GOVERNMENT POLYTECHNIC, NAYAGARH

FLUID MECHANICS

4TH SEMESTER, MECHANICAL ENGG.

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Chapter-01 Properties of fluid

Fluid:-

Defination:-

A fluid is a substance which is capable of flowing, or which substance which deform contineously when subjected to external shearing force.

* Fluid means Liquid and Gas.

Liquid:-

It a fluid which posses a definite volume and assume as incompressibley.

Gas:-

It posses no definite volume and is compressibley.

Characterstics:-

* It has no definite shape of its own but it will take the shape of the container in which it is stored.

* A small amount of shear force causes a deformation.

Classification:-

Fluid is bludy classified in to two group.

① Ideal fluid
② Real fluid

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Describe:-

① **Ideal Fluid:-**

A ideal fluid is one which has no viscosity, surface tension and is incompressible. Actually no ideal fluid exist.

② **Real Fluid:-**

A real fluid is one which has viscosity, surface tension and compressibility in addition to density.

Fluid Mechanics:-

Fluid Mechanics is defined as that branch of engineering science which deals with the study of behaviour of fluid under the condition of rest or motion.

* It may be divided in to three groups:-
  1. Fluid Statics
  2. Fluid Kinematics
  3. Fluid Dynamics

① Fluid Static:-

It deals with the study of fluid at rest.

* The study of in-compressible fluid under static condition is known as hydrostatic.
* The study of compressible fluid under static condition is known as Aerostatic.

② Fluid Kinematics:-

The study of fluid in motion where pressure forces are not considered is known as Fluid Kinematics.

③ Fluid Dynamics:-

The study fluid in motion where pressure forces are considered is known as fluid Dynamics.

Density or Mass Density (ρ)

Density or Mass Density is defined as the ratio of Mass of fluid to its Volume. It is also defined as mass / unit volume. It is denoted by 'ρ' (row).

Mathematically,

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V}$$

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Unit :-

Kg / m^3 in S.I. system.

ρ_w (Density of water) = 1000 kg/m^3

ρ_{Hg} (Density of Mercury) = 13,600 kg/m^3

Specific Weight / Weight density (w)

Specific weight or Weight density of a fluid is defined as the ratio of weight of the fluid to its volume. It is denoted by 'w'.

Mathematically,

$$w = \frac{W}{V} = \frac{\text{Weight}}{\text{Volume}}$$

$$= \frac{Mg}{V}$$

$$= \left(\frac{M}{V}\right)g$$

$$= \rho g$$

So,

$$\boxed{w = \rho g}$$

Unit :-

N/m^3 in S.I system.

Specific volume (V_s) :-

Specific volume of fluid is defined as the ratio of volume of the fluid to its mass. or volume / unit mass.

Mathematically,

$$V_s = \frac{V}{M} = \frac{1}{\frac{M}{V}} = \frac{1}{\rho}$$

Unit :-

M^3/Kg in S.I system.

Specific Gravity (S) :-

Specific gravity of a fluid is defined as the ratio of specific weight or density of a fluid, to specific weight or density of a standard fluid.

In case of liquid the standard fluid is water and standard fluid is air.

For liquid $\boxed{S = \frac{w_l \text{ or } \rho_l}{w_{water} \text{ or } \rho_{water}}} = S = \frac{w_l}{w_w} = \frac{\rho_l g}{\rho_w g} = \frac{\rho_l}{\rho_w}$

$$S_l = \frac{\text{density or specific weight of a liquid}}{\text{density or specific weight of a air}}$$

$$\Rightarrow S_l = \frac{\rho_l}{1000}$$

$$\Rightarrow \rho_l = S_l \times 1000$$

For gas.

$$S_g = \frac{\text{density or specific weight of a gas}}{\text{density or specific weight of air}}$$

$$\boxed{S_g = \frac{\rho_g}{\rho_{air}}}$$

* It is unit less.

<u>Questions:-</u>

① Calculate the specific weight, density and specific gravity of 1 liter of a liquid which weighs 7 newton.

Ans:-

Given,

$$\text{Weight} = 7\,N$$
$$\text{Volume} = 1\text{ lit} = 10^{-3}\,m^3$$

To find out,

$$w = \frac{W}{V}$$
$$\rho = ?$$
$$S = ?$$

So,

$$\therefore w = \frac{\text{Weight}}{\text{Volume}} = \frac{7}{10^{-3}} = 7000\ N/m^3$$

$$\therefore \rho = \frac{w}{g}$$

$$= \frac{7000}{9.81} = 713.557\ Kg/m^3$$

$\therefore S = \frac{\rho_l}{\rho_w}$

$= \frac{713.557}{1000}$

$= 0.713557$ (Unit less)

② Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of $6m^3$ and weight of 44 KN.

ANS:-

Given,

Volume $= 6m^3$

weight $= 44$ KN $= 44 \times 10^3$ N

To find out,

$w = \frac{W}{V}$

$\rho = ?$

$V_s = ?$

$S = ?$

So,

$\therefore w = \frac{W}{V} = \frac{44 \times 10^3}{6} = 7333.\overline{3}\ N/m^3$

$\rho = \frac{w}{g}$

$= \frac{7333.33}{9.81} = 747.536\ kg/m^3$

So,

$\therefore$ Specific mass $=$

$\rho = \frac{M}{V}$

$\Rightarrow M = \rho V$

$\Rightarrow M = 747.536 \times 6 = 4485.216$ Kg

$\rho = \frac{M}{V}$

$= \frac{4485.216}{6}$

$= 747.536\ kg/m^3$

∴ Specific Volume =

$$\frac{1}{\rho}$$

$$= \frac{1}{747.536} = 1.337728216 \times 10^{-3} \ m^3/kg$$

∴ Specific Gravity =

$$\frac{\rho_c}{\rho_w}$$

$$= \frac{747.536}{1000}$$

$= 0.747$ (unit less)

③ 1 lit of crude oil weighs 9.6 N. Calculate its specific weight density and specific gravity?

Ans:- Given,

Volume = 1 lit $= 10^{-3} m^3$

Weight = 9.6 N

To find out,

Specific weight density = ?

Specific gravity = ?

∴ Specific weight density

$$w = \frac{W}{V}$$

$$= \frac{9.6}{10^{-3}} = 9600 \ N/m^3$$

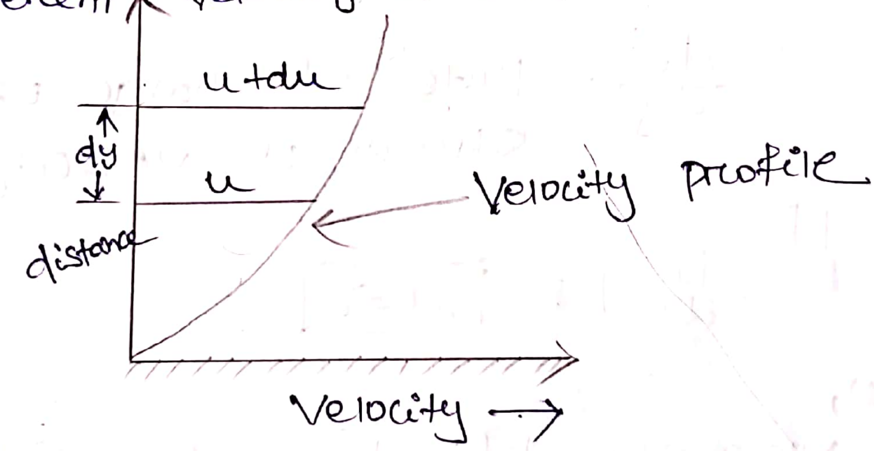
∴ Specific gravity = $\frac{\rho}{\rho_w} = \frac{978.59}{1000} = 0.97$ (unitless)

$$\boxed{\rho = \frac{w}{g} = \frac{9600}{9.81} = 978.59}$$

Viscosity:-

* Viscosity is defined as that property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid.

Let's consider, two layer of liquid at a distance 'dy' apart move one over the other. with different velocity "u" and "u+du".



(Velocity distribution over a solid boundary.)

* The viscosity with the relative velocity between the fluid layer causes a shear stress acting between the layer.

* The shear stress which the top layer acted on the lower layer and also the lower layer causes shear stress on the top layer.

* The shear stress thus develop is proportional to the rate of change of velocity with respect to distance.

$$\tau \propto \frac{du}{dy}$$

$$\Rightarrow \tau = \mu \frac{du}{dy}$$

$$\Rightarrow \mu \frac{du}{dy} = \tau$$

$$\Rightarrow \mu = \frac{\tau}{\frac{du}{dy}}$$

Where,

μ = dynamic viscosity or viscosity or co-efficient of viscosity

τ = Shear Stress

$\frac{du}{dy}$ = rate of change of shear strain or velocity gradiant.

If

$\frac{du}{dy} = 1$ $\boxed{\mu = \tau}$

So,

Viscocity is defined as the shear stress required to produce unit force of shear strain.

$$\mu = \frac{\tau}{du/dy} = \frac{N/m^2}{\frac{M/s}{M}} = Ns/m^2 \text{ (S.I)}$$

In CGS system unit $\frac{\text{Dyne.s}}{cm^2}$ = 1 poise

M.K.S system unit = $\frac{\text{kgf.s}}{m^2}$

1 Centipoise = $\frac{1}{100}$ Poise

* Viscocity of water at 20°C is 0.01 poise or 1 centipoise.

$$\boxed{1\ Ns/m^2 = 10\ Poise}$$

Kinematic Viscosity (ν) :-

* Kinematic viscosity is defined as the ratio between dynamic viscosity and density of fluid.
* It is denoted by 'ν'.

$$\boxed{\nu = \frac{\mu}{\rho}}$$

$$S.I\ Unit = \frac{Ns/m^2}{Kg/m^3} = \frac{Kg\ m/s^2 \times s/m^2}{Kg \times m^3}$$

$$= m^2/s$$

$$C.G.S\ unit = cm^2/s$$

* In C.G.S ~~unit~~ system = $1\ cm^2/s = 1$ stoke

$$1\ \text{centistoke} = \frac{1}{100}\ \text{stoke}$$

$$1\ m^2/sec = 10^4\ \text{stoke}$$

Newton's Law of Viscocity:-

It states that the shear stress in a fluid element where is directly proportional to the rate of shear strain.

* The constant of proporsnality is known as co-efficient of viscocity.

$$\tau = \mu \frac{du}{dy}$$

Where, μ = Co-efficient of viscocity or viscocity.

Types of Fluid :-

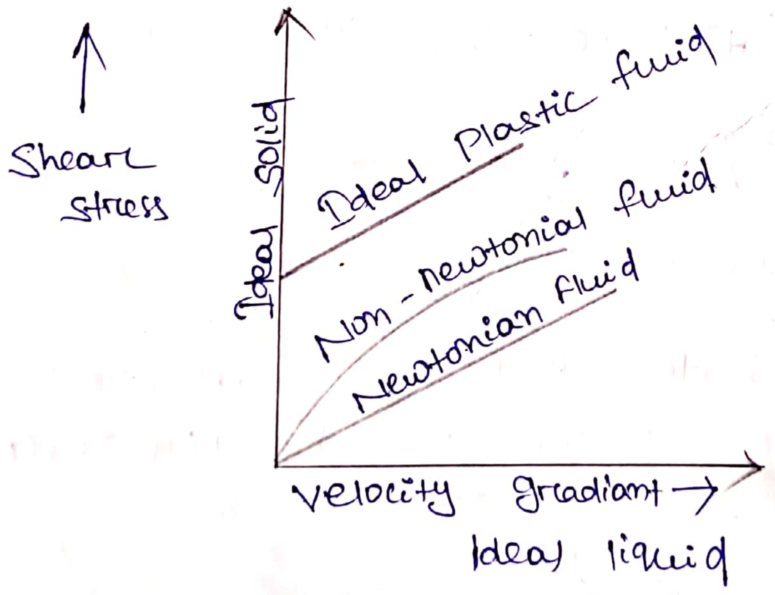
1. Ideal Fluid
2. Real Fluid
3. Newtonial Fluid
4. Non-newtonial fluid
5. Ideal Plastic fluid.

1. Ideal Fluid :-

* The fluid which has no viscosity is known as ideal fluid.
* It is a imaginary fluid.

2. Real fluid :-

The fluid which has viscosity is known as real fluid.



③ Newtonial fluid:-

The fluid which obey the newtons law of viscocity is known as newtonian fluid.

④ Non-newtonial fluid:-

The real fluid which doesn't obey the newton's law of viscocity is known as non-newtonial fluid.

⑤ Ideal Plastic fluid:-

The fluid which has a shear stress more than the yeild value (fixed value) and proportional to the rate of shear strain is known as ideal plastic fluid.

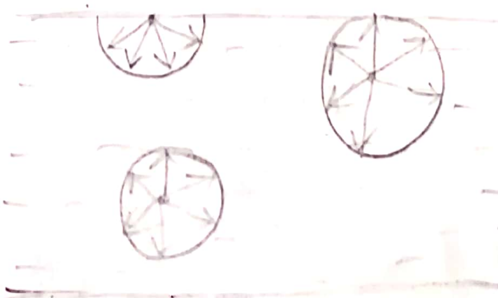
Surface tension:-

Surfacetension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquid such that the contact surface behaves like a membrane under tension.

* It is denoted by sigma 'σ'.
* It is represented as force / unit length or Energy / unit area.

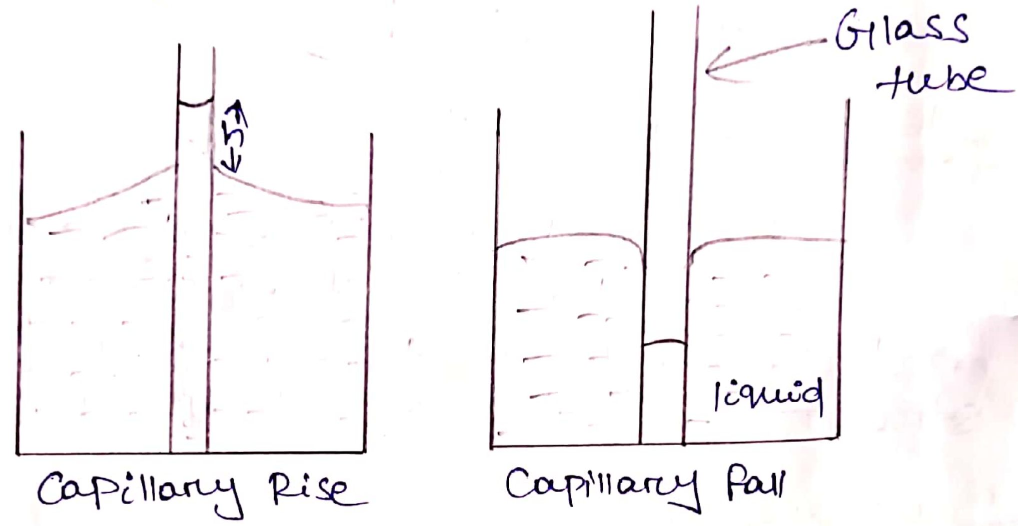
Unit:-

$\frac{N}{m}$ in SI



Capillarity:-

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjcent general level of liquid. When the tube is held vertically in the liquid.



* The rise of liquid surface in the tube is known as capillary rise, while the fall of liquid surface is known as capillary fall or depression.
* It is expressed interms of cm or mm of liquid.
* Its value depends upon the specific weight, diameter of the tube and surface tension of the liquid.

Fluid pressure:-

When a fluid a contained in a vessel is exerts force on all its sides and bottom. This force/unit area is known as pressure or fluid pressure.

If,

f = Force exerted by the ~~fluid~~ fluid.

