= 36.05 \text{ V}$$

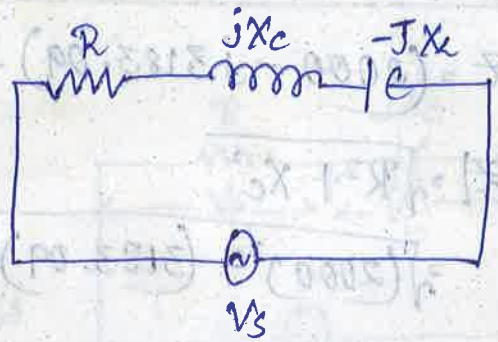
$$\theta = \tan^{-1} \left(\frac{V_c}{V_R} \right)$$

$$= \tan^{-1} \left(\frac{30}{20} \right)$$

$$= 56.30^\circ$$



* RLC Series Circuit:



- Series RLC circuit is the combination of resistance, inductance & capacitance.
- Inductive reactance, X_L is indicated on +ve j-axis.
- In capacitive reactance, X_C is indicated on -ve j-axis.
- The magnitude & type of reactance in a series RLC circuit is the difference of two reactances.

Impedance,

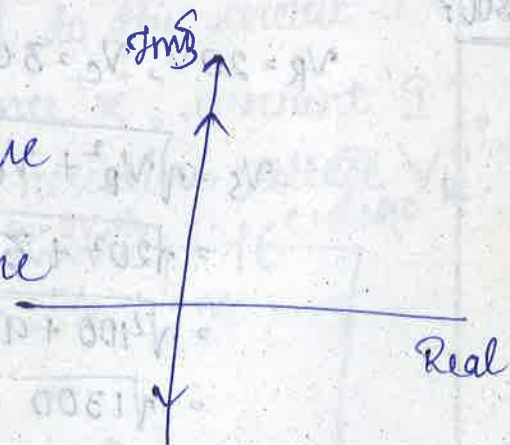
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Phase angle,

$$\theta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

- (i) If $X_L > X_C \rightarrow +ve \rightarrow$ Inductive
- (ii) If $X_C < X_L \rightarrow -ve \rightarrow$ Capacitive

1)



Solⁿ:

$$X_L = 2\pi fL =$$

$$X_C = \frac{1}{2\pi fc} = \cancel{31830.98} \Omega$$

$$= 318.5 \Omega$$

$$Z = R + j(X_L - X_C)$$

$$= 10 + j(157 - 318.5)$$

$$= 10 - j161.5$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{10^2 + (-161.5)^2}$$

$$= 161.80$$

$$\theta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$= \tan^{-1} \left(\frac{-161.5}{10} \right)$$

$$= -86.45$$

$$I = \frac{V_s}{|Z|} = \frac{10}{161.80} = 0.3 \text{ A}$$

$$V_R = IR = 0.3 \times 10$$

$$= 3 \text{ V}$$

$$V_L = IX_L$$

$$= 95.5 \text{ V}$$

$$V_C = IX_C$$

$$= 47.1 \text{ V}$$

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* Determine the impedance and phase angle and current in the circuit.

Soln:

$$Z = \sqrt{R^2 + X_L^2}$$

$$X_L = 2\pi fL$$
$$= 2\pi \times 50 \times 10^{-3} \times$$

$$R_{eq} = R_1 + R_2$$
$$= 10 + 20$$

$$\boxed{R_{eq} = 30 \Omega}$$

$$15.7 + 31.41$$
$$= 47.11 \Omega$$

$$Z = (10 + 20) + j(15.7 + 31.41)$$

$$Z = \sqrt{30^2 + 47.11^2}$$

$$= (30 + j47.11) \Omega$$

$$= \sqrt{30^2 + 47.11^2}$$

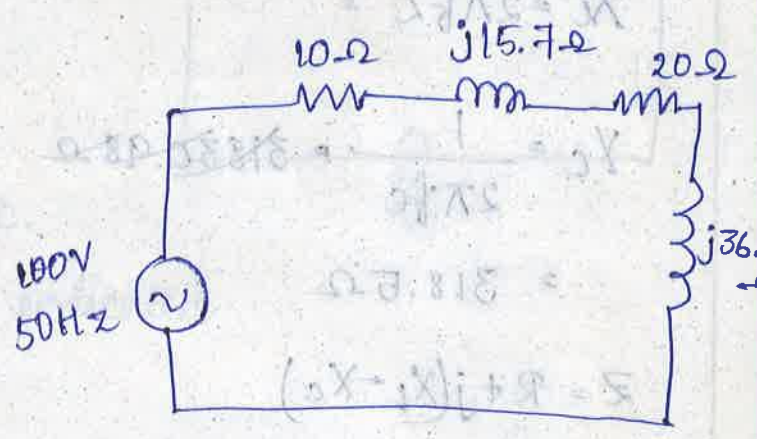
$$= 55.85 \Omega$$

$$\text{phase angle} = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$= \tan^{-1} \left(\frac{47.11}{30} \right)$$

$$= 57.51^\circ$$

$$I = \frac{V}{Z} = \frac{100}{55.85} = 1.79 \text{ A}$$



Satyabrata Sahoo Lect. EE

current find the Parallel R.C. Circuit total current, phase angle & total impedance.

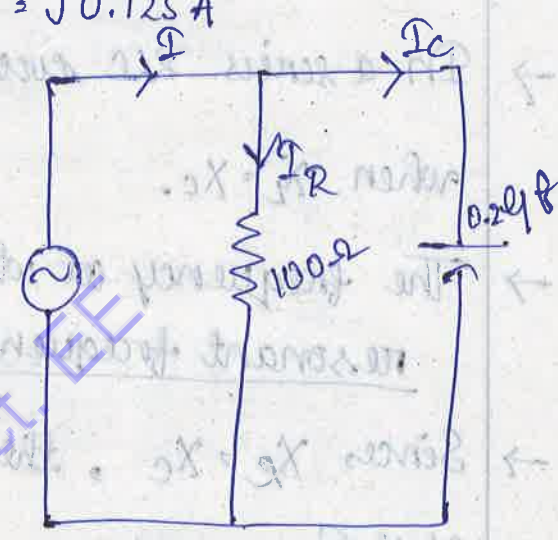
$$I_R = \frac{V}{R} = 0.2 \text{ A}$$

$$I_C = \frac{V}{X_C} = \frac{V}{\left(\frac{1}{j2\pi fC}\right)} = j2\pi fC \cdot V = j0.125 \text{ A}$$

$$I = I_R + jI_C = 0.2 + j0.125$$

$$= \sqrt{0.2^2 + 0.125^2} = 0.23 \text{ A}$$

$$Z = \frac{V}{I} = \frac{20}{(0.2 + j0.125)} = (71.91 - j44.94) = 84.79 \angle -32.005$$



Find the total current, phase angle & total impedance.

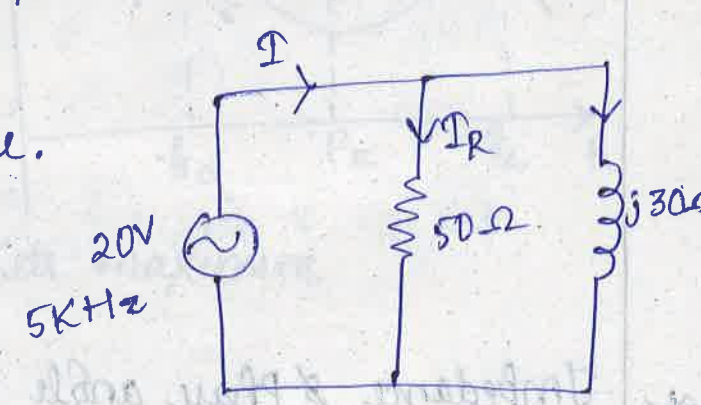
$$I_R = \frac{V}{R} = 0.4 \text{ A}$$

$$I_C = \frac{V}{X_C} = \frac{V}{j2\pi fC}$$

$$= j0.66 \text{ A}$$

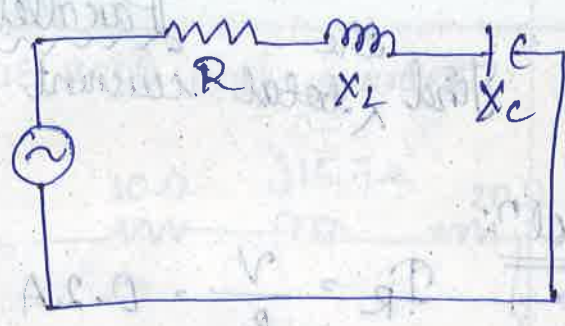
$$Z = \frac{V}{I} = \frac{20}{0.4 - j0.66} = 25.91 \angle 58.7^\circ \Omega$$

$$I = I_R + I_C = 0.4 - j0.66 = \sqrt{(0.4)^2 + (0.66)^2} = 0.77$$



27.01/02/2022

Resonance:



* Series resonance:

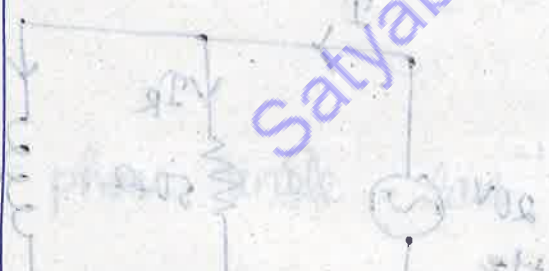
→ The circuit is said to be in resonance if the current is in phase with the applied voltage.