A = Area.

p = Pressure or intensity of pressure

Then.

$$p = \frac{F}{A}$$

Unit = N/m^2 in SI system

$Dyne/m^2$ in CGS system

$kg.f/m^2$ in MKS system

$\boxed{1 \text{ kg.f} = 10 \text{ Newton}}$

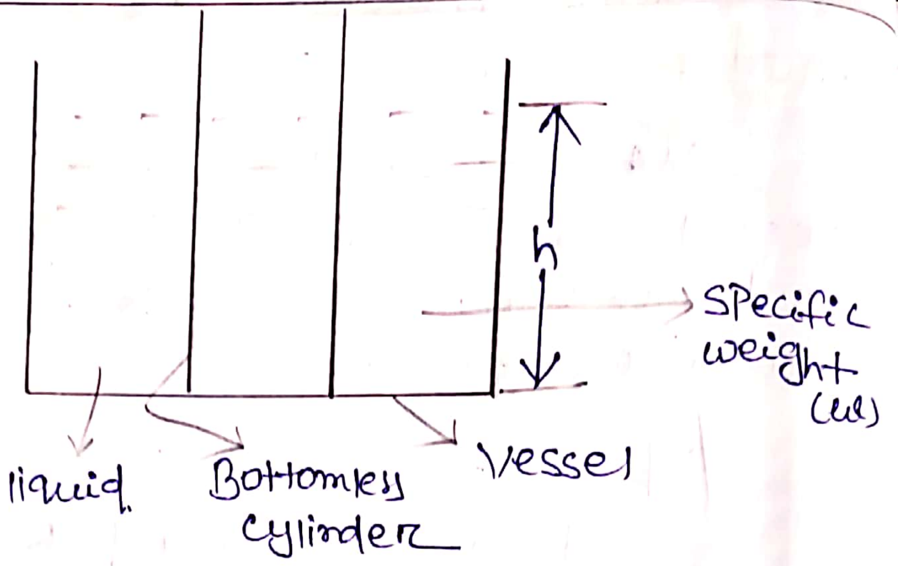
$\boxed{1 \ N/m^2 = 1 \text{ Pascal}}$ $\boxed{1 \text{ bar} = 10^5 \text{ pascal}}$

* It is also expressed in pascal.

$\boxed{1 \text{ Pascal} = 1 \ N/m^2}$

Pressure Head:-

* A liquid is subjected to pressure due to its own weight, this pressure increases with the increase of depth of the liquid.



Let

w = Specific weight of the liquid is in the cylinder

A = Cross sectional area of the cylinder.

h = Height of the liquid in the cylinder.

$$P = \frac{F}{A}$$

$$= \frac{\text{Weight of the liquid in the cylinder}}{\text{Area of cylinder}}$$

$$= \frac{W}{A} = \frac{w \times V}{A}$$

$$\Rightarrow P = \frac{\rho g \times \cancel{A} \times h}{\cancel{A}}$$

$$\Rightarrow P = \boxed{\rho g \times h}$$

$$\Rightarrow P = \boxed{\rho g h}$$

$$\boxed{h = \frac{P}{\rho g}} \longrightarrow \text{pressure head}$$

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* The liquid pressure depends on the depth of the liquid or directly proportional to the depth of the liquid coloumn.

Pascal's Law:-

* It state's that the pressure or intensity of pressure at any point in a fluid at rest is same in all directions.

Q
1. Calculate the pressure due to a coloumn of 0.3 m of
   (i) water
   (ii) An oil of specific gravity 0.8
   (iii) Mercury of specific gravity 13.6.

Take density of water is = 1000 kg/m^3.

Ans

$P = \rho_w g h$

$= 1000 \times 9.81 \times 0.3$

$= 2943 \ N/m^2.$

(ii) Oil of specific gravity 0.8.

$S = \dfrac{\rho_{oil}}{\rho_w}$

$\Rightarrow 0.8 = \dfrac{\rho_{oil}}{1000}$

$\Rightarrow 0.8 \times 1000 = \rho_{oil}$

$\Rightarrow \rho_{oil} = 0.8 \times 1000 = 800 \ kg/m^3$

$\therefore P = \rho g h$

$= 800 \times 9.81 \times 0.3$

$= 2354.4 \ N/m^2$

(iii) Mercury,

$$S = \frac{\rho_{mercury}}{\rho_{water}}$$

$$\Rightarrow 13.6 = \frac{\rho_{mercury}}{1000}$$

$$\Rightarrow 13.6 \times 1000 = \rho_{mercury}$$

$$\Rightarrow \rho_{mercury} = 13600 \ Kg/m^3$$

$$\therefore P = \rho g h$$

$$= 13600 \times 9.81 \times 0.3$$

$$= 40025 \ N/m^2$$

(2) The pressure intencity at a point in a fluid is given $3.924 \ N/cm^2$. Find the corresponding height of fluid when the fluid is,

(i) Water

(ii) Oil of specific gravity = 0.9

Ans Given,

$$\text{Pressure} = 3.924 \ N/cm^2$$

$$= 3.924 \times 10^4 \ N/m^2$$

We know that,

$$P = \rho g h$$

(i) Water,

So,

$$h_w = \frac{P}{\rho_w g}$$

$$= \frac{3.924 \times 10^4}{1000 \times 9.81} = 4 \ m \text{ of water} \leftarrow \text{(ANS)}$$

(ii) Oil,

$$S = \frac{\rho_{oil}}{\rho_w}$$

$$\Rightarrow 0.9 = \frac{\rho_{oil}}{1000}$$

$$\Rightarrow \frac{\rho_{oil}}{1000} = 0.9$$

$$\Rightarrow \rho_{oil} = 0.9 \times 1000 = 900$$

So,

$$h_o = \frac{P}{\rho_o g}$$

$$= \frac{3.924 \times 10^4}{900 \times 9.81} = 4.44 \text{ m of oil.}$$

(Ans)

(3) An oil of specific gravity 0.9 is contains in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at that point.

Ans:- Given,

$$S_{oil} = 0.9$$

$$h_o = 40 \text{ m}$$

$$S_{oil} = \frac{\rho_{oil}}{\rho_{water}}$$

$$\Rightarrow 0.9 = \frac{\rho_{oil}}{\rho_{water}}$$

$$\Rightarrow 0.9 = \frac{\rho_{oil}}{1000}$$

$$\Rightarrow \rho_{oil} = 0.9 \times 1000 = 900 \text{ kg/m}^3$$

$\therefore P = \rho_o g h$

$= 900 \times 9.81 \times 40$

$= 353160 \ N/m^2$

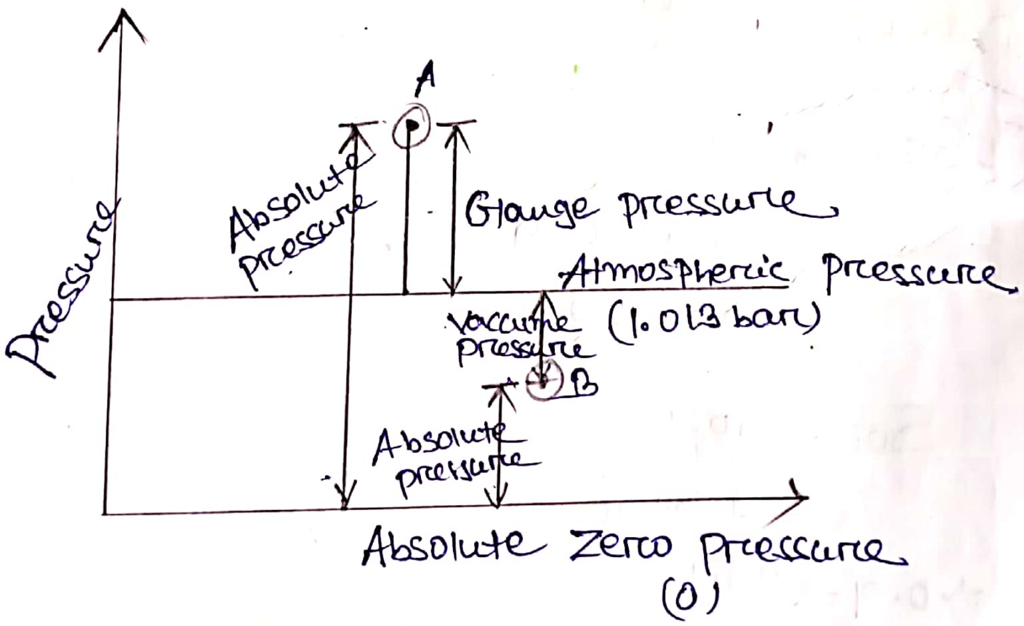
Then,

$$h_w = \frac{P}{\rho_w g}$$

$$= \frac{353160}{1000 \times 9.81}$$

$= 36$ m of water —— (ANS)

Concept of atmospheric pressure, gauge pressure, vaccume pressure, absolute pressure.



$\therefore P_{abs} = P_{atm} + P_{gauge}$

$\therefore P_{vaccume} = P_{atm} - P_{abs}$

Atmospheric Pressure:-

* The atmospheric air exerts normal pressure upon all surfaces with which it is a contact and this pressure is known as Atmospheric pressure.

Absolute Pressure:-

* It is defined as the pressure which is measured with referance to absolute '0' pressure or absolute vaccume pressure

Gauge Pressure:-

* It is defined as the pressure which is measured with the help of a measuring instrument in which atmospheric pressure is taken as referance. Atmospheric pressure on scale is taken as '0'.

Vaccume Pressure:-

* It is defined as the pressure below atmospheric pressure.

Pressure Measuring Instruments:-

1. Manometers.
2. Mechanical gauges.

① Manometers:-

* Mano meters are defined as the devices used for measuring the pressure at the point in a fluid by balancing the coloumn of fluid by the same or

another coloumn of fluid.

* They are classified as

① Simple Manometer.

② Differential Manometer

② Mechanical Gauges:-

* Mechanical Gauges are defined as the devices used for measuring the pressure by balancing the fluid coloumn by the spring or dead weight.

* The commonly used mechanical pressure gauges are :- ① Diaphragm pressure gauge

② Bourdon tube pressure gauge

③ Dead weight pressure gauge

④ Bellow type pressure gauge.

Describe:-

① Simple Manometer:-

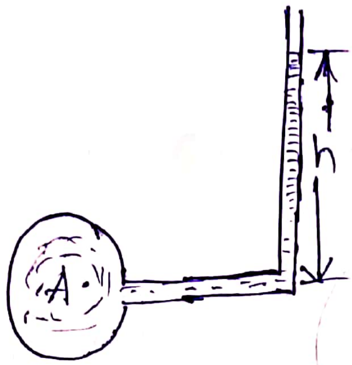
* Simple manometer consists of a glass tube whose one end is connected to the point where the pressure is to be measured and the other end is open to the atmosphere.

* Simple manometer are of three types:-

① P.

1. Piezometer
2. U-tube manometer
3. Single-column manometer.

① Piezometer:-



* It is the simplest form of manometer.
* One end of this manometer is connected to a point where another end is open to atmosphere.
* The ~~[illegible]~~ rise of the liquid gives the pressure head at that point.

If 'A' is the point where the pressure (P_A) is to be measured.

$$\boxed{P_A = \rho g h}$$

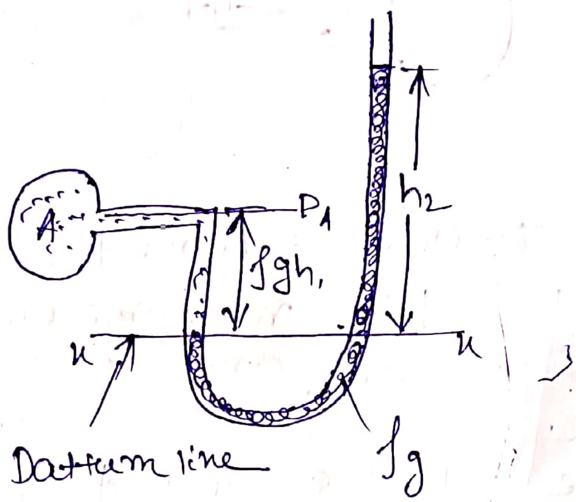
② U-tube manometer:-

* U-tube manometer consist of glass tube paint in 'U' shape.
* One end of the tube is connected into a point where the pressure is to be measured and other end remains open to

the atmosphere.

* The tube generally contains Mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

For gauge pressure:- (Above in atmosphere)



Pressure in the left coloumn,

$$P_A + \rho g h_1 \quad \text{—— (I)}$$

Pressure in the right column,

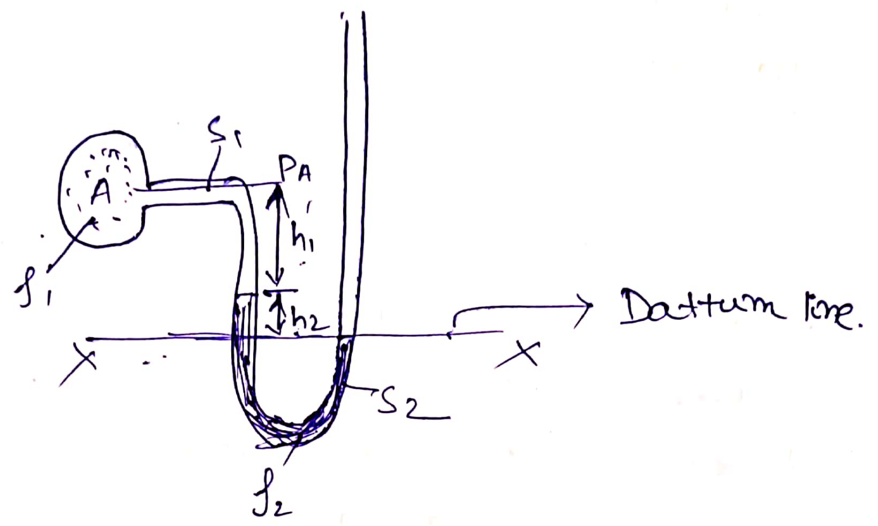
$$\rho_g g h_2 \quad \text{—— (II)}$$

Equating both equation

$$P_A + \rho g h_1 = \rho g h_2$$

$$\Rightarrow \boxed{P_A = \rho g h_2 - \rho g h_1}$$

For Vaccume Pressure:- (Below in Atmosphere):-



Let,

h_1 = Height of the pipe liquid

h_2 = Height of Hg heavy liquid in left size.

ρ_1 = Density of liquid in pipe 'A'

ρ_2 = Density of heavy liquid 'Hg'

S_1 = Specific gravity of pipe liquid.

S_2 = Specific gravity of heavy liquid.

Pressure in the left coloumn (above the dattum line x-x)

$$P_A + \rho_1 g h_1 + \rho_2 g h_2 \text{ --- (I)}$$

Pressure in the right Column (above the dattum line x-x)

$$= 0 \text{ --- (II)}$$

Equating both pressure,

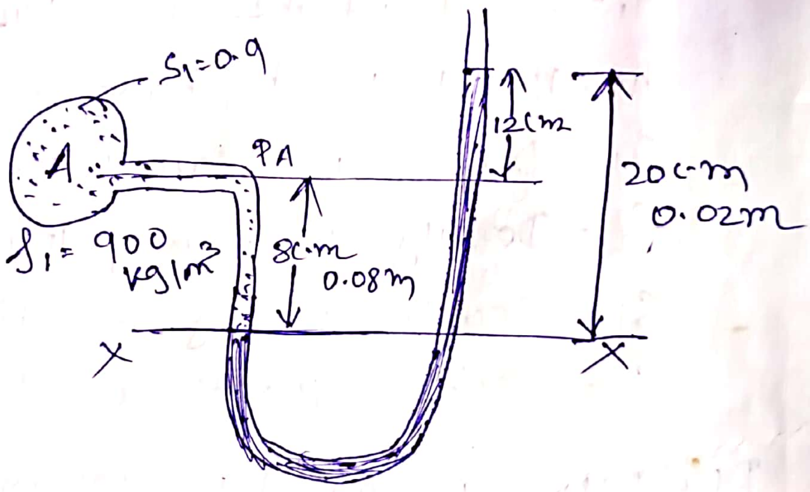
$$P_A + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$\Rightarrow \boxed{P_A = -(\rho_1 g h_1 + \rho_2 g h_2)}$$

Questions:-

① The right column of a simple U-tube manometer containing mercury in open to the atmosphere. While the left column is connected to a pipe in which a fluid of specific gravity 0.9 is flowing the centre of the pipe is 12 mm below the level of mercury in right link. Find the pressure of fluid in the pipe to the different of mercury level is the two column in 20 c.m.

ANS:-



∴ Pressure in the left column above the dattum line X-X

$$= P_A + \rho_1 g h$$

$$= P_A + 900 \times 9.81 \times 0.08$$

$$= P_A + 706.32 \quad \text{———} \; ①$$

∴ Pressure in the right column above the dattum line X-X

$$\rho_2 g h$$

$$= 13600 \times 9.81 \times 0.02 \text{ mm} = 2668.32$$

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Equating the both equation,

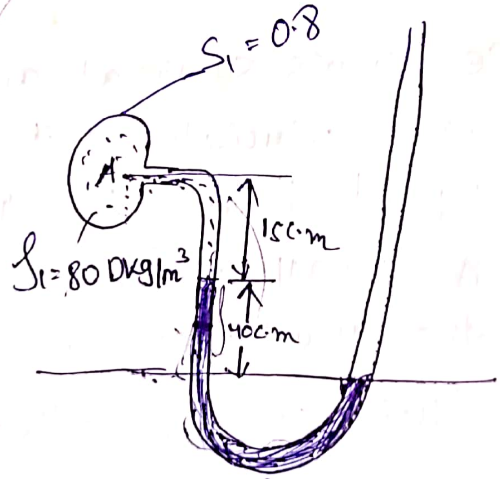
$$P_A + 706.32 = 2668.32$$

$$\Rightarrow P_A = 2668.32 - 706.32$$

$$\Rightarrow P_A = 1962 \ N/m^2$$

② A simple U-tube manometer containing 'Hg' is connected to a pipe in which a fluid of specific gravity 0.8 and having vaccum pressure is flowing. Other end of manometer is open to atmosphere, find the vaccum pressure in pipe in different of mercury level the two column in 40 cm and height of the fluid in the left column the center of the pipe is 15 cm. below.

Ans:-



∴ pressure in the left column above the dattum line x-x.

$$P_A + \rho_1 g h_1 + \rho_2 g h_2$$

$$= P_A + 800 \times 9.81 \times 0.15 + 13600 \times 9.81 \times 0.4$$

$$= P_A + 1177.2 + 53366.4$$

$$= P_A + 54543.6$$

∴ Pressure in right column above the datum line x-x

$= 0$

Equating both equation

$$P_A + 54543.6 = 0$$

$$\Rightarrow \boxed{P_A = -54543.6 \ N/m^2}$$

Single column manometer:-

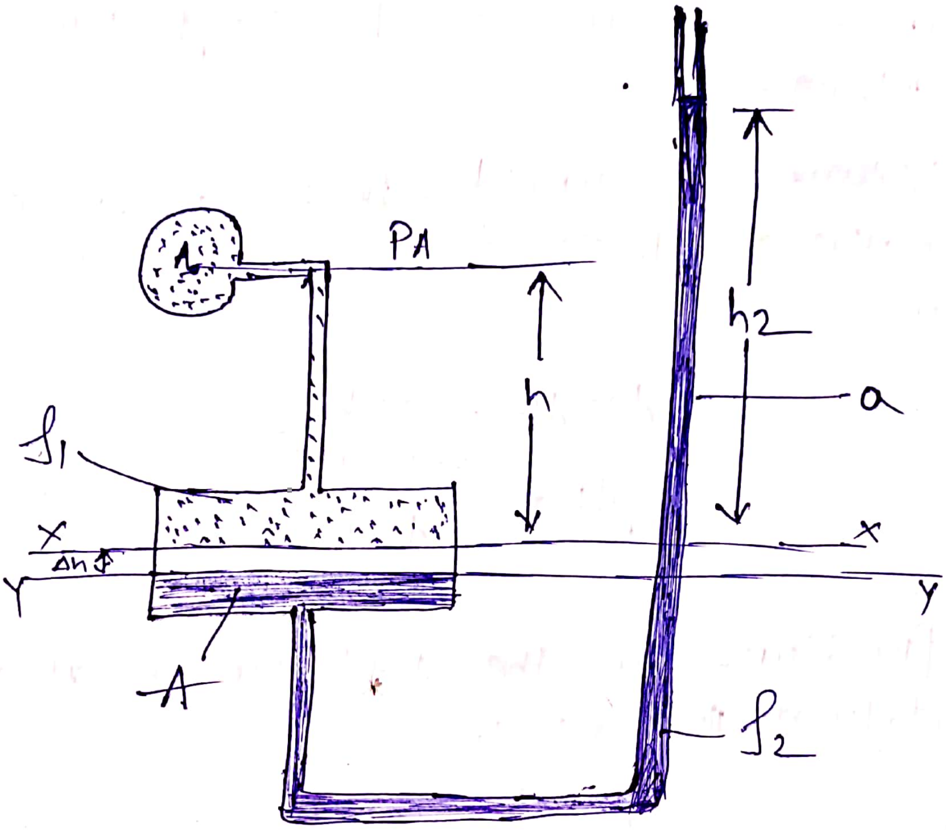
* It is a modify form of U-tube manometer in which a reservoir having large cross-sectional area (about 100 times) as compare to the area of the tube is connected to one of the column of the manometer.

* Due to large cross sectional area of the reservoir, any variation in pressure, the change in liquid level in the reservoir will be very small which may be neglete and hence the pressure is given by the height of liquid in the other column.

* There are two types of single column manometer.

(1) Vertical single column manometer
(2) Inclind single column manometer

Vertical single column manometer:-



Let,

① x-x is the line, when the reserviour isn't connected to the pipe.

② Y-Y is the datum line when the reserviour is connected to the pipe.

③ Δh = fall of heavy liquid in the reserviour

④ h_2 = Raise the heavy liquid in the right column in the X-X.

⑤ h_1 = Height of center of the pipe above the X-X.

⑥ P_A = Pressure at A.

⑦ A = Cross sectional area of the reserviour

⑧ a = cross sectional area of the tube.

(9) S_1 = Specific gravity of the heavy liquid.

* Fall of heavy liquid in the reserviour will case the raise of heavy liquid in righ column.

* Volume of liquid fall in reserviour = The volume of liquid raise in the right column.

$$A \times \Delta h = a \times h_2$$

$$\Rightarrow \boxed{\Delta h = \frac{a \times h_2}{A}}$$

(i) Pressure in the left column above the datum line Y-Y

$$P_A + \rho_1 g (h_1 + \Delta h)$$

(ii) Pressure in the right column above the ~~dd~~ datum line Y-Y

$$\rho_2 g (h_2 + \Delta h)$$

Equating both the pressure.

$$P_A + \rho_1 g (h_1 + \Delta h) = \rho_2 g (h_2 + \Delta h)$$

$$\Rightarrow P_A + \rho_1 g h_1 + \rho_1 g \Delta h = \rho_2 g h_2 + \rho_2 g \Delta h$$

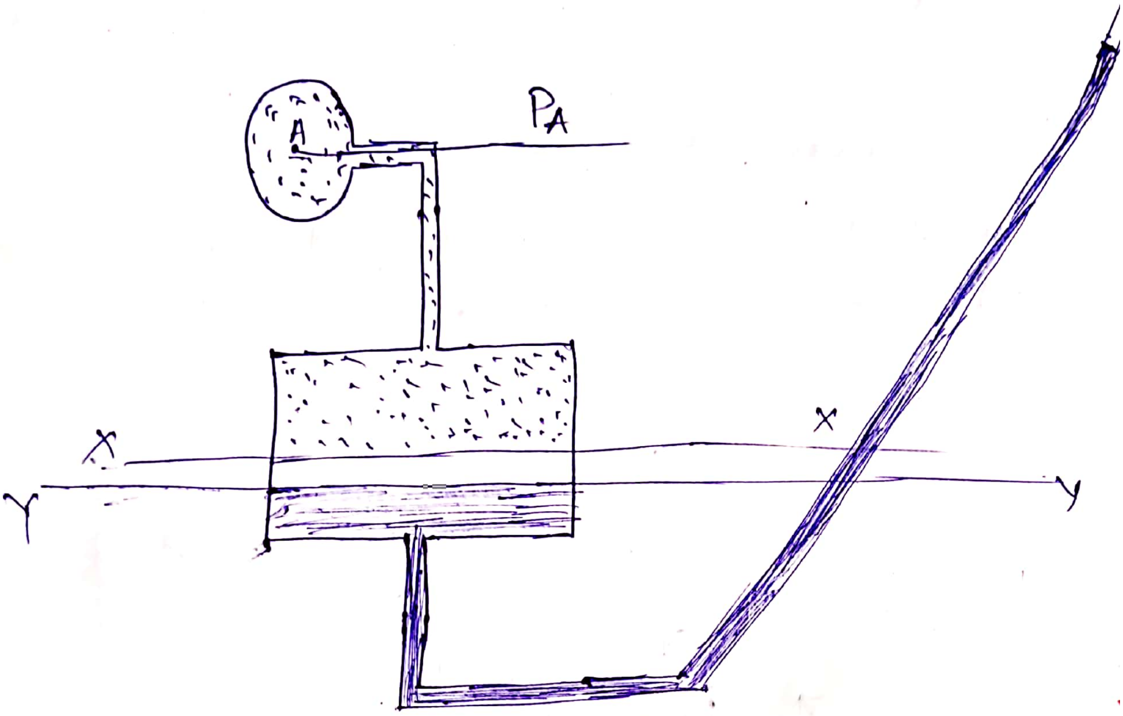
$$\Rightarrow P_A = \rho_2 g h_2 + \rho_2 g \Delta h - \rho_1 g h_1 - \rho_1 g \Delta h$$

$$\Rightarrow P_A = \rho_2 g h_2 - \rho_1 g h_2 + \Delta h (\rho_2 g - \rho_1 g)$$

As Δh is very small it may be neglated

$$\Rightarrow P_A = \rho_2 g h_2 - \rho_1 g h_1$$

Inclind single column manometer:-



Let,

θ = Angle made by right column with horizontal.

L = Length of the heavy liquid in right column.

h_2 = Vertical raise of the heavy liquid in right column.

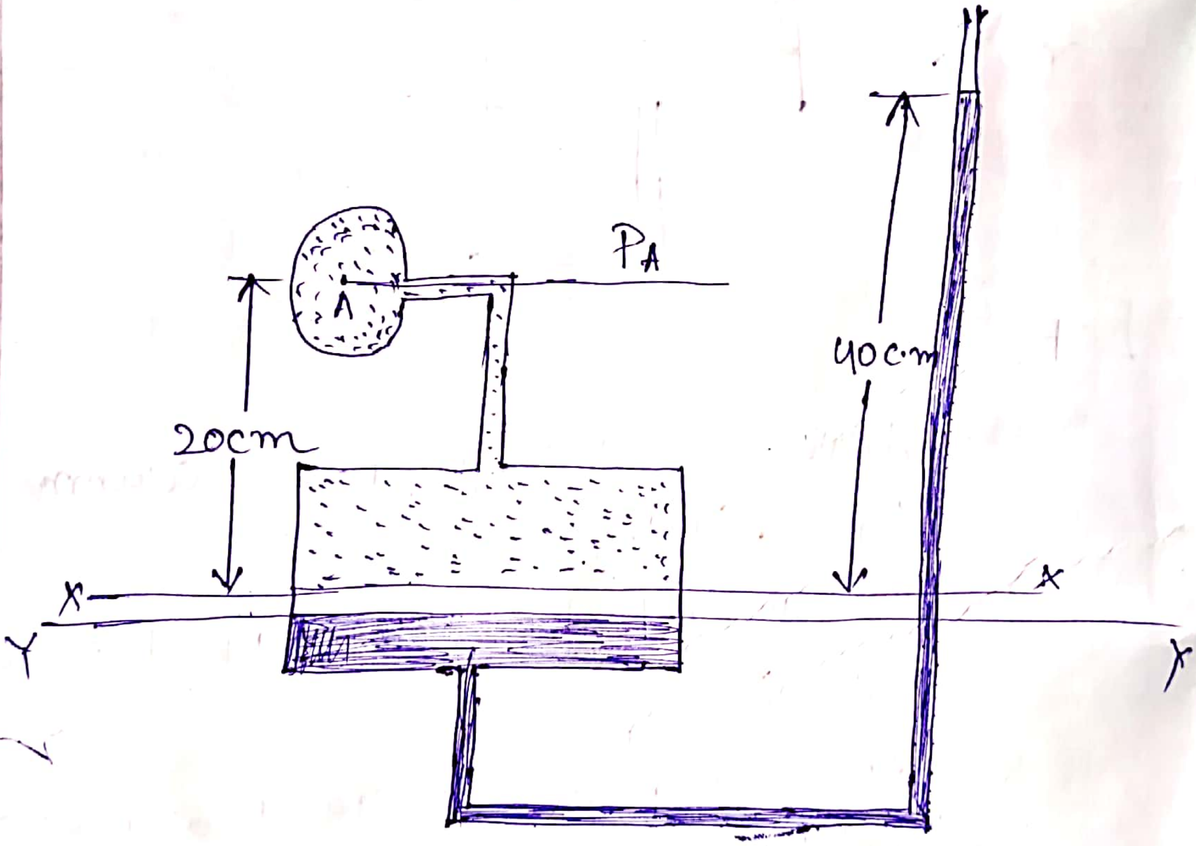
$h_2 = L\sin\theta$

Then,

$P_A = L\sin\theta\, g L\sin\theta - \rho_1 g h_1 + A h \; (L\sin\theta\, g - \rho_1 g$

Question:-

A Single column manometer is connected to a pipe containing a liquid of specific gravity 0.9. Find the pressure in the pipe if the area of reservoir is 100 times the area of tube for the manometer reading shown in the fig. Specific gravity of mercury 13.6.



Given data,

$h_1 = 20\text{ cm} = 0.2\text{ m}$

$h_2 = 40\text{ cm} = 0.4\text{ m}$

Specific gravity $= 0.9$ (S_1)

Specific gravity of mercury $= 13.6$ $(S_2$

So,

$\rho_1 = S_1 \times 1000$

$= 0.9 \times 1000 = 900\ kg/m^3$

$\rho_2 = S_2 \times 1000$

$= 13.6 \times 1000 = 13600\ Kg/m^3$

$\therefore \Delta h = \frac{a \times h_2}{(A)} = \frac{1 \times 0.4}{100} = \frac{1 \times 0.4}{100} = \frac{0.4}{100} = 4 \times 10^{-3}$

$= \frac{a \times h_2}{100 \times a}$

(i) Pressure in the left column in the datum line 'Y-Y'

$P_A + \rho_1 g (h_1 + \Delta h)$

$= P_A + 900 \times 0.9 (0.2 + 4 \times 10^{-3})$

$= P_A + 810 \times 0.204$

$= P_A + 165.24$

(ii) Pressure in the right column in the datum line 'Y-Y'

$\rho_2 g (h_2 + \Delta h)$

$= 13600 \times 9.81 (0.4 + 4 \times 10^{-3})$

$= 13416 \times 0.404$

$= 53900.064$

Equating the both pressure

$P_A + 165.24 = 53900.064$

$\Rightarrow P_A = 53900.064 - 165.24$

$= 53734.824\ N/m^2$ —ANS)

Differential Manometer:-

Differential Manometer is a device used to measure the difference of pressure at two points in a pipe or at two different pipes.