→ In a series RLC circuit resonance occurs, when $X_L = X_C$.

→ The frequency at which resonance occurs is called resonant frequency.

→ Since $X_L = X_C$, the impedance is purely resistive.

→ Voltage across capacitance & inductance are equal in magnitude.



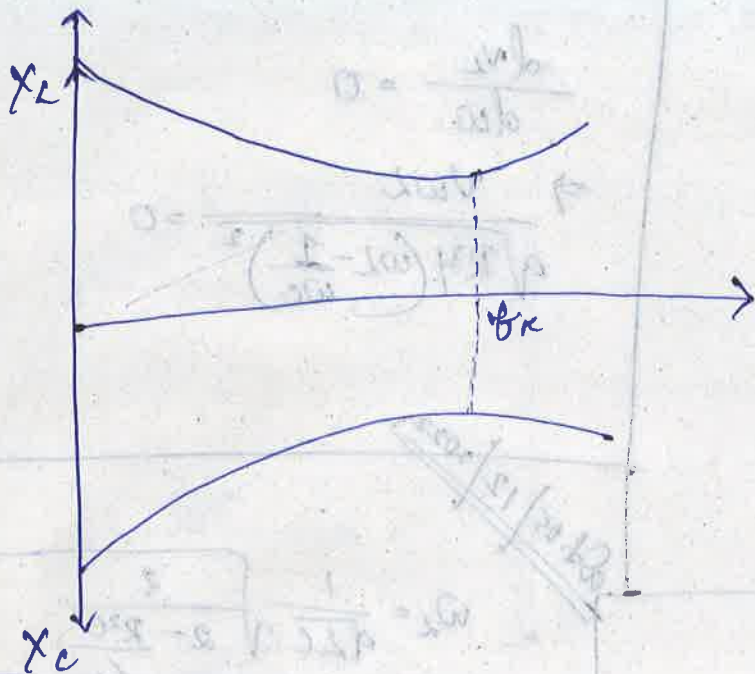
* Impedance & Phase angle of a series Resonant Circuit:

$$X_L = \omega L = 2\pi fL$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

$$\boxed{Z = R + j(X_L - X_C)} \quad \text{--- +ve.}$$

$$f_L = 0 \quad f_C = \infty \quad \left(\because Z = R + j(X_L^\circ - X_C^\infty) \right) \quad \text{--- -ve.}$$

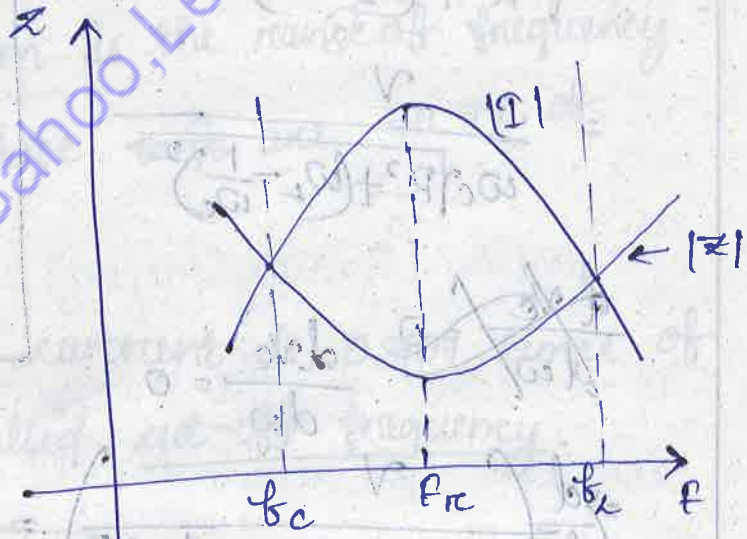


* Voltage & Current Series Resonance:

$$f_{rc} = X_L = X_C$$

$$Z = R + j$$

$$I_1 = \frac{V}{Z_m}$$



→ The drop across resistance is maximum

Similarly,

Maximum voltage across inductor occurs at

$$f = f_x$$

$$\begin{aligned}
 V_L &= I X_L \\
 &= I \omega L \\
 &= \frac{V \omega L}{Z} \\
 &= \frac{V \omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{dV_L}{d\omega} &= 0 \\
 \Rightarrow \frac{V \omega L}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} &= 0
 \end{aligned}$$

20/05/22

$$\begin{aligned}
 V_C &= I X_C \\
 &= \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \cdot \frac{1}{\omega C} \\
 &= \frac{V}{\omega C \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}
 \end{aligned}$$

$$\frac{dV_C}{d\omega} = 0 \quad \frac{dV_C}{d\omega} = 0$$

$$\Rightarrow \frac{d}{d\omega} \left(\frac{V}{\omega C \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \right) = 0$$

$$\omega_L = \frac{1}{\sqrt{LC}} \sqrt{\frac{2}{2 - \frac{R^2 C}{L}}}$$

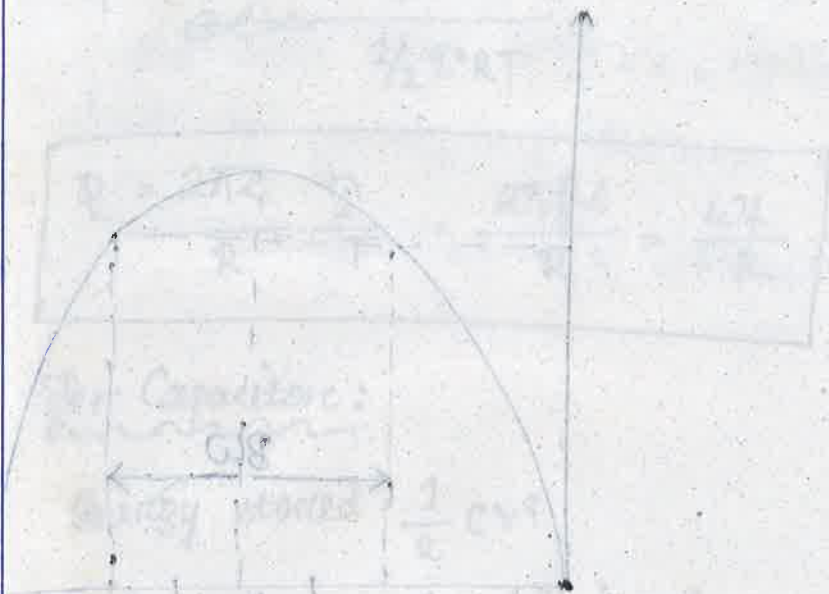
$$f_L = \frac{1}{2\pi \sqrt{LC}} \sqrt{\frac{1}{1 - \frac{R^2 C}{L}}}$$

frequency at which
voltage across inductor
is maximum.

$$\omega_C = \sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$f_C = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

frequency at which
voltage across capacitor is
max.



Band-width of Resonant Circuit:

Band-width of any system is the range of frequency for which currents voltage ~~are~~ are 70.7% of max. value.

The frequency at which current is 0.707 times of maxi. value are also called cut-off frequency.

$$BW = f_2 - f_1$$

Unit = Hertz (Hz).

$$BW = \frac{R}{2\pi L}$$

$$f_1 = f_r - \frac{BW}{2}$$

$$f_2 = f_r + \frac{BW}{2}$$

At upper & lower cut-off frequency power absorbed is half of max. power.