* This is basically classified in to two types

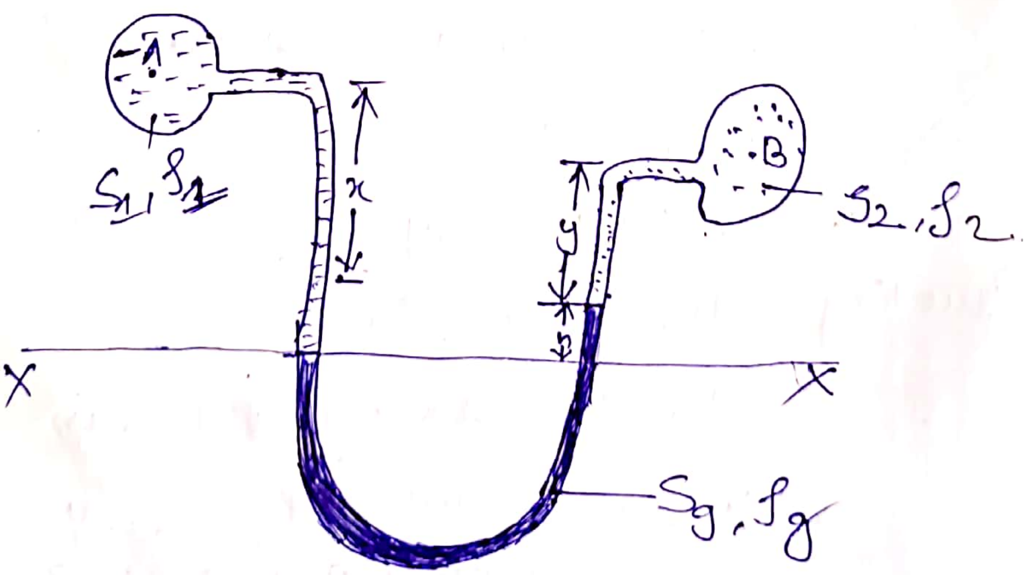
(I) U-tube differential manometer.

(II) Inverted U-tube differential manometer.

U-tube differential Manometer:-

Case 1:-

Pressure measured at two points in different level and also contain liquids of different specific gravity.



(4)

The two points A and B are connected to the differential U-tube manometer.

Let,

(i) The pressure and A and B at P_A and P_B.

(ii) h = Difference of mercurey level in U-tube

(iii) x = Height of centere of pipe 'A' from level of mercury in right limb

(iv) y = Height of center of pipe 'B' from level of mercury in right limb.

(v) S_1 = Specific gravity of liquid in the pipe 'A'.

(vi) S_2 = Specific gravity of the liquid 'B'.

(vii) S_g = Specific gravity of the heavy liquid in the manometer.

Pressure in the left column above datum X-X = Pressure in the right column above datum X-X.

(i) Pressure in left column above datum line X-X

$$P_A + \rho_1 g (x+h)$$

(ii) Pressure in right column above datum line X-X

$$P_B + \rho_2 g y + \rho_g g h$$

Equating both pressure.

$$P_A + \rho_1 g(x+h) = P_B + \rho_2 g y + \rho_g g h$$

$$\Rightarrow P_A + \rho_1 g x + \rho_1 g h = P_B + \rho_2 g y + \rho_g g h$$

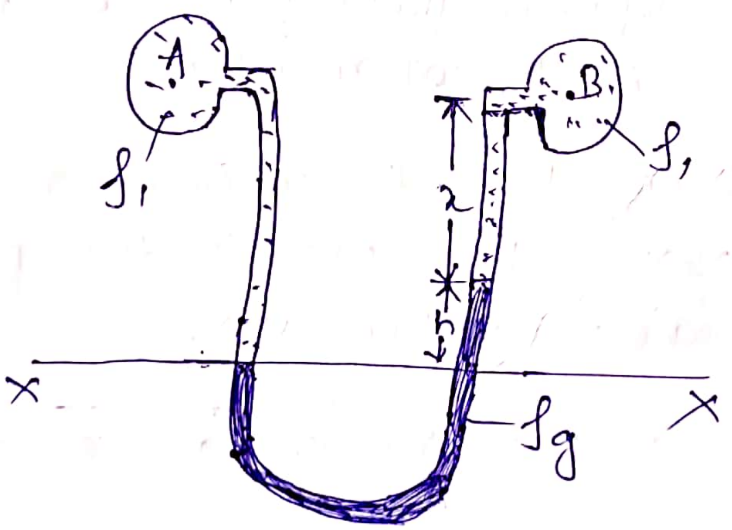
$$\Rightarrow P_A - P_B = \rho_2 g y + \rho_g g h - (\rho_1 g x + \rho_1 g h)$$

$$\Rightarrow P_A - P_B = \rho_2 g y + \rho_g g h - \rho_1 g x - \rho_1 g h$$

$$\Rightarrow P_A - P_B = \rho_2 g y - \rho_1 g x + g h(\rho_g - \rho_1)$$

Case 2

The two points A and B are at same level and contains the same liquid



Pressure in the left column above datum X–X = pressure in the right column above datum X–X.

(i) Pressure in the left column above datum X–X

$$P_A + \rho_1 g (x+h)$$

(ii) Pressure in the right column above datum X–X

$$P_B + \rho_1 g x + \rho_g g h$$

Equating the both pressure

$$P_A + \rho_1 g (x+h) = P_B + \rho_1 g x + \rho_g g h$$

$$\Rightarrow P_A - P_B = \rho_1 g x + \rho_g g h - \{\rho_1 g (x+h)\}$$

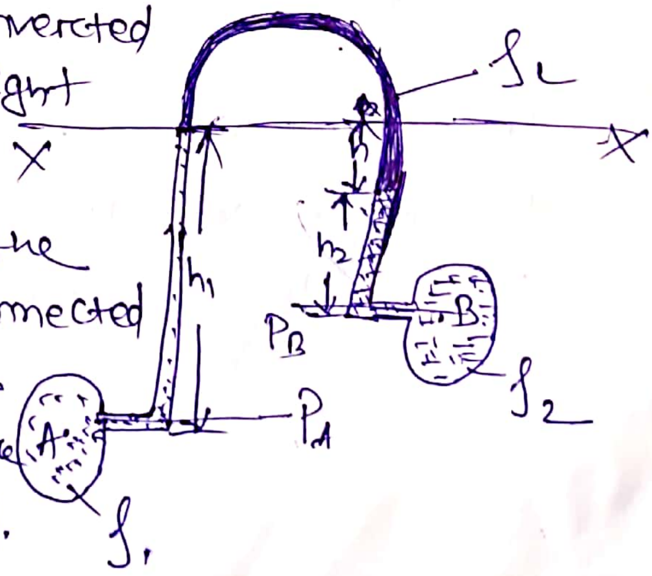
$$\Rightarrow P_A - P_B = \rho_1 g x + \rho_g g h - (\rho_1 g x + \rho_1 g h)$$

$$\Rightarrow P_A - P_B = \cancel{\rho_1 g x} + \rho_g g h - \cancel{\rho_1 g x} - \rho_1 g h$$

$$\Rightarrow P_A - P_B = \rho_g g h - \rho_1 g h$$

$$\Rightarrow P_A - P_B = g h (\rho_g - \rho_1)$$

Inverted U-tube Differential Manometer :-



* It consists of an inverted U-tube containing light liquid.

* The two ends of the manometer are connected to the points whose difference of pressure is to be measured.

* It is used for measuring difference of low pressures.

* Let,

The pressure at A is more that the pressure at B.

(i) h_1 = Height of liquid 'A' from the center of point to the level of light liquid in left limb.

(ii) h_2 = Difference of light liquid level.

(iii) h_2 = Height of liquid 'B' from the center of the pipe to the level of light liquid in the right limb.

(iv) ρ_1 = Density of liquid in pipe 'A'.

(v) ρ_2 = Density of liquid in pipe 'B'.

(vi) ρ_l = Density of light liquid.

Pressure in the ~~right~~ left limb below datum X-X = pressure in the right limb below datum X-X.

(i) Pressure in the left limb below datum X–X.

$$= P_A - \rho_1 g h_1$$

(ii) Pressure in the right limb below datum X–X

$$= P_B - \rho_2 g h_2 - \rho_1 g h$$

Equating ~~the~~ both the pressure.

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_1 g h$$

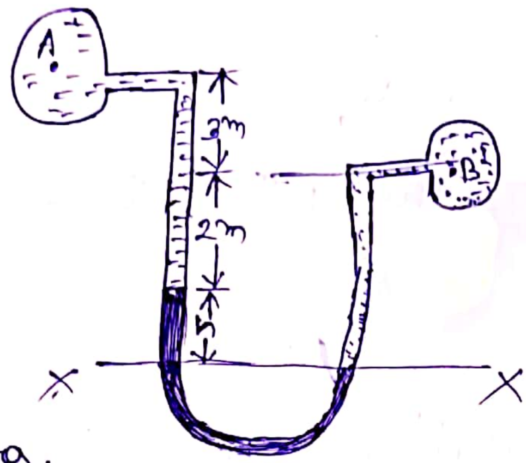
$$\Rightarrow P_A - P_B = -\rho_2 g h_2 - \rho_1 g h + \rho_1 g h_1$$

$$\cancel{\Rightarrow P_A - P_B = -\rho_2 g h_2 - \rho_1 g (h + h_1)}$$

$$\Rightarrow P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_1 g h$$

Question:-

A differential manometer is connected at the two points A and B of two pipes as shown in the figure. The pipe 'A' contains A liquid of specific gravity 1.5 while pipe B contains a liquid of specific of 0.9. The pressure at A and B are 1 kgf/cm^2 and 1.8 kgf/cm^2 respectively. Find the difference of mercury level in the differential manometer.



Given data,

Specific gravity = 1.5 (S_1)

$$\rho_1 = 1.5 \times 1000 = 1500 \ kg/m^3$$

Specific gravity = 0.9 (S_2)

$$\rho_2 = 0.9 \times 1000 = 900 \ kg/m^3$$

Pressure at A = $1 \ kgf/cm^2$

$$= \frac{1 \ kgf}{cm^2}$$

$$= \frac{1 \times 9.81 \ N}{(10^{-2} m)^2}$$

$$= \frac{1 \times 9.81 N}{10^{-4} m^2}$$

$$= 9.81 \times 10^4 \ N/m^2 = 98100 \ N/m^2$$

Pressure at B = $1.8 \ kgf/m^2$

$$= \frac{1.8 \ kgf}{m^2}$$

$$= \frac{1.8 \times 9.81 N}{(10^{-2} m)^2}$$

$$= \frac{1.8 \times 9.81}{10^{-4} m^2}$$

$$= 17.658 \times 10^4 \ N/m^2$$

$$= 176580 \ N/m^2$$

Pressure in the left limb above the dattum line x-x.

$$\cancel{P_A + \rho_1 g h + \rho_2 g h.}$$

$$\cancel{= 98100 + 1500 \times 9.81 \times 5}$$

$$P_A + \rho_1 g h_1 + \rho_g g h$$

$$= 98100 + 1500 \times 9.81 \times 5 + 13{,}600 \times 9.81 \times h$$

$$= 98100 + 73575 + 133416 h$$

$$= 171675 + 133416 h$$

Pressure in the right limb above the dattum line x-x.

$$P_B + \rho_2 g (2 + h)$$

$$= P_B + \rho_2 g \times 2 + \rho_2 g h$$

$$= P_B + 900 \times 9.81 \times 2 + 900 \times 9.81 \times h$$

$$= 176580 + 17658 + 8829 h$$

$$= 194238 + 8829 h$$

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Equating the both Pressure

$$171675 + 133416h = 194238 + 8829h$$

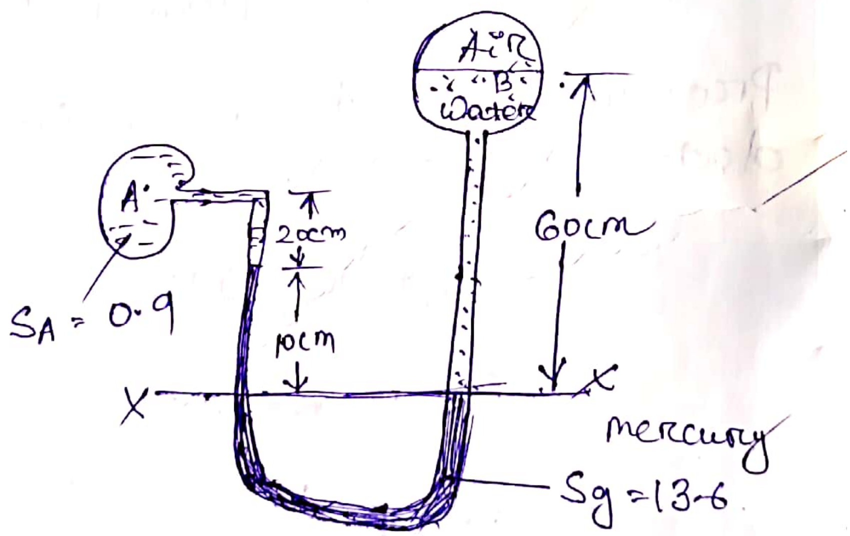
$$\Rightarrow 133416h - 8829h = 194238 - 171675$$

$$\Rightarrow h(133416 - 8829) = 22563$$

$$\Rightarrow h \; 124587 = 22563$$

$$\Rightarrow h = \frac{22563}{124587} = 0.18\,m.$$

② A differential manometer is connected A and B as shown in fig. at B air pressure is $9.81\ N/cm^2$ (Abs). find the Absolute Pressure at A.



Given data,

$$P_B = 9.81\ N/cm^2$$

$$= \frac{9.81\ N}{cm^2}$$

$$= \frac{9.81}{(10^{-2}m)^2} = \frac{9.81}{10^{-4}m^2} = 9.81 \times 10^4\ N/m^2$$

$$= 98100\ N/m^2.$$

$S_A = 0.9$

$\rho_A = 0.9 \times 1000$

$= 900\ kg/m^3$

$\rho_B = 1000\ kg/m^3$

$\rho_g = 13600\ kg/m^3$

pressure at ~~righ~~ left limb above the dattum line X-X

$P_A + \rho_A g (20) + \rho_g g\, 10$

$= P_A + 900 \times 9.81 \times 0.2 + 13600 \times 9.81 \times 0.1$

$\cancel{= P_A + 26487.0}$

$= P_A + 1765.8 + 13341.6$

$= P_A + 15107.4$

Pressure at right limb above the dattum line X-X.

$P_B + \rho_B g \times 0.6$

$= 98100 + 1000 \times 9.81 \times 0.6$

$= 98100 + 5886$

$= 103986$

Equating the both pressure.

$P_A + 15107.4 = 103986$

$\Rightarrow P_A = 103986 - 15107.4$

$\Rightarrow P_A = 88878.6\ N/m^2$ (abs)

Chapter - 3 Hydrostatics

Introduction:-

* Hydrostatics means fluid at rest.
* There is no relative motion between adjcent fluid layers.
* The velocity gradiant will be zero as well as shear stress.
* The forcess acting on the fluid particles will be

(i) Pressure acting normal to the surface.

(ii) Gravity (self weight of the fluid particles)

Total Pressure and Center of pressure on immersed bodies:-

① Total Pressure:-

It is defined as the total force exerted by a static fluid on a surface which may be plane or curve, when the fluid comes in contact with the surface

This force always acts normal to the surface.

② Center of pressure:-

It is defined as the point of applic of total pressure on the surface.

There are four cases of submerged surfaces :-

(i) Vertical plane surface.
(ii) Horizontal plane surface.
(iii) Inclind plane surface.
(iv) Curved surface.

Vertical plane surface submerged in a liquid ~~surface~~:-

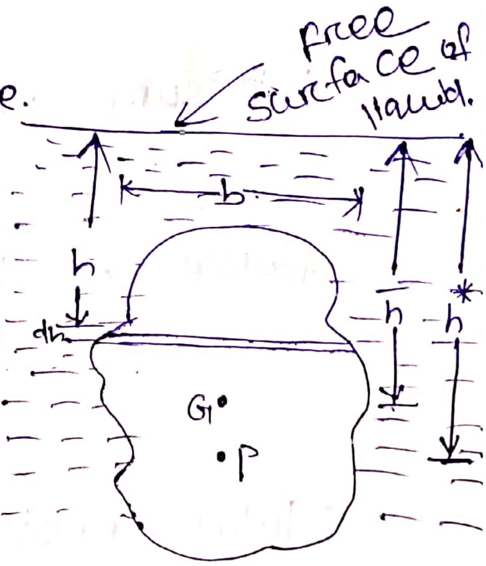
Consider a plane vertical surface of arbitary shape immersed in a liquid.

Let,

A = Total area of surface.

$\bar{h}$ = Distace of center of gravity from free surface of liquid.

h^* = Distance of center of pressure from free surface of liquid.



G = Center of gravity of the plance surface.

P = Center of pressure of the plane surface.

Total Pressure due to the liquid:-

The total pressure on the surface may be determine by dividing the entare surface in to a number of small strips

The force on entary surface is the calculated by intigrating the force on small strips.

Consider a small strip of thickness 'dh' and width 'b' at a depth of 'h' from free surface of liquid.

Pressure on the strip

$$P = \rho g h$$

Force on the strip.

$$dF = P \times dA$$

$$= \rho g h \times b \times dh$$

Total pressure force on the surface =

$$F = \int dF$$

$$= \int \rho g h \times b \times dh$$

$$F = \rho g \int h \times b \times dh$$

$$= \rho g \boxed{\int h \times dA}$$

= Area of the surface × distance of C.G from free surface of liquid.

$= A \times \bar{h}$

$$\boxed{F = \rho g A \bar{h}}$$

Centre of Pressure (h^*)

Centre of Pressure is calculated by using Principle of moments: Which state that the moment of resultant force about an axis is equals to the some of the ~~move~~ moments of components about the Same axis.

The resultant force 'F' is acting at 'P' at a distance 'h^*' from the free Surface of liquid. Hence moment of force 'F' about free surface liquid is equals to $F \times h^*$.

The moment of the component 'dF' about free surface of liquid is equals to $dF \times h$.

Some of the moments of all such forces about free surfaces of liquid is equals to $\int dF \times h$.

$$= \int P \times dA \cdot \times h$$

$$= \int \rho g h \times dA \times h$$

$$= \rho g \int h \times dA \times h$$

$$= \rho g \int h^2 \times dA$$

$\int h^2 \times dA = I$

Moment of inertia of the surface area

about free surface of liquid.

$$\int h^2 dA = I_0$$

So,

$$\rho g \int h^2 dA$$

$$= \rho g I_0$$

$$\Rightarrow F \times h^* = \rho g I_0$$

$$\Rightarrow \rho g A \bar{h} \times h^* = \rho g I_0$$

$$\Rightarrow h^* = \frac{\rho g I_0}{\rho g A \bar{h}}$$

$$\Rightarrow \boxed{h^* = \frac{I_0}{A \bar{h}}}$$

According to parallel axis theorem

$$\boxed{I_0 = I_G + A \bar{h}^2}$$

Where,

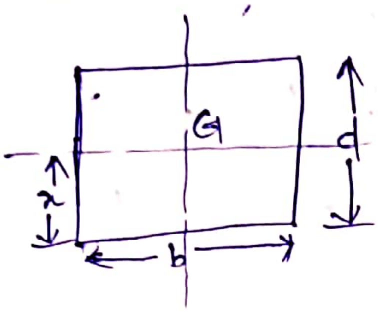
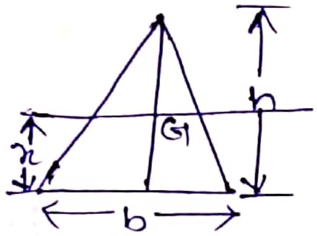
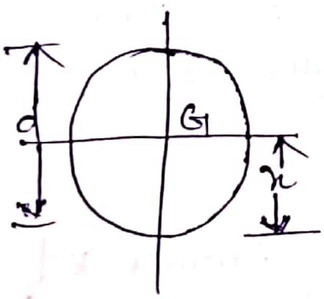
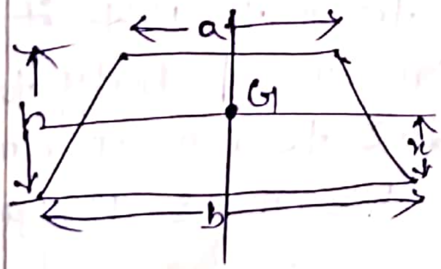
I_G = Moment of inertia of the surface about an axis passing through center of gravity and parallel to free surface of liquid.

$$h^* = \frac{I_0}{A \bar{h}}$$

$$\Rightarrow h^* = \frac{I_G + A \bar{h}^2}{A \bar{h}}$$

$$\Rightarrow \boxed{h^* = \frac{I_G}{A \bar{h}} + \bar{h}}$$

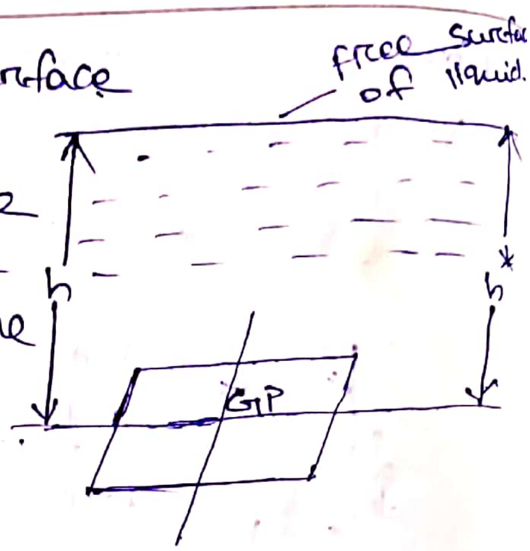
* Center of gravity and value of I_G for some common surfaces :-

Surfaces	C.G. from base	Area	I_G	I_0
	$d/2$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
	$x = h/3$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	
	$x = \frac{(2a+b)h}{(a+b)3}$	$\frac{(a+b)h}{2}$	$\left[\frac{a^2+4ab+b^2}{36(a+b)}\right] h^3$	

Horizontal Plane surface immerged in a Static fluid :-

Consider a horizontal surface submerged in a liquid

* At every point of the surface is at the same depth from the free surface of liquid. The pressure intensity is equal on the entaire surface.



$$F = \rho g \cancel{A} h^*$$

$$h^* = \bar{h}$$

Archimedis Principle:-

When a body is immerresed in a fluid either wholly or partially, it is buoyed or lifted up by a force, which is equal to the weight of fluid displaced by the body.

Buoyancy:-

Whenever a body is immersed wholly or partialy in a fluid it is subjected to an upwards froce which tends to lift it up this tendency for an immersed body to be lifted up in the fluid due to an upward force opposite gravity is known as Buoyancy This upward force is known as force of Buoyancy.

Centre of Buoyancy:-

It is defined as the paint, through which the force of Buoyancy is supposed to out. The force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body.

* Centre of Buoyancy will be the centre of gravity of the fluid displaced.

Problem :- 1 :-

Find the vaccume of the water displaced and position of centre of ~~boure bo~~ buoyancy force a wooden block of width 2.5m and ~~the~~ of depth 1.5m when it floats horizontally in water. The density of wooden block is 650 kg/m³ and its length 6.0m.

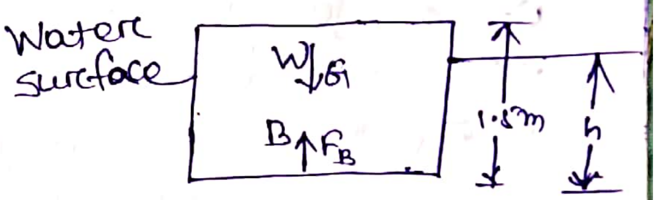
ANS

Density of wooden block = 650 kg/m³

Width = 2.5m

Depth = 1.5m

Length = 6m



Volume of the block

$= 2.5 \times 1.5 \times 6$

$= 22.50 \text{ m}^3$

Wt. of water displaced

$= \rho g \times V$

$= 650 \times 9.81 \times 22.50$

$= 143471 \text{ N}$

Volume of water displaced.

$= \dfrac{\text{Weight}}{\rho_w \times g}$

$= \dfrac{143471}{1000 \times 9.81} = 14.625$

Position of centre of ~~Bo~~ Buoyancy

Volume of wooden block in water

= Volume of water displac

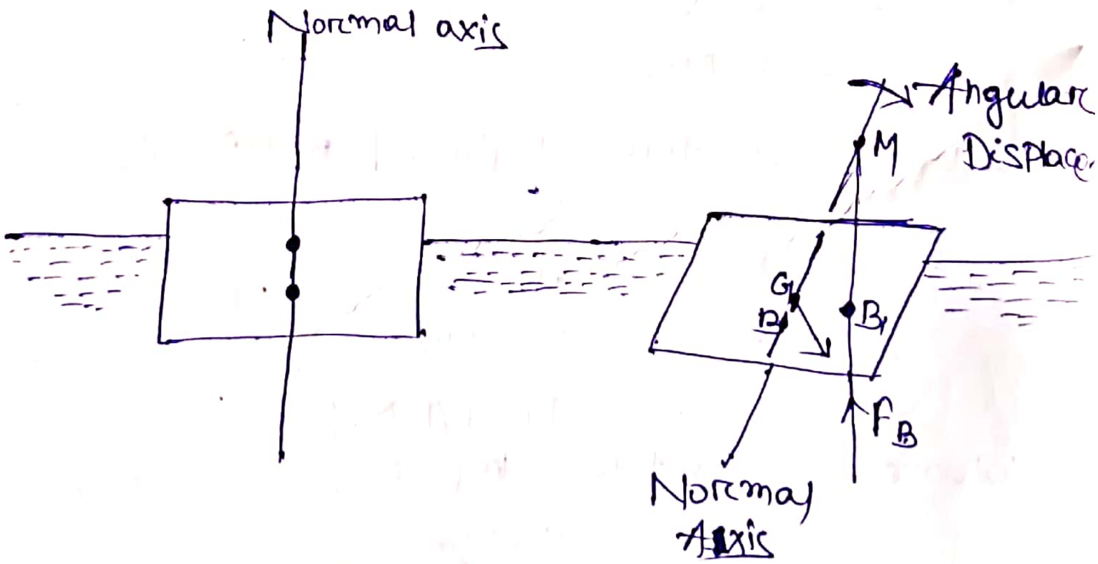
$2.5 \times 6 \times h = 14.625$

$\Rightarrow h = \frac{14.625}{2.5 \times 6} = 0.975 \ m$

Centre of ~~bo~~ buoyancy $= \frac{0.975}{2} = 0.4875 \ m$ from base.

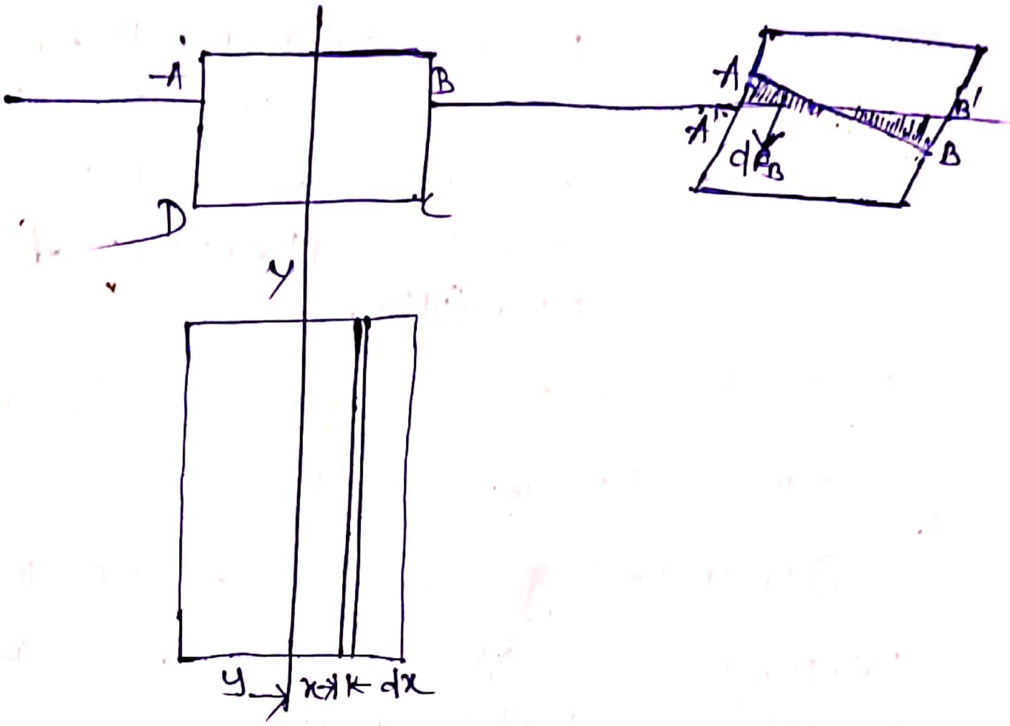
Meta-centre:-

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta centre may also be defined as the point at which the line of action of the force of buoyancy ~~while~~ will melt the normal axis of the body when the body is given a small angular displacement.



M. Meta centre

Meta centric Height :-



The distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height i.e. the distance MG.

Concept of floatation :-

When a body is immersed in any fluid, it expressed 2 forces.

- Weight of the body acting vertically downward.
- Buoyancy force F_B acting vertically upward.

In case $W > F_B$ = The body will in the fluid.

$W = F_B$ = The body will equillibrium at any level.

$W < F_B$ = The body will move upward in the fluid until the fluid displaced by its submerged part is equal to its weight (W). The body in this situation is said to be floating and this phenomenon is known as floatation.

Ways to make the body to float:—

The body can float.

(i) Decreasing the weight of the body by keeping the volume same. for example making a body hollow.

(ii) Increasing the volume of the body by keeping the weight same equillibrium attaching life jacket to a person keeps the person floating.

Types of equillibrium of floating body:—

* Stable equillibrium
* Unstable equillibrium
* Neutral equillibrium.

Stable equillibrium:—

When a body is given a small angular displacement by some external force and then it returns back to its original position due to internal force such an equillibrium is called stable equillibrium.

Unstable equillibrium:—

If the body doesn't return to its original position from the slightly displaced angular position and heals farther away when given a small angular displace, such an equillibrium is called unstable equillibrium.

Neutral equillibrium:—

If a body when given a small angular displacement occupies a new position and remains at rest in this new position it is said to possess a neutral equillibrium.

Chapter:- 04 Kinematics of flow

Types of flow:-

* Steady and unsteady flow.
* Uniform and Non-uniform flow.
* Laminar and turbulent flow.
* Compressible and Incompressible flow.
* Rotational and Irrotational flow.
* One, two and three dimensional flow.

Steady and Unsteady flow:-

(i) Steady flow:-

It is defined as that type of flow in which the fluid characteristics like velocity, pressure and density etc, at a point don't change with time.

$$\left(\frac{\partial V}{\partial t}\right)_{(x_0,y_0,z_0)} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{(x_0,y_0,z_0)} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{(x_0,y_0,z_0)} = 0$$

(ii) Unsteady flow:-

It is defined as that type of flow in which the fluid characteristics like velocity, pressure and density etc, at a point changes with time.

$$\left(\frac{\partial v}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{(x_0,y_0,z_0)} \neq 0$$

Uniform and Non-uniform flow:-

(i) Uniform flow:-

Uniform flow is that type of flow in which the velocity at any given time doesn't change with respect to space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{constant}} = 0$$

(ii) Non-uniform flow:-

It is that type of flow in which the velocity at any given time changes with respect to space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=\text{Constant}} \neq 0$$

Laminar and turbulent flow:-

(i) Laminar flow:-

Laminar flow is that type of flow in which the fluid particles move along a we define path or streamlines and all streamlin as Straight and Parallal.

(ii) Turbulent flow:-

It is that type flow in which the fluid Particles moves in a zig-zag mannar.

For a pipe flow the ~~type~~ type of flow is determine "Reynold's Number". (Re).

$$Re = \frac{VD}{\nu}$$ Where V = Velocity of flow.
D = Diamiter of Pipe
ν = Kinematics ~~viso~~ viscosity.