Hence, these are called half power frequency.

$$P = V \cdot I$$

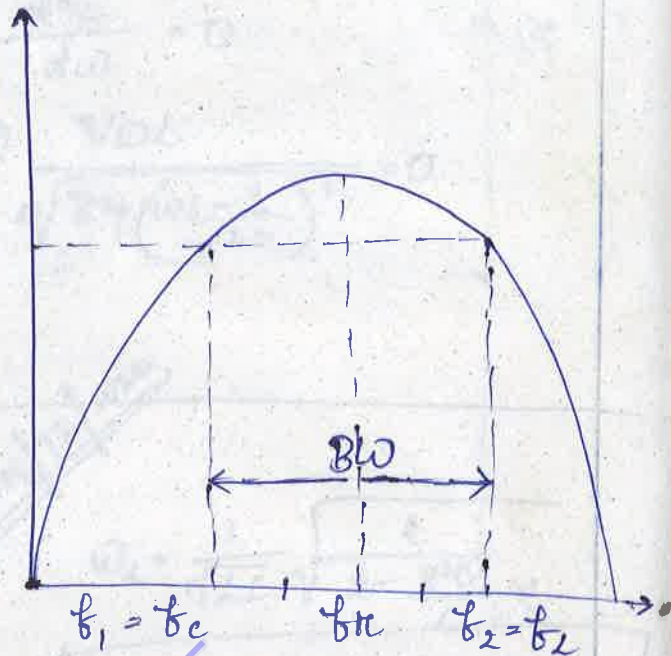
$$= \frac{V}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} = \frac{VI}{2}$$

$$= \frac{P}{2}$$

$$I^2 R$$

$$= \left(\frac{I}{\sqrt{2}} \right)^2 \cdot R$$

$$= \frac{I^2 R}{2} = \frac{P}{2}$$



* Relation between Bandwidth & Q-factor:

$$BW = \frac{f_c}{Q}$$

* Q-Factor (Quality Factor):

→ Q-Factor is the ratio of reactive power in inductor & capacitor to

$$Q = 2\pi \times \frac{\text{max. energy stored}}{\text{Energy dissipated / cycle}}$$

In Inductor:

$$\text{Energy stored} = \frac{1}{2} LI^2$$

$$\text{Energy dissipated in Resistance} = \left(\frac{I}{\sqrt{2}} \right)^2 \times R \times T$$

$$= \frac{I^2 RT}{2}$$

$$Q = \frac{2\pi \times \frac{1}{2} LI^2}{\frac{1}{2} I^2 RT}$$

$$Q = \frac{2\pi L}{R} \cdot \frac{I}{T} = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

For Capacitors:

$$\text{Energy stored} = \frac{1}{2} CV^2$$

$$= \frac{1}{2} C \left(\frac{1}{\omega C} \right)^2 \times I_2^2$$

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* A series circuit with $R = 10\ \Omega$, $L = 0.1\ \text{H}$, $C = 50\ \text{mF}$ has an applied voltage, $V = 50\ \text{V}$ with variable frequency, find the resonant frequency at which voltage across inductor & capacitor are max.

Solⁿ:

$$L = 0.1\ \text{H}$$

$$R = 10\ \Omega$$

$$C = 50\ \mu\text{F}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 50 \times 10^{-6}}}$$

$$= 71.18\ \text{Hz}$$

$$f_c = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{1}{0.1 \times 50 \times 10^{-6}} - \frac{10^2}{2 \times 0.1}} = 71.08\ \text{Hz}$$

$$f_L = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2}{2L}}$$

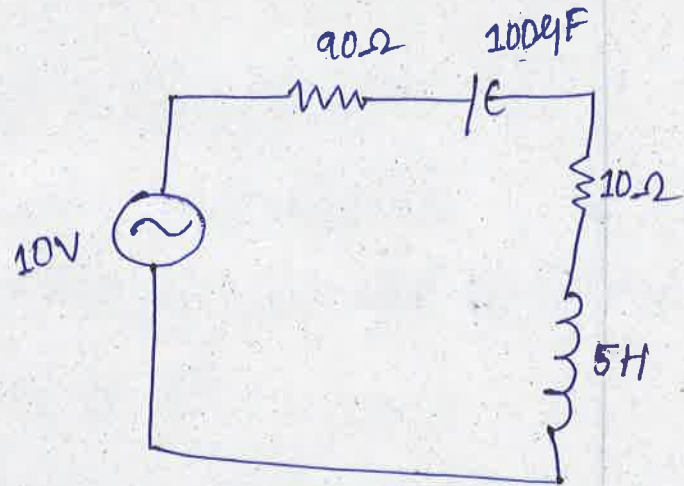
$$= 71.18 \sqrt{1 - 2 \times 0.1}$$

$$= 72.18$$

= 50 mF
ble
which
max.

Determine the value of Q -factor of ~~resonance~~ ^{bandwidth} of the circuit.

$$Q = \frac{2\pi f_{\pi} L}{R} = 2.24$$



$$f_{\pi} = \frac{1}{2\pi\sqrt{LC}} = 7.12 \text{ Hz}$$

$$BW = \frac{f_{\pi}}{Q}$$

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21/12/2022

Magnetic Circuits

* Magnetic flux (Φ):

- The total number of lines of forces existing in a particular magnetic field is called magnetic flux.
- The unit of flux is weber and flux is denoted by symbol (Φ). The unit weber is denoted by Wb.

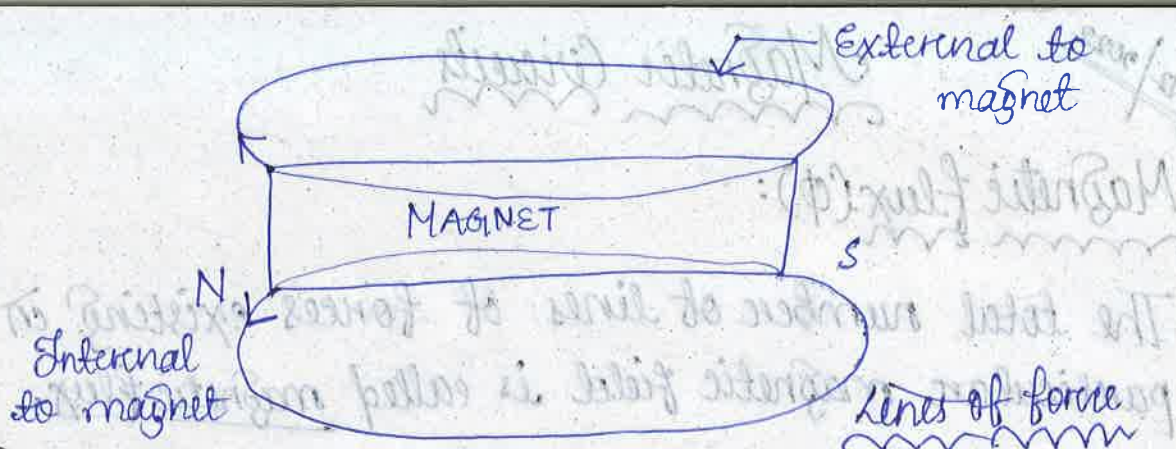
$$1 \text{ weber} = 10^8 \text{ lines of force}$$

* Properties of lines of force:

- Though the lines of force are imaginary, with the help of them various magnetic effects can be explained very conveniently.

Various properties of lines of force:

- Lines of force always originate on a N-pole and terminate on a S-pole, external to the magnet.
- Each line forms a closed loop.
- Lines of force never intersect each other.
- The lines of force, are like stretched rubberbands & always try to contract in length.
- The lines of force, which are parallel & travelling in the same direction repel each other.
- Magnetic lines of force always prefer a path offering least opposition.