Re < 2000 then the flow is Laminar flow

ν = Stoke CGS unit

Re > 2000 then the flow is Turbulent flow

2000 < Re < 4000 may be Laminar or Turbulent flow

Compressible and In compressible flow:-

(i) Compressible flow:-

Compressible flow is that type of flow in which the density of the fluid Changes from point to point that is density is not Constant for the fluid flow.

$\boxed{\cancel{\rho \neq 0}}$ $\boxed{\rho \neq \text{Constant}}$

(ii) In Compressible flow:-

It is that type of flow in which the density is constant for the fluid flow.

$$\boxed{\rho = \text{constant}}$$

* Liquids are generally incompressible but gases are compressible.

Rotational and Ir-rotational flow:-

(i) Rotational flow:-

Rotational flow is that type of flow in which the fluid particles while flowing in a stream lines also rotate about their own axis.

(ii) Ir-rotational Flow:-

It is that type of flow in which the fluid particles while moving along stream line don't rotate about their own axis.

One, two and three dimensional flow:-

(i) One dimensional flow:-

It is that type of flow in which the flow parameters like velocity is a function of time and one space co-ordinate

$U(x) \neq 0$ $\quad U = f(x)$
$V(y) = 0$
$W(z) = 0$

X, Y, Z are the directions and U, V, W are the velocities in those directions respectively.

(ii) Two dimensional flow:-

It is that type of flow in which the flow parameters like velocity is a function of time and two space co-ordinate.

$U = f(x)$ $\quad U(x) \neq 0$
$V = f(y)$ $\quad V(y) \neq 0$
$W = 0$ $\quad W(z) = 0$

(iii) Three dimensional flow:-

It is that type of flow in which the flow parameters like velocity is a function of time and three space co-ordinate

$U = f(x)$ $\quad U(x) \neq 0$
$V = f(y)$ $\quad V(y) \neq 0$
$W = f(z)$ $\quad W(z) \neq 0$

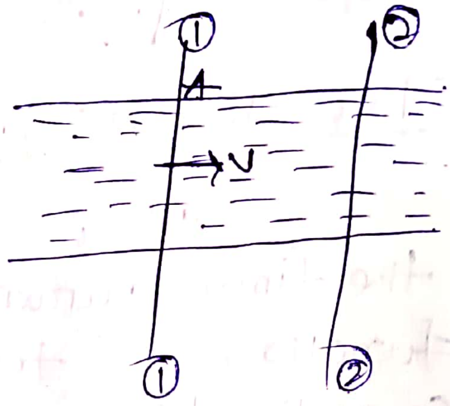
Rate of flow or Discharge (Q)

* Rate of flow or Discharge is defined as the quantity of flowing per second through a section of pipe or channel.

For in Compressible fluid (Liquid):-

* In case of liquid discharge is expressed as the volume of liquid flowing across the section per second.

$$Q = A.V$$



For Compressible fluid :-

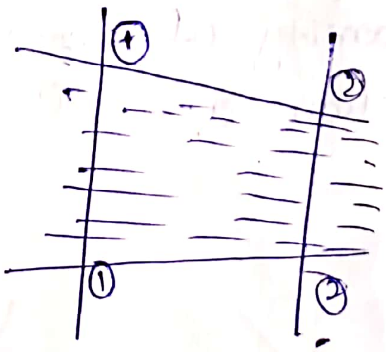
* For compressible fluid discharge is express as the weight of the fluid flowing across a section per second.

$$Q = Mass \times g / sec.$$

Continuity Equation:-

* The equation that derive from the principle of conservation of mass is called continunity equation. Thus for a fluid through the pipe all the cross section quantity of fluid per second is constant.

* Consider a pipe of decresing area. Let,
  section (1,1) and section (2,2) in the pipe



Let,

A_1 = Cross sectional are at section 1-1.

V_1 = Avg. velocity at section 1-1.

ρ_1 = Density of liquid at section 1-1.

Similarly

A_2, V_2, ρ_2 are the corresponding values at section 2-2.

* According to law of conservation of mass flow rate at section 1-1 = Mass flow rate at section 2-2.

$\rho_1 \times$ (Volume/sec) at section 1-1 =

$\rho_2 \times$ (Volume/sec) at section 2-2.

$$\Rightarrow \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

In case of liquids

$$\rho_1 = \rho_2$$

so,

$$A_1 V_1 = A_2 V_2 = A_3 V_3 \text{ ---- } A_n V_n$$

$$\boxed{AV = \text{Constant}}$$

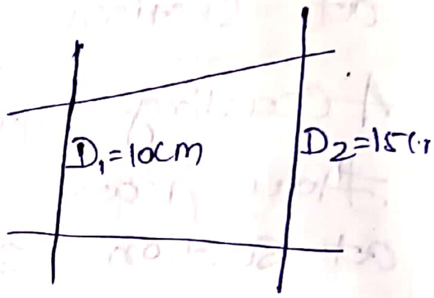
Problem 5.1:-

The diameters of a pipe at the sections 1 and 2 are 10 c.m and 15 c.m respectively. Find the dis-charge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.

Ans:-

Diameter of section 1 = 10 cm
= 0.1 m

Diameter of section 2 = 15 cm
= 0.15 m



Area of section 1 $= \frac{\pi}{4} \times d^2$
$= \frac{\pi}{4} \times (0.1)^2$
$= 7.853 \times 10^{-3}\ m^2$

Area of section 2 $= \frac{\pi}{4} \times d^2$
$= \frac{\pi}{4} \times (0.15)^2$
$= 0.01767\ m^2$

Velocity ~~[illegible]~~ of section 1 = 5 m/s

Continuity Equⁿ

$A_1 V_1 = A_2 V_2$

$\Rightarrow 7.853 \times 10^{-3} \times 5 = 0.01767 \times V_2$

$\Rightarrow 0.01767 \times V_2 = 7.853 \times 10^{-3} \times 5$

$\Rightarrow V_2 = \frac{7.853 \times 10^{-3} \times 5}{0.01767} = 2.22\ m/s.$

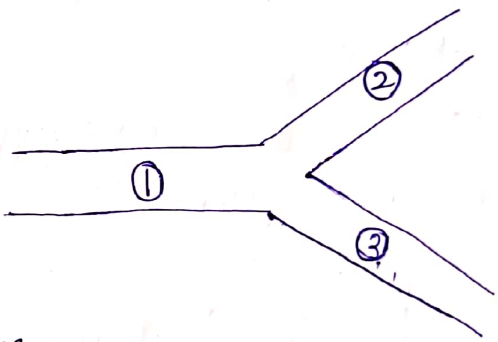
$\therefore Q_1 = A_1V_1 = 7.853 \times 10^{-3} \times 5 = 0.039265\ m^3/s$.

$\therefore Q_2 = A_2V_2 = 0.01767 \times 2.22 = 0.0392274\ m^3/s$.

Problem 5.2:-

A 30cm diameter pipe, conveying water, branches into two pipes of diameters 20cm and 15cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

Ans:-



Diameter of pipe 1 = 30cm
$= 0.3m$

Diameter of pipe 2 = 20cm
$= 0.2m$

Diameter of pipe 3 = 15cm
$= 0.15m$

Velocity of pipe 1 = 2.5 m/s.

Velocity of pipe 2 = 2 m/s

Area of pipe 1 $= \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times (0.3)^2$

$= 0.07068\ m^2$

Area of pipe 2 $= \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times (0.2)^2$

$= 0.0314\ m^2$.

Area of pipe 3 $= \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times (0.15)^2 = 0.01767\ m^2$

$\therefore Q_1 = A_1 V_1 = 0.07068 \times 2.5 = 0.1767\ m^3/s$

$\therefore Q_2 = A_2 V_2 = 0.0314 \times 2 = 0.0628\ m^3/s$

We know that

$Q_1 = Q_2 + Q_3$

$\Rightarrow Q_1 = 0.0628 + A_3 V_3$

$\Rightarrow 0.1767 = 0.0628 + 0.01767 \cdot V_3$

$\Rightarrow 0.1767 - 0.0628 = 0.01767 \cdot V_3$

$\Rightarrow 0.1139 = 0.01767 \cdot V_3$

$\Rightarrow \frac{0.1139}{0.01767} = V_3$

$\Rightarrow 6.44 = V_3 \quad \Rightarrow V_3 = 6.44\ m/s$

Bernoulli's Theorem:-

Statement:-

It states that for a steady, ideal flow of an in-compressible fluid the total energy at any point in the fluid remains constant. The total energy consist of Pressure energy, kinetic energy and Potential energy.

Energies per unit wt. are

① Kinetic energy / unit wt $= \frac{V^2}{2g}$

② Pressure energy / unit wt $= \frac{P}{\rho g}$

③ Potential energy / unit wt $= Z$

Then Bernoulli's equation will be

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}}$$

Assumptions:-

* The fluid is in-compressible that is density in constant.
* The flow is steady.
* The flow is ideal.
* The flow is ir-rotational.

Total Energy:-

① Pressure energy:-

$$\boxed{\begin{aligned} P &= \frac{F}{A} \\ \Rightarrow P \times A &= F \end{aligned}}$$

Energy = Workdone

= force x displacement

= P x (A x displacement)

= P x V

Pressure E / unit wt.

$$\Rightarrow \frac{PV}{mg} = \frac{P}{\frac{m}{V} \times g} = \frac{P}{\rho g} \qquad \left(\because \rho = \frac{M}{V} \right)$$

② Kinetic energy:-

$$\frac{1}{2} mv^2$$

Kinetic energy / unit wt.

$$\frac{\frac{1}{2} mv^2}{mg} = \frac{\frac{v^2}{2}}{g}$$

$$= \frac{v^2}{2} \times \frac{1}{g}$$

$$= \frac{v^2}{2g}$$

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③ Potential Energy:-

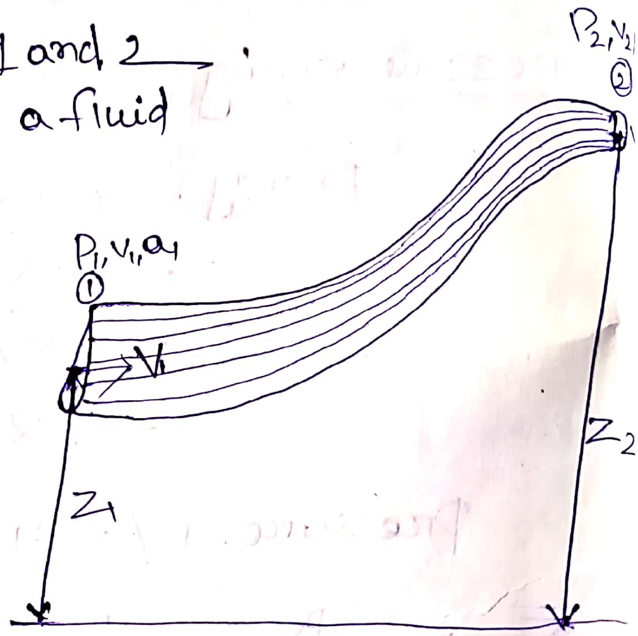
$$mgz$$

$$\text{Potential energy / unit wt} = \frac{mgz}{mg}$$

$$= z$$

Proof:-

Consider two sections 1 and 2 in a pipe in which a fluid is flowing.



Let,

P_1 = Pressure at section 1

V_1 = Velocity at section 1

a_1 = Area of section 1

Z_1 = Height of section 1 from datum.

Similarly P_2, V_2, a_2, Z_2 are the corresponding values at section 2.

Change in Pressure Energy:-

Work is done by the pressure ener in increasing kinetic and potential ener

$P_1V_1 - P_2V_2$

$V_1 = V_2$ - (constant)

(According to continuity eqn)

$(P_1 - P_2)V$

Change in kinetic energy:-

$\frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$

$\frac{1}{2}m(V_2^2 - V_1^2)$

Change in Potential energy:-

$mgz_2 - mgz_1$

$= mg(z_2 - z_1)$

Gain in energy = Lose ~~of~~ in energy

(According to low of Conservation of energy)

$\Rightarrow$ Gain in KE + Gain in potential energy = Lose in Pressure energy

$\Rightarrow \frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1) = (P_1 - P_2)V$

$\Rightarrow \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2 + mgz_2 - mgz_1 = P_1V - P_2V$

$\Rightarrow P_1V + \frac{1}{2}mV_1^2 + mgz_1 = P_2V + \frac{1}{2}mV_2^2 + mgz_2$

(By dividing mg on both side)

$\Rightarrow \frac{P_1V}{mg} + \frac{\frac{1}{2}\cancel{m}V_1^2}{\cancel{m}g} + \frac{\cancel{mg}z_1}{\cancel{mg}} = \frac{P_2V}{mg} + \frac{\frac{1}{2}V_2^2}{g} + \frac{\cancel{mg}z_2}{\cancel{mg}}$

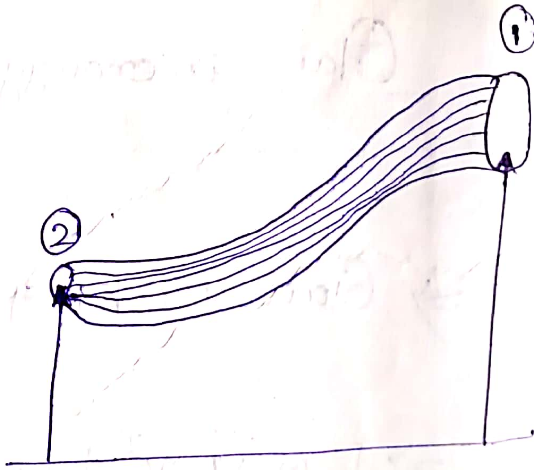
$\Rightarrow \frac{P_1}{\frac{m}{V}\times g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\frac{m}{V}\times g} + \frac{V_2^2}{mg} + z_2$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2.$$

$$\boxed{\therefore \frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{Constant}} \rightarrow \text{Bernoulli's Equation.}$$

<u>Problem:- 6.4:-</u>

The water is flowing through a pipe having diameter 20 cm and 10 cm at section 1 and 2 respectively. The rate of flow through pipe is 35 l/sec. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 39.24 N/cm². Find the intensity of pressure at section 2.



Ans:- Given data,

Diameter of ~~Pipe~~ Section 1 = 20 cm
= 0.2 m

Diameter of ~~Pipe~~ Section 2 = 10 cm
= 0.1 m

Area of ~~Pipe~~ Section 1 $= \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times (0.2)^2$

$= 0.0314 \, m^2.$

Area of ~~Pipe~~ Section 2 $= \frac{\pi}{4} \times d^2$

$= \frac{\pi}{4} \times (0.1)^2$

$= 7.853 \times 10^{-3} \, m^2$

Height of the section 1 = 6 m
Height of the section 2 = 4 m

Pressure at section 1 = $39.24\ N/cm^2$

$$= 39.24 \frac{N}{cm^2}$$

$$= 39.24 \frac{N}{(0.01)^2}$$

$$= 39.24 \frac{N}{10^{-4}}$$

$$= 39.24 \times 10^4$$

$$= 392400\ N/m^2$$

Pressure at section 2 = ?