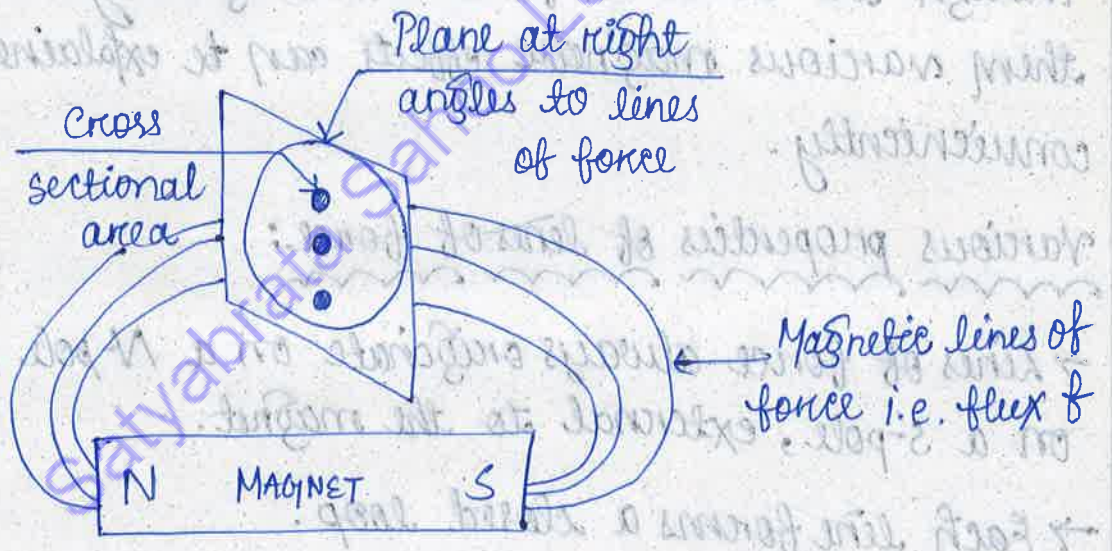


* Magnetic Flux density (B): complete the closed path

It can be defined as, the flux per unit area (a) in a plane at right angles to the flux is known as "flux density".

Mathematically,

$$B = \frac{\Phi}{a} \frac{Wb}{m^2} \text{ or Tesla}$$



Concept of magnetic flux density

* Magnetic Field strength (H):

- This gives quantitative measure of strength or weakness of the magnetic field.
- This can be defined as the force experienced by a unit N-pole when placed at any point inside a magnetic field.

nal to magnet

It is denoted by H & its unit is newton per weber i.e. N/Wb or ampere per meter (A/m) or ampere turns per meter (AT/m).

of force closed path

The mathematical expression for calculating magnetic field strength.

$$H = \frac{\text{ampere turns}}{\text{length}}$$

(as in a x density)

$$\therefore H = \frac{NI}{l} \text{ AT/m}$$

Permeability:

Permeability is the measure of the ease, with which magnetic lines of force pass through a given material.

The ability of a material to concentrate magnetic flux is called permeability and its symbol is the Greek lower case letter μ . Any material that is easily magnetized tends to concentrate magnetic flux. i.e. flux

Because soft iron is easily magnetized, so it has a high permeability. The permeability of a material is a measure of how easy it is for flux lines to pass through it.

Numerical values of μ for different materials are assigned by a unit by comparing their permeability with the permeability of air or vacuum.

→ Mathematically, μ can be defined as the ratio of flux density to magnetizing force.

$$\mu = \frac{B}{H}$$

* Absolute Permeability:

→ The permeability of free space, μ_0 , is

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

→ The absolute permeability of another material can be expressed relative to the permeability of free space.

Then,

$$\mu = \mu_0 \mu_r$$

→ Where is the dimensionless quantity called relative permeability.

* Relative Permeability:

→ The relative permeability of a magnetic material, designated μ_r , is the ratio of the all practical purposes, equal to unity.

→ The μ_r of a non-magnetic material such as air, copper, wood, glass & plastic is, for all practical purposes, equal to unity.

μ can be defined as the ratio of magnetic flux density to magnetizing force. Iron, steel & their alloys are ferromagnetic & their μ are not constant.

$$\mu = \frac{B}{H}$$

Permeability:

of free space, μ_0 is

$$= 4\pi \times 10^{-7} \text{ H/m}$$

the property of the substance, which is the ratio of magnetic flux through it.

is similar to the resistance in an electrical circuit, denoted by the letter "S" and is measured in ampere per weber, (AT/Wb).

Permeability of another material is the ratio of the permeability of that material to the permeability of free space.

$$\mu_r = \frac{\mu}{\mu_0}$$

For any part of a magnetic circuit, the reluctance is the ratio of the drop in magnetomotive force (MMF) to the flux produced in that part of the circuit.

Reluctance is a dimensionless quantity.

$$S = \frac{\text{m.m.f.}}{\text{flux}} = \frac{NI}{\phi} \text{ AT/Wb}$$

Reluctance:

Reluctance of a magnetic circuit is the ratio of the MMF to the flux.

Reluctance of a magnetic circuit depends upon

the permeability of the material, i.e. μ , the length of the magnetic path, i.e. 'l'

the cross-sectional area of the material, i.e. 'a'

the length of the magnetic path, i.e. 'l'

→ It is denoted by H & its unit is newton per weber i.e. (N/Wb) or ampere per metre (A/m) or ampere turns per metre (AT/m) .

→ The mathematical expression for calculating magnetic field strength.

$$H = \frac{\text{ampere turns}}{\text{length}}$$

$$\therefore H = \frac{NI}{l} \text{ AT/m}$$

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→ Mathematically, μ can be defined as the ratio of flux density to magnetizing force.

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→ Where is the dimensionless quantity called relative permeability.

* Relative Permeability:

→ The relative permeability of a magnetic material, designated μ_r , is the ratio of the all practical purposes, equal to unity.

→ The μ_r of a non-magnetic material such as air, copper, wood, glass & plastic is, for all practical purposes, equal to unity.

→ On the otherhand, the perc of magnetic materials such as cobalt, nickel, iron, steel & their alloys are far greater than unity & are not constant.

* Reluctance (S):

→ Reluctance is the property of the substance, which opposes the flow of magnetic flux through it.

→ It is analogous to the resistance in an electrical circuit. It is denoted by the letter "S" and is measured in ampere-turn per weber, (AT/Wb).

→ Reluctance of any part of a magnetic circuit is also defined as the ratio of the drop in magnetomotive force to the flux produced in that part of the circuit.

∴ Reluctance,

$$S = \frac{\text{m.m.f.}}{\text{flux}} = \frac{NI}{\phi} \text{ AT/Wb}$$

→ Reluctance of magnetic circuit depends upon

(I) Nature of the material, i.e. μ or μ_r

(II) Length of the material magnetic path,
i.e. 'l'

(III) Cross-sectional area of the material, i.e., 'a'

Mathematically,

$$S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A} \quad \text{AT/Wb}$$

where

μ_0 = Absolute permeability in H/m.

μ_r = Relative permeability

Problem 1: A current of 2A is flowing through each of the conductors in a coil containing 15 such conductors. If a point pole of unit strength is placed at a perpendicular distance of 10cm from the coil, determine the field intensity at that point.

Soln: Given data,

$$I = 2A$$

$$N = 15$$

$$d = 10 \text{ cm} = 0.1 \text{ m}$$

$$H = \frac{NI}{2\pi d} = \frac{15 \times 2}{2\pi \times 0.1}$$

$$H = 47.74 \text{ AT/m.}$$

Problem 2: A toroidal core has a magnetic path length of 33cm & a magnetic field strength of 650 A/m. The coil current is 250 mA. Determine the no. of coil turns.

Solⁿ:

Given data,

$$H = 650 \text{ A/m}$$

$$I = 250 \text{ mA} = 0.25 \text{ A}$$

$$l = 33 \text{ cm} = 0.33 \text{ m}$$

$$H = \frac{NI}{l}$$

$$650 = \frac{N \times 0.25}{0.33}$$

$$N = \frac{650 \times 0.33}{0.25}$$

$$= 858 \text{ turns.}$$

Problem 3: Determine the m.m.f. required to generate a total flux of $100 \mu\text{Wb}$ in an air gap 0.2 cm long. The cross-sectional area of the air gap is 25 cm^2 .

Solⁿ: Given data,

$$\Phi = 100 \mu\text{Wb}$$

$$= 100 \times 10^{-6} \text{ Wb}$$

$$A = 25 \times 10^{-4} \text{ m}^2$$

$$l = 0.2 \times 10^{-2} \text{ m}$$

$$\text{Flux density, } B = \frac{\Phi}{A} = \frac{100 \times 10^{-6}}{25 \times 10^{-4}} = 4 \times 10^{-2} \text{ Wb/m}^2$$

$$\text{Magnetising force, } H = \frac{B}{\mu_0} = \frac{4 \times 10^{-2}}{4\pi \times 10^{-7}} = 3.18 \times 10^4 \text{ AT/m}$$

Now, $H = \frac{MMF}{l}$

$$MMF = H \times l = 3.18 \times 10^4 \times 0.2 \times 10^{-2}$$

$$= 63.7 \text{ AT}$$

Problem 4: An air-cored toroidal has 3000 turns of carries a current of 0.1 A. The cross-sectional area of the coil is 4 cm^2 and the length of the magnetic circuit is 15 cm .

Solⁿ:

Given data,

$$N = 3000 \text{ turns}$$

$$I = 0.1 \text{ A}$$

$$A = 4 \times 10^{-4} \text{ m}^2$$

$$l = 15 \times 10^{-2} \text{ m}$$

Determine the magnetic field strength, the flux density and the total flux within the coil and the reluctance.

Magnetic field strength,

$$H = \frac{NI}{l} = \frac{3000 \times 0.1}{15 \times 10^{-2}} = 2000 \text{ AT/m}$$

$$\text{Flux density, } B = \mu_0 H = 4\pi \times 10^{-7} \times 2000$$

$$= 2.5 \times 10^{-3} \text{ Wb/m}^2$$

Total flux,

$$\phi = B \times A$$

$$= 2.5 \times 10^{-3} \times 4 \times 10^{-4}$$

$$= 1 \times 10^{-6} \text{ Wb}$$

$$= 1 \mu\text{e Wb}$$

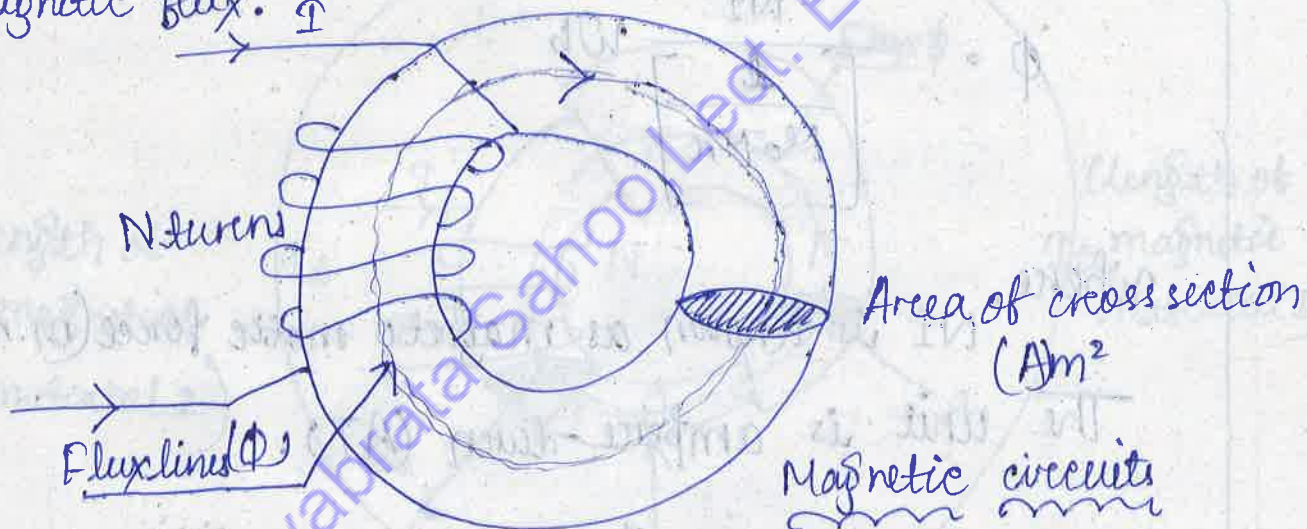
$$\text{Reluctance, } S = \frac{\text{MMF}}{\phi} = \frac{NI}{\phi}$$

$$S = \frac{3000 \times 0.1}{1 \times 10^{-6}}$$

$$S = 300 \times 10^6 \text{ A}\cdot\text{t/Wb}$$

* Magnetic Circuits:

→ Magnetic circuit is the path which is followed by magnetic flux. Φ



→ Consider a toroidal iron ring having a magnetic path of l metre, area of cross section $(A) \text{ m}^2$ and a coil of N turns carrying current (I) ampere.

Field strength inside the ring

$$H = \frac{NI}{l} \text{ AT/m}$$

where

$$B = \mu_0 \mu_r H$$

$$\therefore B = \mu_0 \mu_r \left(\frac{NI}{l} \right) \text{ Wb/m}^2$$

The total flux

= flux density \times area

$$\phi = B \times A$$

$$\therefore = \left\{ \left(\frac{\mu_0 \mu_r N I}{l} \right) A \right\}$$

$$\text{Flux, } \phi = \left\{ \frac{\mu_0 \mu_r N I A}{l} \right\} \text{ Wb}$$

(Core)

$$\phi = \frac{N I}{\left[\frac{l}{\mu_0 \mu_r} \right]} \text{ Wb}$$

where,

$N I$ is known as magnetomotive force (m.m.f.).

The unit is ampere-turn (AT).

$\mu_0 = 4\pi \times 10^{-7}$ Absolute permeability

$\mu_r =$ relative permeability [$\mu_r = 1$ for air & vacuum]

and

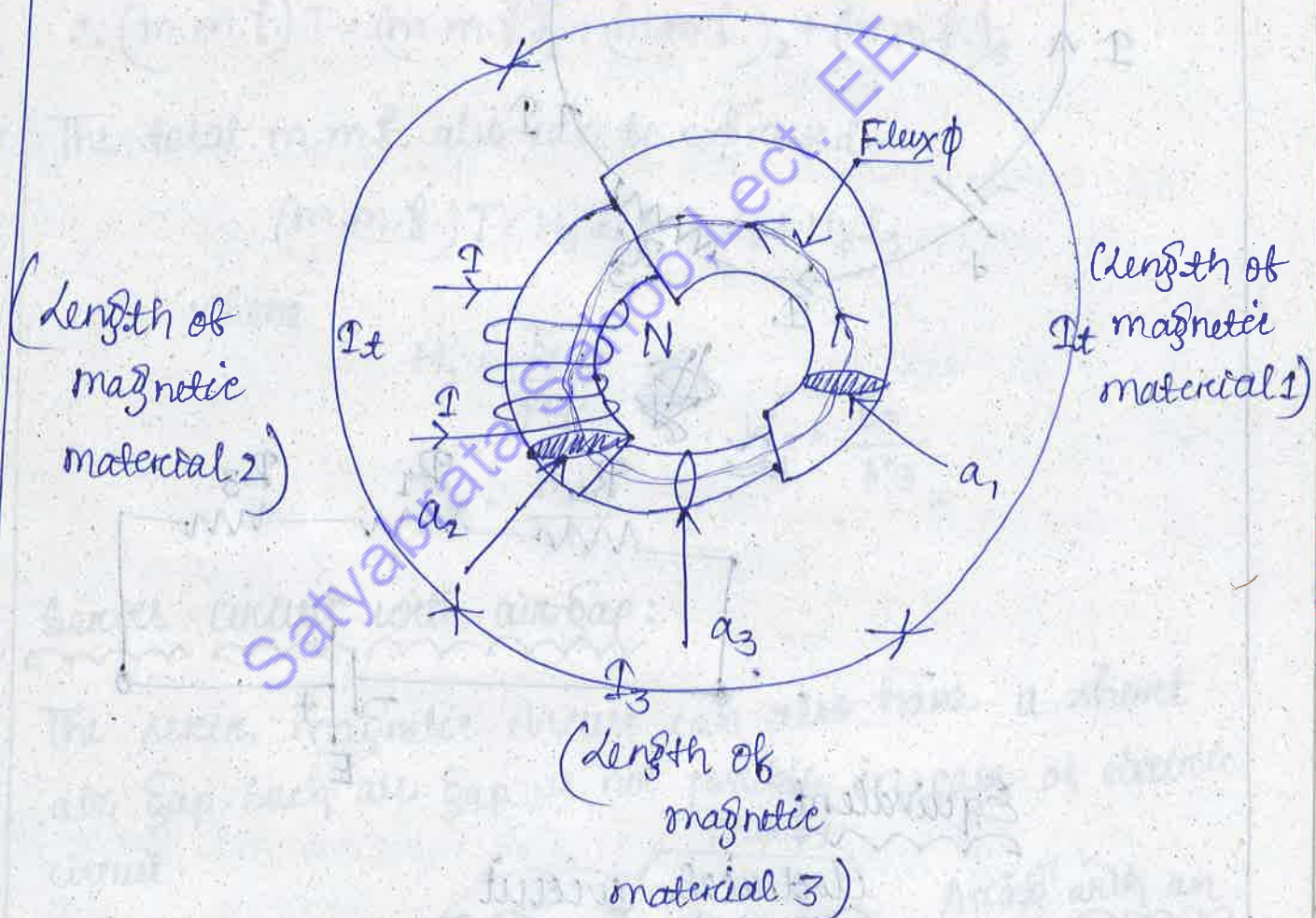
$\frac{l}{(\mu_0 \mu_r A)}$ is called the reluctance of the magnetic circuit.

$$\therefore \text{flux} = \frac{\text{m.m.f.}}{\text{Reluctance}} = \frac{\text{m.m.f.}}{S}$$

* Series magnetic circuits:

→ In practice magnetic circuit may be composed of various materials of different permeabilities, of diff. lengths & of diff. cross-sectional areas. Such a circuit is called composite magnetic circuit.