Rate of flow

$$Q = 35\ l/sec = \frac{35}{1000} = 0.035\ m^3/s$$

$\therefore\ Q = A_1 V_1$

$\Rightarrow 0.035 = 0.0314 \times V_1$

$\Rightarrow 0.0314 \times V_1 = 0.035$

$\Rightarrow V_1 = \frac{0.035}{0.0314} = 1.114\ m/s$

$\therefore\ Q = A_2 V_2$

$\Rightarrow 0.035 = 7.853 \times 10^{-3} \times V_2$

$\Rightarrow 7.853 \times 10^{-3} \times V_2 = 0.035$

$\Rightarrow V_2 = \frac{0.035}{7.853 \times 10^{-3}} = 4.456\ m/s$

Applying Bernoulli's theorem,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{392400}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + y$$

$$\Rightarrow 40 + \frac{1.24}{19.62} + 6 = \frac{P_2}{9810} + \frac{19.85}{19.62} + y$$

$$\Rightarrow 40 + 0.0632 + 6 = \frac{P_2}{9810} + 1.011 + y$$

$$\Rightarrow 46.06 = \frac{P_2}{9810} + 5.011$$

$$\Rightarrow \frac{P_2}{9810} + 5.011 = 46.06$$

$$\Rightarrow \frac{P_2}{9810} = 46.06 - 5.011$$

$$\Rightarrow \frac{P_2}{9810} = 41.049$$

$$\Rightarrow P_2 = 41.049 \times 9810 = 40.69 \; N/m^2$$

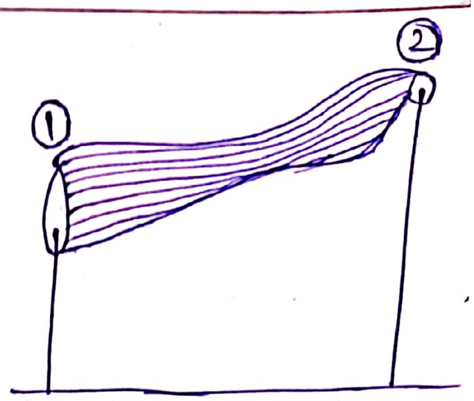
Problem :- 6.5 :-

Water is flowing through a pipe havi diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the differen in dattum head if the rate of flow through the pipe 40L/sec.

Ans:- Given data,

Diameter of bottom section = 300 mm

$= 0.3 m$



Diameter of upper section = 200 mm

$= 0.2 m$

Area of bottom section $= \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.3)^2 = 0.070 \, m^2$

Area of upper section $= \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \, m^2$

Pressure at the bottom end $= 24.525 \frac{N}{cm^2}$

$= 24.525 \frac{N}{(0.01)^2}$

$= 24.525 \frac{N}{10^{-4}}$

$= 24.525 \times 10^4$

$= 245250 \, N/m^2.$

Pressure at the upper end $= 9.81 \frac{N}{cm^2}$

$= 9.81 \frac{N}{(0.01)^2}$

$= 9.81 \frac{N}{10^{-4}}$

$= 9.81 \times 10^4$

$= 98100 \, N/m^2.$

Rate of flow (Q) = 40 l/sec

$= \frac{40}{1000} = 0.040 \, m^3/s.$

So,

(i) $Q = A_1 V_1$

$\Rightarrow 0.040 = 0.070 \times V_1$

$\Rightarrow 0.070 \times V_1 = 0.040$

$\therefore V_1 = \frac{0.040}{0.070} = 0.57 \, m/s$

(ii) $Q = A_2 V_2$

$\Rightarrow 0.040 = 0.0314 \times V_2$

$\Rightarrow 0.0314 \times V_2 = 0.040$

$\Rightarrow V_2 = \frac{0.040}{0.0314} = 1.27$ m/s

Applying Bernoulli's theorem.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{245250}{1000 \times 9.81} + \frac{(0.57)^2}{2 \times 9.81} + Z_1 = \frac{98100}{1000 \times 9.81} + \frac{(1.27)^2}{2 \times 9.81} + Z_2$$

$\Rightarrow 25 + 0.016 + Z_1 = 10 + 0.08 + Z_2$

$\Rightarrow 25.016 + Z_1 = 10.08 + Z_2$

$\Rightarrow 10.08 + Z_2 = 25.016 + Z_1$

$\Rightarrow Z_2 - Z_1 = 25.016 - 10.08$

$\Rightarrow Z_2 - Z_1 = 14.92$ —— (ANS)

Application of Bernoulli's Theorem:—

Bernoulli's equation appered to the measuring devices

1. Venturimeter
2. Orificemeter
3. pitot - tube

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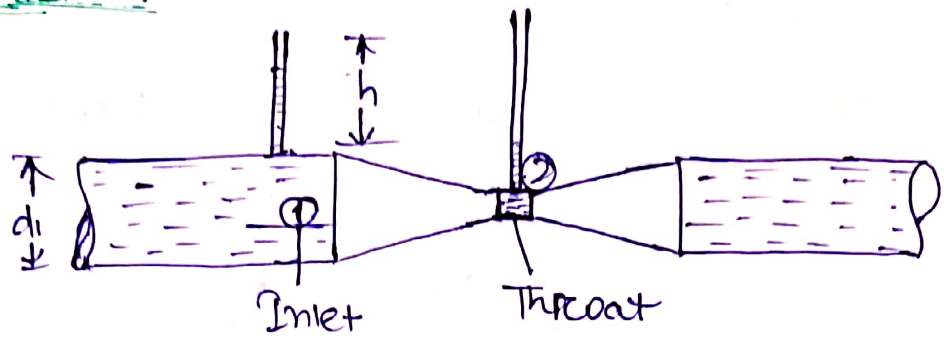
Venturimeter:-



A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe.

(a) A Short Converging Part.
(b) Throat.
(c) Diverging part.

Expression for rate of flow:-

Consider a venturimeter fitted in a horizontal pipe.

Let d_1 = Dia at inlet or at section (1).
P_1 = Pressure at section (1)
V_1 = Velocity of fluid at section (1)
a = Cross sectional area at Section (1) $= \frac{\pi}{4} \times d_1^2$

and d_2, P_2, V_2, a_2 are corresponding values at section (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As pipe is horizontal hence $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Difference of pressure heads at sections ① and ② it is equal to h or $\frac{P_1 - P_2}{\rho g} = h$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$a_1 V_1 = a_2 V_2$$

$$\Rightarrow V_1 = \frac{a_2 V_2}{a_1}$$

$$\Rightarrow h = \frac{V_2^2}{2g} - \frac{(a_2 V_2 / a_1)^2}{2g}$$

$$\Rightarrow h = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$\Rightarrow h = \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$\Rightarrow \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right] = h$$

$$\Rightarrow V_2^2 = 2gh\left[\frac{a_1^2}{a_1^2 - a_2^2}\right]$$

$$\Rightarrow V_2 = \sqrt{2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right)}$$

$$\Rightarrow V_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q = a_2 V_2$$

$= \frac{a_2 a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$ (Ideal discharge) (Put the value of V_2)

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

C_d = Co-efficient of venturimeter.

Value of 'h' given by differential U-tube manometer.

Case 1

Let the differential U-tube manometer contains a liquid which is heavier than the liquid flowing through the pipe.

Let S_h = SP gravity of heavy liquid

S_b = SP gravity of liquid flowing through pipe.

x = Difference of the heavne

$$P_A - P_B = gx(\rho_g - \rho_o)$$

$$\Rightarrow \frac{P_A - P_B}{\rho_1 g} = \left(\frac{\rho_g}{\rho_o} - 1\right)$$

$$\Rightarrow h = x\left(\frac{\rho_g}{\rho_o} - 1\right)$$

$$\Rightarrow h = x \cdot \left(\frac{S_g}{S_o} - 1\right)$$

Case II

If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe.

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

Case-III

Inclined venturimeter with differential U-tube manometer:

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_0} - 1\right]$$

Case-IV

Inclined venturimeter with differential U-tube manometer.

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_0}\right]$$

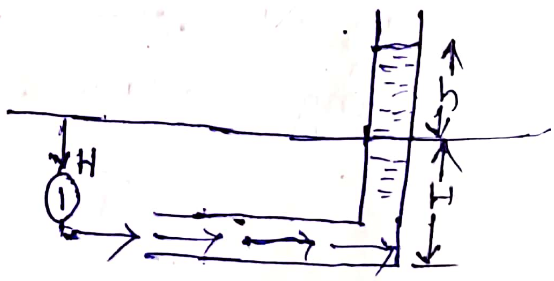
Limitations:-

(1) Bernoulli's equation has been derived under the assumption that no external force except the gravity force is acting on the ~~fluid~~ liquid. But in actual parctice some external force always acting on the liquid when effect the flow of liquid.

(2) If the liquid is flow in a curved path the energy due to centrifugal force should also be taken in to account.

Pitot tube:-

It is a device used for measuring the velocity of flow at any point in a pipe or a ~~chem~~ channel.



It is based on the Principle that if the velocity of flow at a point becomes zero the pressure there is increased due to conversion of the kinetic energy in to pressure energy.

The Pitot tube Consists of a glass tube bent an right angles.

Consider two points ① and ② at the same level such a way that ② is at the inlet of pitot tube and ① is far away from the tube.

Let

P_1 = Pressure at ①
V_1 = Velocity of fluid at pt ①
P_2 = Pressure at ②
V_2 = Velocity at pt ②
H = Depth of tube in the liquid
h = Rise of the liquid in the tube above the free surface.

Applying Bernoulli's equation:-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{\rho g} = H \qquad \frac{P_2}{\rho g} = (h+H)$$

$$\cancel{H} + \frac{V_1^2}{2g} = h + \cancel{H}$$

$$\Rightarrow V_1 = \sqrt{2gh} \quad \Rightarrow V_{act} = C_V \times \sqrt{2gh}$$

C_V = Co efficient of velocity.

Chapter-05 Orifices, notches and weirs

Definition of Orifices:-

* Orifice is a small opening of any cross-section (such as (circular, triangular, rectangular) etc. On the side or at the bottom of a tank through which a fluid is flowing.

* Orifices are used for measuring the rate of flow of fluid.

Classification:-

The orifices are classified on the basis of their size, shape, nature of the discharge and shape of the up stream edge.

(1) According to the size of orifice

- Small orifice
- Large orifice

(2) According to the shape of orifice

- Circular orifice
- Triangular orifice
- Rectangular orifice
- Square orifice.

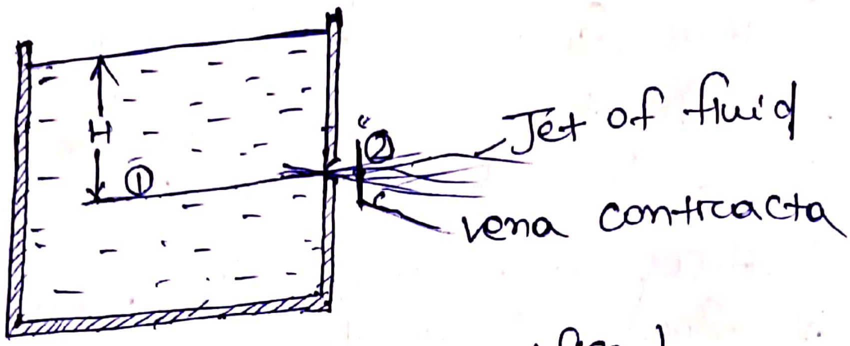
(3) According the nature of discharge.

- Free discharging orifices
- Drowned or submerged orifice.

(4) According to the shape of upstream edge

- Sharp edged orifice
- Bell mouthed orifice

Flow through an orifice:-



(Tank with an orifice)

Consider a tank fitted with a circular ~~circu~~ orifice in one of its side.

* The liquid flowing through the orifice forms a jet of liquid whose area is less than the area of orifice.

* The section where the area of jet is minimum and the stream lines are straight and parallel to each other, is known as vena contracta.

Consider two points ① and ②

Point ① ~~in~~ is inside the tank

Point ② is at vena contracta

Let the flow is steady and H is the head of liquid above the centre of the orifice.

Applying Bernoulli's equation at points 1 and 2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$z_1 = z_2$, $\frac{P_1}{\rho g} = H$ and $\frac{P_2}{\rho g} = 0$, V_1 = very small as compared to V_2 so neglected.

Substituting all values in the equation

$$\Rightarrow H + 0 = 0 + \frac{V_2^2}{2g}$$

$$\Rightarrow V_2 = \sqrt{2gH}$$

$$a_2 V_2 = Q$$

Hydraulic Co efficients (orifice co efficients)

The orifice co-efficients are

1. Co-efficients of velocity C_v
2. Co-efficient of contraction C_c
3. Co-efficient of discharge C_d

Co-efficient of velocity (C_v) :-

Co-efficient of velocity is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by Cv.

Mathematically,

$$C_v = \frac{\text{Actual velocity of jet at vena contracta}}{\text{Theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}$$

The value of C_v varries from 0.95 to 0.99
Generally C_v is taken as 0.98

Co-efficient of Contraction (C_c):-

It is defined as the ratio of the area of the jet at vena contracta to the area of the orifice.

* It is denoted by C_c.

Let

a = area of orifice

a_c = area of jet at vena-contracta

$$C_c = \frac{\text{area of jet at vena contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a}$$

The value of C_c varies from 0.61 to 0.69 in general the value of C_c may be taken 0.64.

Co-efficient of discharge (C_d)

It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d.

Let

Q = actual discharge.

Q_{th} = Theoretical discharge.

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

$$= \frac{\text{Actual area} \times \text{actual velocity}}{\text{Theoretical area} \times \text{Theoretical velocity}}$$

$$= \frac{\text{Actual area}}{\text{Theoretical area}} \times \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$$

$= C_c \times C_v$

$\boxed{C_d = C_c \times C_v}$

The value of C_d varies from 0.61 to 0.65. In general C_d is taken as 0.62.

Problem:-

1) The head of water over an orifice of diameter 40mm is 10m. Find the actual discharge and actual velocity of the jet at vena contracta. Take $C_d = 0.6$ and $C_v = 0.98$.

Ans Given data,

Head of water = 10m

Diameter of orifice = 40mm = 0.04 m

$$\text{Area} = \frac{\pi}{4} \times d^2$$
$$= \frac{\pi}{4} \times (0.04)^2 = 1.256 \times 10^{-3}\ m^2$$

$C_d = 0.6$

$C_v = 0.98$

$$\text{Actual velocity} = C_v \sqrt{2gH}$$
$$= 0.98 \times \sqrt{2 \times 9.81 \times 10}$$
$$= 13.72$$

$$\text{Actual discharge} = C_d \times \text{Theoretical discharge}$$
$$= C_d \times (a_{th} \times V_{th})$$
$$= 0.6 \times \left(1.256 \times 10^{-3} \times \sqrt{2gH}\right)$$
$$= 0.6 \times \left(1.256 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 10}\right)$$
$$= 0.01055\ m^3/s \text{ (ANS)}.$$

(2) The head of water over the centre of an orifice of diameter 20mm is 1 m. The actual discharge through the orifice is 0.85 lit/s. Find the co-efficient of discharge?

Ans:- Given data:-

Diameter = 20mm = 0.02m

Actual discharge = 0.85 lit/s = 0.85×10^{-3} m^3/s

$H = 1m$

$\therefore$ Area $= \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.02)^2 = 3.1415 \times 10^{-4}\ m^2$

So,

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}}$$

$$= \frac{0.85 \times 10^{-3}}{a \times V}$$

$$= \frac{0.85 \times 10^{-3}}{3.1415 \times 10^{-4} \times \sqrt{2gH}}$$

$$= \frac{0.85 \times 10^{-3}}{3.1415 \times 10^{-4} \times \sqrt{2 \times 9.81 \times 1}}$$

$= 0.610$ (ANS)

Notches and Weirs:-

Notch:- A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank.

* It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

Weir:- A weir is a concrete or masonary structure placed in an open channel over which the flow occurs.

* It is generally in the form of vertical wall with sharp edge at the top.

* The notch is of small size while the weir is of a bigger size.

* Notch is generally made of metallic plate while weir is made of concrete or masonary structure.

Nappe or vein :-

The sheet of water flowing through a notch or over a weir is called Nappe or vein.

Crest or Sill:-

The bottom edge of a notch or a top of a weir over which the water flows is known as the crest or sill.

Classification of notches and weirs:-

The notches are classified as

1. According to the shape of the opening.
   - (a) Rectangular notch.
   - (b) Triangular notch.
   - (c) Trapezoidal notch.
   - (d) Stepped notch.
2. According to the effect of the sides on the nappe.
   - (a) Notch with end contraction.
   - (b) Notch without end contraction.

The weirs are classified as

1. According to the shape of the opening
   - (a) Rectangular weir.
   - (b) Triangular weir.
   - (c) Trapezoidal weir.
2. According to the shape of the crest
   - (a) Sharp crested weir.
   - (b) Broad crested weir
   - (c) Narrow crested weir
   - (d) Ogee-Shaped weir.
3. According to the effect of sides on the emerging nappe.
   - (a) Weir with end contraction.
   - (b) Weir without end contraction.