→ When such parts are connected one after the other, the circuit is called series magnetic circuit.



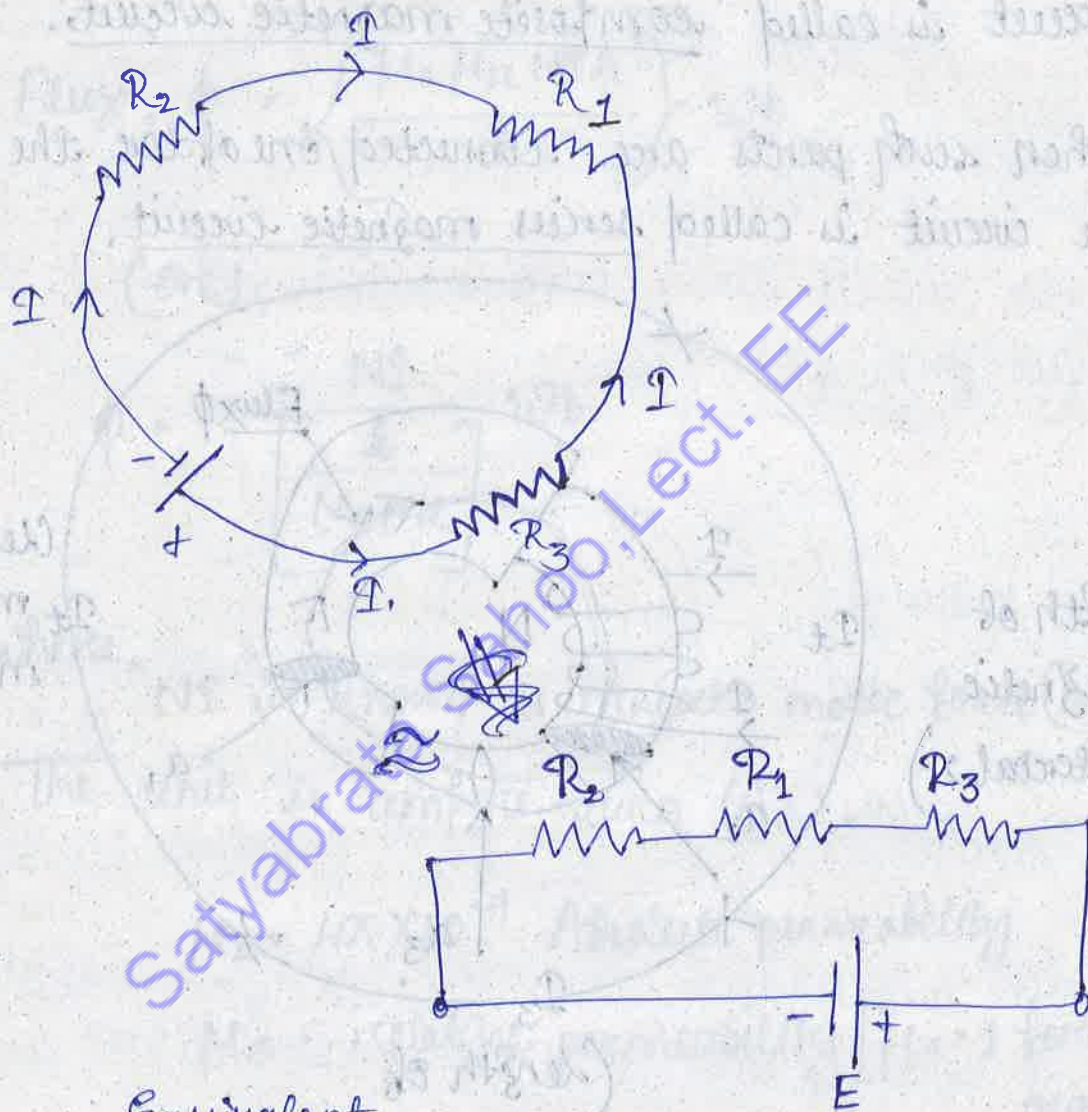
A series magnetic circuit

→ Let a coil wound on a ring has N turns carrying a current of I amperes.

The total m.m.f. available is NI ampere turns.

→ This will set the flux " Φ " which is same through all the three elements of the circuit.

→ Its analogous electric circuit.



Equivalent

electrical circuit

→ The total resistance of the electric circuit is $R_1 + R_2 + R_3$. Similarly the total reluctance of the magnetic circuit.

$$\text{Total } S_{tc} = S_1 + S_2 + S_3$$

$$= \frac{l}{\mu_1 a_1} + \frac{l}{\mu_2 a_2} + \frac{l}{\mu_3 a_3}$$

$$\begin{aligned} \therefore \text{Total } \phi &= \frac{\text{Total m.m.f.}}{\text{Total reluctance}} \\ &= \frac{NI}{S_T} \\ &= \frac{NI}{(S_1 + S_2 + S_3)} \end{aligned}$$

$$\begin{aligned} \therefore NI &= S_T \phi \\ &= (S_1 + S_2 + S_3) \phi \end{aligned}$$

$$\therefore (\text{m.m.f.})_T = (\text{m.m.f.})_1 + (\text{m.m.f.})_2 + (\text{m.m.f.})_3$$

→ The total m.m.f. also can be expressed.

$$(\text{m.m.f.})_T = H_1 l_1 + H_2 l_2 + H_3 l_3$$

where

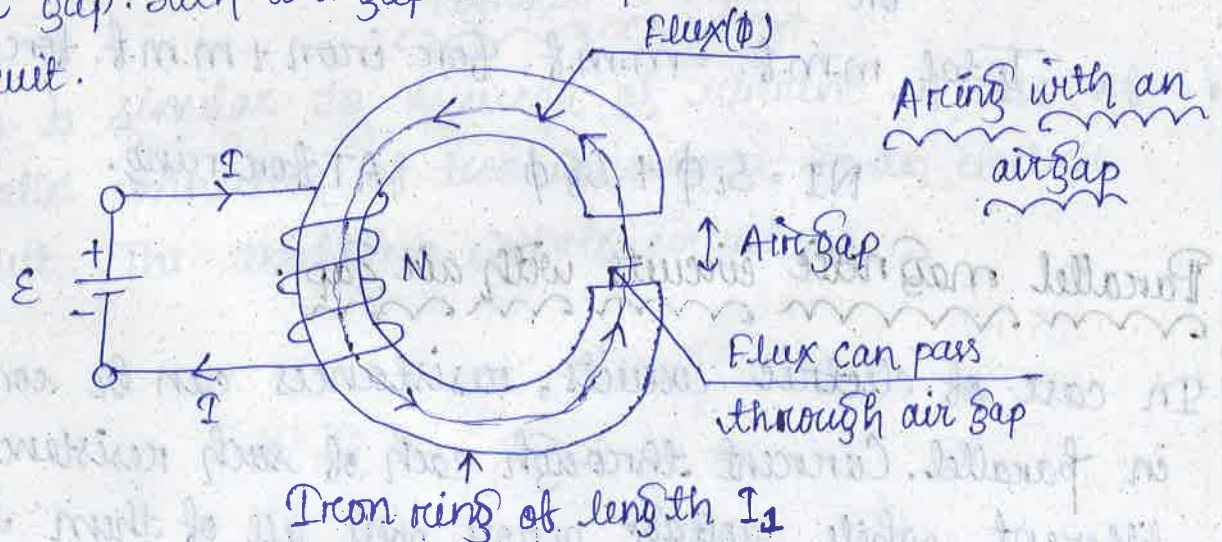
$$H_1 = \frac{B_1}{\mu_1}$$

$$H_3 = \frac{B_3}{\mu_3}$$

$$H_2 = \frac{B_2}{\mu_2}$$

● Series circuit with air gap:

→ The series magnetic circuit can also have a short air gap. Such air gap is not possible in case of electric circuit.



→ Consider a ring having mean length of iron part as " l_i ".

$$\text{Total m.m.f.} = NI AT$$

Total reluctance,

$$S_T = S_i + S_g$$

where,

S_i = Reluctance of iron path

S_g = Reluctance of air gap

$$\therefore S_i = \frac{l_i}{\mu a_i}$$

$$S_g = \frac{l_g}{\mu_0 a_i}$$

The cross-sectional area of air gap is assumed to be equal to area of the iron ring.

$$\therefore S_T = \frac{l_i}{\mu a_i} + \frac{l_g}{\mu_0 a_i}$$

$$\phi = \frac{\text{m.m.f.}}{\text{Reluctance}}$$

$$= \frac{NI}{S_T}$$

or

Total m.m.f. = m.m.f. for iron + m.m.f. for air gap

$$NI = S_i \phi + S_g \phi \quad \text{AT for ring.}$$

* Parallel magnetic circuits with air gap:

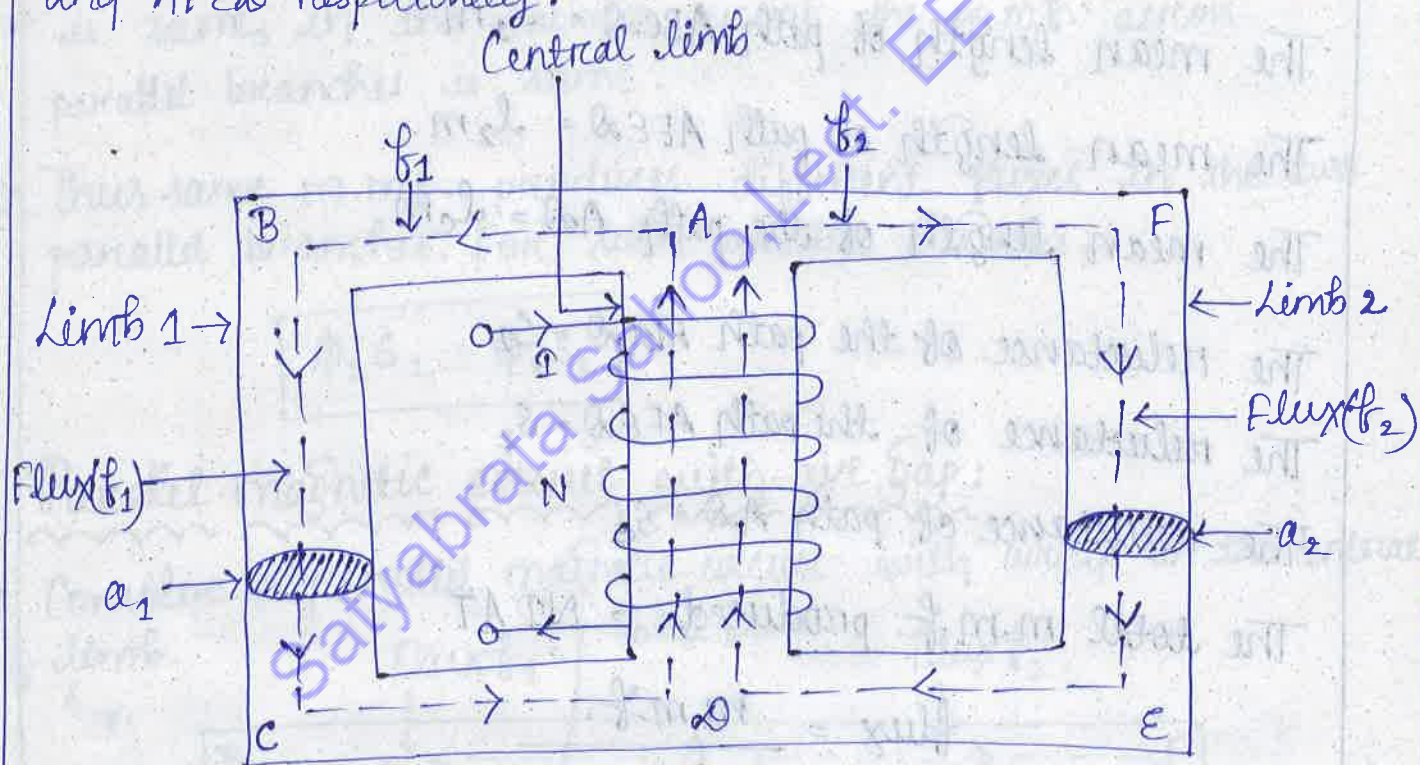
→ In case of electric circuits, resistances can be connected in parallel. Current through each of such resistances is different while voltage across all of them is same.

→ Similarly different reluctances may be in parallel in case of magnetic circuits. A magnetic circuit which has more than one path for the flux is known as a parallel magnetic circuit.

→ Consider a magnetic circuit. At point A the total flux ϕ , divides into two parts ϕ_1 & ϕ_2 .

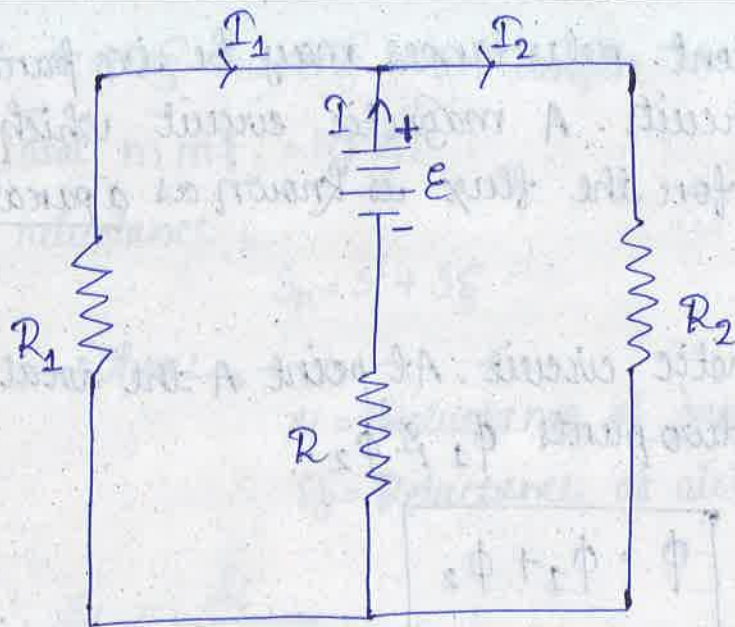
$$\therefore \phi = \phi_1 + \phi_2$$

→ The flux ϕ_1 & ϕ_2 have their paths completed through ABCD and AFECD respectively.



Magnetic Circuit

→ This is similar to division of current in case of parallel connection of two resistances in an electric circuit. The analogous electric circuit.



Equivalent electrical circuit.

The mean length of path ABCD = l_1 m

The mean length of path AFED = l_2 m

The mean length of the path AD = l_c m

The reluctance of the path ABCD = S_1

The reluctance of the path AFED = S_2

The reluctance of path AD = S_c

The total m.m.f. produced = $NIAT$

$$\text{flux} = \frac{\text{m.m.f.}}{\text{reluctance}}$$

$$\therefore \text{m.m.f.} = \phi \times S$$

$$\therefore \text{For path ABCDA, } NI = \phi_1 S_1 + \phi S_c$$

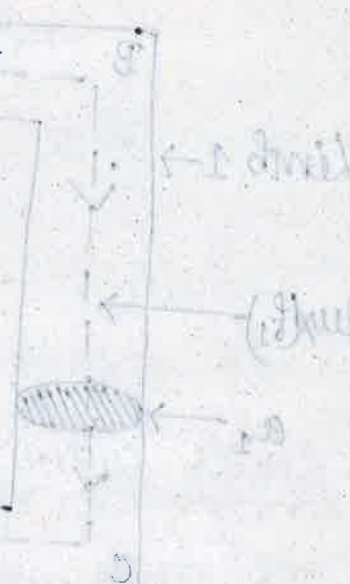
$$\text{For path AFEDA, } NI = \phi_2 S_2 + \phi S_c$$

where

$$S_1 = \frac{l_1}{\mu a_1}$$

$$S_c = \frac{l_c}{\mu a_c}$$

$$S_2 = \frac{l_2}{\mu a_2}$$



Generally,

$$a_1 = a_2 = a_c = \text{Area of cross-section}$$

→ For parallel circuit,

$$\text{Total m.m.f.} = \left[\text{m.m.f. required by central limb} \right] + \left[\text{m.m.f. required by any one of outer limbs} \right]$$

$$NI = (NI)_{AOD} + (NI)_{ABCO} \text{ or } (NI)_{AFEO}$$

$$NI = \phi S_c + [\phi_1 S_1 \text{ or } \phi_2 S_2]$$

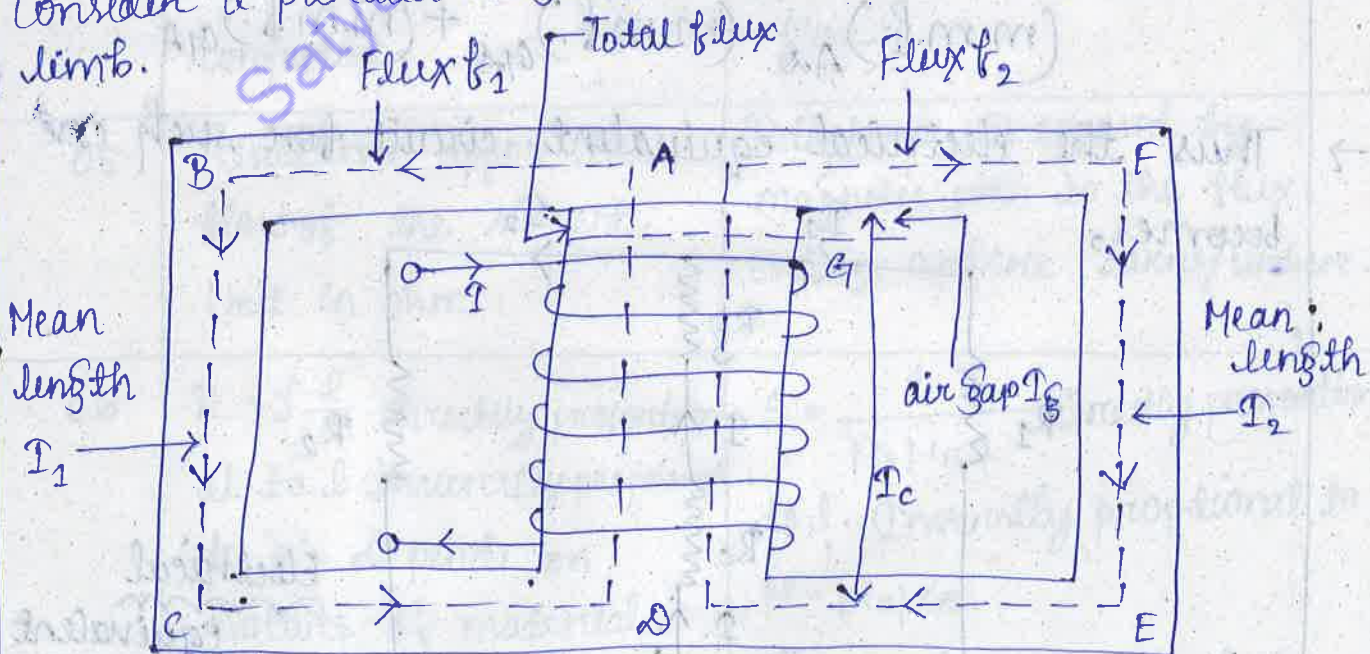
→ As in the electric circuit e.m.f. across parallel branches is same, in the magnetic circuit the m.m.f. across parallel branches is same.

→ Thus same m.m.f. produces different fluxes in the two parallel branches. For such parallel branches,

$$\phi_1 S_1 = \phi_2 S_2$$

● Parallel magnetic circuit with air gap:

→ Consider a parallel magnetic circuit with airgap in the central limb.



Parallel circuit with air gap

→ The analysis of this circuit is exactly similar to the parallel circuit.

The only change is the analysis of central limb. The central limb is series combination of iron path and air gap. The central limb is made up of,

$$\text{path } G_1 D = \text{Iron path} = l_c$$

$$\text{path } G_1 A = \text{Air gap} = l_g$$

→ The total flux produced is ϕ . It gets divided at A into ϕ_1 and ϕ_2 .

$$\therefore \boxed{\phi = \phi_1 + \phi_2}$$

The reluctance of central limb is now

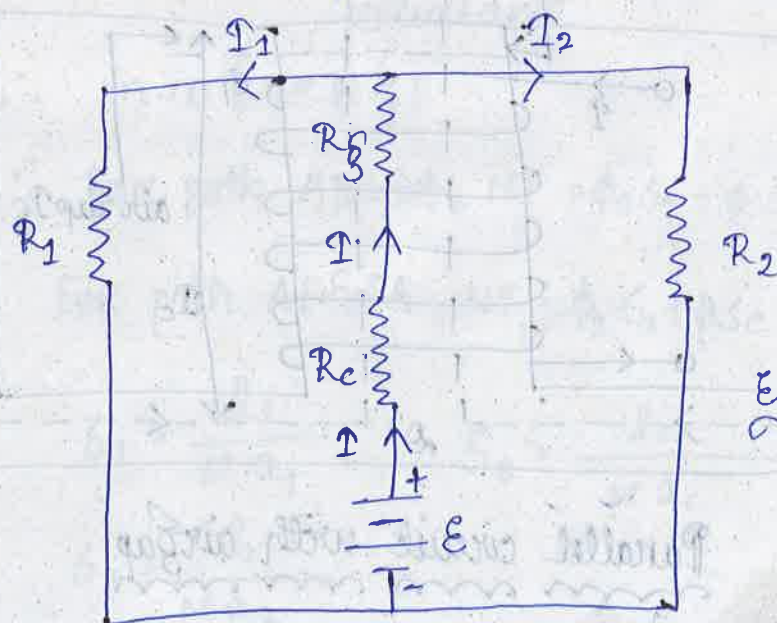
$$S_c = S_i + S_g$$

$$= \frac{l_c}{\mu_0 \mu_r a c} + \frac{l_g}{\mu_0 a c}$$

Hence, m.m.f. of central limb is now

$$(\text{m.m.f.})_{A D} = (\text{m.m.f.})_{G_1 D} + (\text{m.m.f.})_{G_1 A}$$

→ Thus, the electrical equivalent circuit for such case becomes,



Electrical
equivalent
circuit

Similarly, there may be air gaps in the side limbs but the method of analysis remains the same.

* Comparison of magnetic circuit & electric circuit:

Sl.No.	Electric Circuit	Magnetic Circuit
01	Path traced by the current is called electric circuit.	Path traced by the magnetic flux is defined as magnetic circuit.
02	E.M.F. is the driving force in electric circuit, the unit is volts.	M.M.F. is the driving force in the magnetic circuit. The unit of which is ampere turns.
03	There is current I in the electric circuit measured in amperes.	There is flux ϕ in the magnetic circuit measured in webers.
04	The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
05	Resistance oppose the flow of the current. Unit in ohm.	Reluctance is opposed by magnetic path to the flux. Unit is ampere turns/weber.
06	$R = \rho \frac{l}{a}$ Directly proportional to l inversely proportional to a . Depends on nature of material.	$S = \frac{1}{\mu_0 \mu_r \frac{l}{a}}$ Directly proportional to l . Inversely proportional to $\mu = \mu_0 \mu_r$. Inversely proportional to area a .

Sl. No.

Electric Circuit

Magnetic Circuit

07

The current,
$$I = \frac{\text{e.m.f.}}{\text{resistance}}$$

The flux, Φ
$$\Phi = \frac{\text{m.m.f.}}{\text{reluctance}}$$

08

The current density,
$$\delta = \frac{I}{a} \text{ A/m}^2$$

The flux density,
$$B = \frac{\Phi}{a} \text{ wb/m}^2$$

09

Conductivity is reciprocal of the resistivity.
Conductance = $\frac{1}{R}$

Permeance is reciprocal of the reluctance.
Permeance = $\frac{1}{S}$

10

Kirchhoff's current & voltage law is applicable to the electric circuit.

Kirchhoff's m.m.f. law & flux law is applicable to the magnetic circuit.