Discharge over a triangular notch or weir :-

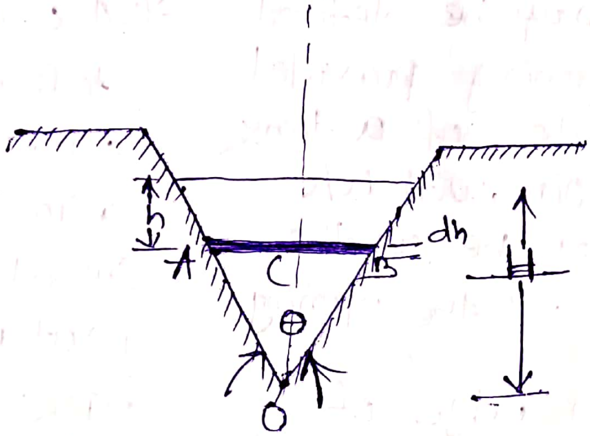
The expression for discharge over a triangular notch or weir is the same.

Let

H = Head of water above the V-notch

θ = Angle of notch.

For finding out the total discharge consider an elementary strip of water of thickness 'dh' at a depth 'h' from free surface of liquid.



$dQ = C_d \times dA \times V$

$= dh \times AB \times \sqrt{2gh}$

$= C_d \times 2 \times (H-h) \times \tan\theta/2 \times \sqrt{2gh} \times dh$

$\int_0^Q dQ = \int_0^H C_d \times 2 \times (H-h) \times \tan\theta/2 \times \sqrt{2gh} \times dh$

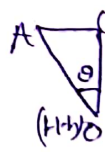
$Q = 2 C_d \times \sqrt{2g} \tan\theta/2 \cdot \int_0^H (H-h)\sqrt{h}\, dh$

$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \int_0^H (Hh^{1/2} - h^{3/2})\, dh$

~~$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \int (Hh^{1/2} - h^{3/2})\, dh$~~

$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$

$\tan\theta/2 = \frac{AC}{OC}$



$= \frac{AC}{H-h}$

$AC = (H-h)\tan\theta/2$

$AB = 2AC$

$= 2(H-h) \times \tan\theta/2$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \left[\frac{H \times H^{3/2}}{3/2} - \frac{H^{5/2}}{5/2} \right]$$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \left[\frac{2H^{5/2}}{3} - \frac{2}{5} H^{5/2} \right]$$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \times \frac{4}{15} H^{5/2}$$

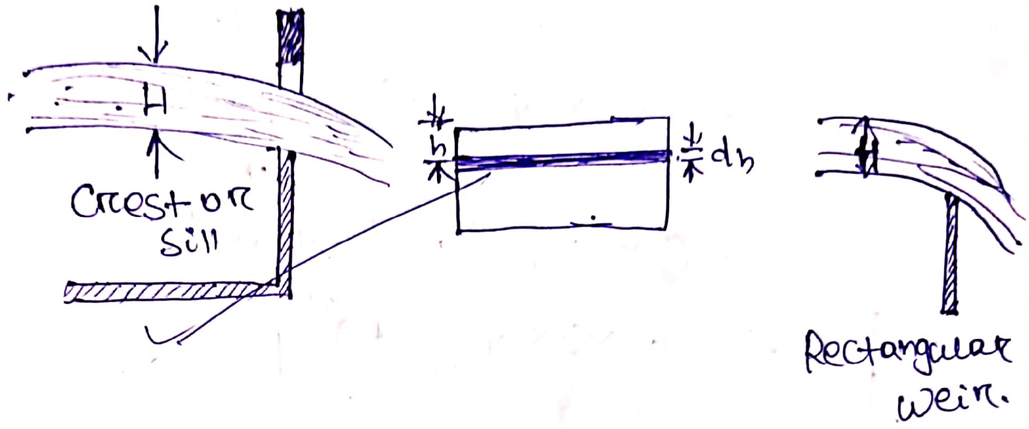
$$= \frac{8}{15} \times C_d \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}$$

Difference between Notch and weir

Notch	Weir
* A notch may be defined as an opening provided in one side of a tank or reservoir, with u/s liquid level below the top edge of the opening.	* A weir may be defined as a structure constructed across a river or canal to store water on the upstream side.
* The bottom edge of notch over which water flows is known as sill or crest.	* The top of the weir over which water flows is known as crest.
* A notch is usually made of a metallic plate.	* A weir is made of cement concrete or masonary.
* Notches are of small size.	* Weirsed are of bigger size.
* A notch is used to measure small discharge of small steam or canal.	* A weir is used to measure large discharge of rivers and large canals.

Discharge over a Rectangular notch:-

The expression for discharge over a rectangular notch or weir is the same.



Consider a rectangular notch or weir provided in a channel carrying water.

Let,

H = Head of water over the crest.

L = Length of the notch or weir.

Consider an elementary horizontal strip of water of thickness 'dh' and length 'L' at a depth 'h' from the free surface of water.

The area of the strip $= L \times dh$

Theoretical velocity of water flowing through th strip $= \sqrt{2gh}$

The discharge dQ through the strip is

$$dQ = C_d \times L \times \sqrt{2gh} \times dh \quad (C_d \times dQ_{th} = C_d \times A_{th} \times V_{th})$$

The total discharge Q for the whole notch or weir is determined by integrating the above equation between the limits 0 and H.

$$Q = \int_0^H C_d . L . \sqrt{2gh}\, dh$$

$$= C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \times \frac{2}{3} \times (H)^{3/2}$$

$$\boxed{Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\, (H)^{3/2}}$$

Problem - 1

Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.

Ans:-

Given data,

Length of the notch = 2m

Head over notch = 300mm = 0.3m

$C_d = 0.60$

Discharge $Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\, (H)^{3/2}$

$= \frac{2}{3} \times 0.60 \times 2 \times \sqrt{2 \times 9.81} \times (0.3)^{\frac{3}{2}}$

$= 0.582\ m^2/s$.

Problem :- 2

Find the discharge over a triangular notch of angle 60°, when the head over the V notch is 0.3m. Assume $C_d = 0.6$.

Ans:-

Given data:-

$\theta = 60°$

$H = 0.3m$

$C_d = 0.6$

so,

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{\frac{5}{2}}$$

$$= \frac{8}{15} \times 0.6 \times \tan\frac{60°}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{\frac{5}{2}}$$

$$= 0.040 \ m^3/s \quad \text{(ANS)}$$

Problem :- 3

Water flows over a rectangular weir 1m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively. Find the depth over the triangular weir.

Ans:- Given data:-

For rectangular,

$H = 150 mm = 0.15 m$

$C_d = 0.62$

$L = 1m$

$$\text{Discharge} = \frac{2}{3} C_d \times L \times \sqrt{2g} \times (H)^{\frac{3}{2}}$$

$$= \frac{2}{3} \times 0.62 \times 1 \times \sqrt{2 \times 9.81} \times (0.15)^{\frac{3}{2}}$$

$$= 0.1063 \ m^3/sec.$$

For triangular:-

$C_d = 0.59$

$H = ?$, $\theta = 90^\circ$

Here,

Rectangular notch = Triangular notch

So,

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times (H)^{\frac{5}{2}}$$

$$\Rightarrow 0.1063 = \frac{8}{15} \times 0.59 \times \tan\frac{90}{2} \times \sqrt{2 \times 9.81} \times (H)^{\frac{5}{2}}$$

$$\Rightarrow 0.1063 = 1.3937 \times (H)^{\frac{5}{2}}$$

$$\Rightarrow 1.3937 \times (H)^{\frac{5}{2}} = 0.1063$$

$$\Rightarrow (H)^{\frac{5}{2}} = \frac{0.1063}{1.3937}$$

$$\Rightarrow (H)^{\frac{5}{2}} = 0.076$$

$$\Rightarrow H = (0.076)^{\frac{2}{5}}$$

$$\Rightarrow H = 0.35 \ m$$ (ANS)

Chapter:- 06 Flow through Pipes

Pipe:- Pipe is defined as a close conduit through which the flow occures.

Energy loss in pipe:- When a ~~Pipe~~ fluid is flowing through a Pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

This loss of energy is classified as

Energy loss

- Major energy loss
  - This is due to friction
  - It can be calculated by using
    - (a) Darcy-Weisbach formula
    - (b) Chezy's formula
- Minor energy loss
  - (a) Sudden enlargement of Pipe
  - (b) Sudden contraction of pipe
  - (c) Bend in Pipe
  - (d) Pipe fitting
  - (e) Obstruction in pipe.

Loss of energy due to friction:-

(a) Darcy - Weisbach formula.
(b) Chezy's formula.

(a) Darcy weisbach formula:-

According to ~~das~~ Darcy-weisbach the loss of energy or head loss due to friction can be calculated by using the formula

$$\boxed{h_f = \frac{4fLV^2}{2gd}}$$

Where

h_f = Head loss due to friction

f = Co-efficient of friction

$= \frac{16}{Re}$ ($Re < 2000$)

$= \frac{0.079}{Re^{\frac{1}{4}}}$ (When Re lies between 4000 to 10^6)

$$\boxed{Re = \frac{Vd}{\nu}}$$

L = Length of the pipe

V = Mean velocity of flow

d = diameter of pipe

ⓑ Chezy's formula:-

According to chezy

$$V = C \times \sqrt{m \times i}$$

$$\boxed{i = \frac{h_f}{L}}$$

Where, V = Mean velocity of flow

C = Chezy's constant

$$m = \frac{A}{P} = \frac{\text{Area of flow}}{\text{Perimeter}} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

= Hydraulic mean depth.

Problem:- 11.1:-

Find the head lost due to friction in a pipe of diameter 300mm and length 50m, through which water is flowing at a velocity of 3m per sec using

① Darcy formula.

② Chezy's formula for which C=60

Take ν for water = 0.01 Stoke.

Ans:- Given data,

Diameter of pipe = 300 mm = 0.3 m

$$\text{Area} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.3)^2 = 0.070\, m^2$$

Length = 50 m

$C = 60$

ν for water = $0.01 = 0.01 \times 10^{-4} = 1 \times 10^{-6}$

Velocity = 3 m/sec.

① For Darcy formula,

$$h_f = \frac{4fLV^2}{2gd}$$

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 900000$$

$$f = \frac{0.079}{(Re)^{1/4}} = \frac{0.079}{(900000)^{1/4}} = 2.56 \times 10^{-3}$$

So,

$$h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times 2.56 \times 10^{-3} \times 50 \times 3^2}{2 \times 9.81 \times 0.3} = 0.7828\, m$$

② For Chezy formula,

$$V = C\sqrt{mi}$$

We know that,

$$m = \frac{\text{Area}}{\text{Perimeter}} = \frac{d}{4} = \frac{0.3}{4} = 0.075$$

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So,

$$i = \frac{hf}{l} = \frac{hf}{50}$$

Then,

$$V = C\sqrt{mi}$$

$$\Rightarrow 3 = 60\sqrt{0.075 \times \frac{hf}{50}}$$

$$\Rightarrow 3 = 60\sqrt{0.075} \times \sqrt{\frac{hf}{50}}$$

$$\Rightarrow 3 = 60 \times 0.273 \times \sqrt{\frac{hf}{50}}$$

$$\Rightarrow 3 = 16.38 \times \sqrt{\frac{hf}{50}}$$

$$\Rightarrow 16.38 \times \sqrt{\frac{hf}{50}} = 3$$

$$\Rightarrow \sqrt{\frac{hf}{50}} = \frac{3}{16.38}$$

$$\Rightarrow \sqrt{\frac{hf}{50}} = 0.1831$$

$$\Rightarrow \frac{hf}{50} = (0.1831)^2$$

$$\Rightarrow \frac{hf}{50} = 0.0335$$

$$\Rightarrow hf = 0.0335 \times 50 = 1.676 \text{ m (ANS)}$$

Problem 11.2:-

Find the diameter of pipe of length 2000 m when the rate of flow of water through ~~the~~ is 200 lit/sec and the head lost due to friction is 4m. Take value of C = 50 in Chezy's formula.

Ans:- Given data,

$$Q = 200 \text{ lit/sec} = \frac{200}{1000} = 0.2 m^3/sec.$$

$$C = 50$$

$$\text{length} = 2000m$$

$$h_f = 4m$$

So,

$$V = C\sqrt{mi}$$

$$m = \frac{d}{4}$$

$$i = \frac{h_f}{l} = \frac{4}{2000} = 2 \times 10^{-3}$$

We know that,

$$Q = aV$$

$$\Rightarrow V = \frac{Q}{a}$$

$$\Rightarrow V = \frac{0.2}{\frac{\pi}{4} \times d^2} = \frac{0.2 \times 4}{\pi d^2}$$

$$V = C\sqrt{mi}$$

$$\Rightarrow \frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times 2 \times 10^{-3}}$$

$$\Rightarrow \frac{0.8}{\pi d^2} = 50 \times \sqrt{\frac{d}{4}} \times \sqrt{2 \times 10^{-3}}$$

$$\Rightarrow \frac{0.8}{\pi d^2} = 50 \times \sqrt{\frac{d}{4}} \times 0.044$$

$$\Rightarrow \frac{0.8}{\pi d^2} = 2.2 \times \sqrt{\frac{d}{4}}$$

$$\Rightarrow \frac{0.8}{\pi} \times \frac{1}{d^2} = 2.2 \times \sqrt{\frac{d}{4}}$$

$$\Rightarrow \frac{0.25}{d^2} = 2.2 \times \sqrt{\frac{d}{4}}$$

$$\Rightarrow 2.2 \times \sqrt{\frac{d}{4}} = \frac{0.25}{d^2}$$

$$\Rightarrow \sqrt{\frac{d}{4}} = \frac{0.25}{d^2 \times 2.2}$$

$$\Rightarrow \sqrt{\frac{d}{4}} = \frac{1}{d^2} \times \frac{0.25}{2.2}$$

$$\Rightarrow \sqrt{\frac{d}{4}} = \frac{1}{d^2} \times 0.113$$

$$\Rightarrow \sqrt{\frac{d}{4}} = \frac{0.113}{d^2}$$

$$\Rightarrow \frac{d}{4} = \left(\frac{0.113}{d^2}\right)^2$$

$$\Rightarrow \frac{d}{4} = \frac{0.012}{d^4}$$

$$\Rightarrow d^5 = 0.012 \times 4$$

$$\Rightarrow d^5 = 0.048$$

$$\Rightarrow d = (0.048)^{\frac{1}{5}} = 0.544 \text{ m} \quad \text{(ANS)}$$

Hydraulic gradient and total energy line:-

Hydraulic gradient Line (HGL):-

It is defined as the line which gives the sum of pressure head $\left(\frac{P}{\rho g}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line.

Or

It is the line which is obtained by joining the top of ~~th~~ all vertical ordinates showing pressure head of a flowing fluid in a pipe from the centre of the pipe.

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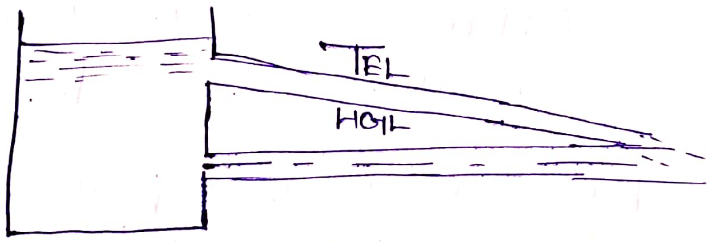
Total energy line (TEL):-

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

or

It is defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.



Impact of Jet

Jet:- It is a stream of fluid issuing from a nozzle with a high velocity.

Impact of Jet:- Impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

The various cases of impact of jet, which are considered are.

ⓐ Force exerted by the jet on stationary plate when

1. Plate is vertical to jet
2. Plate is inclined to the jet.
3. Plate is curved.

ⓑ Force exerted by the jet on a moving plate when.

1. Plate is vertical to the jet
2. Plate is inclined to the jet
3. Plate is curved

Exerted by the jet on a fixed vertical plate:-

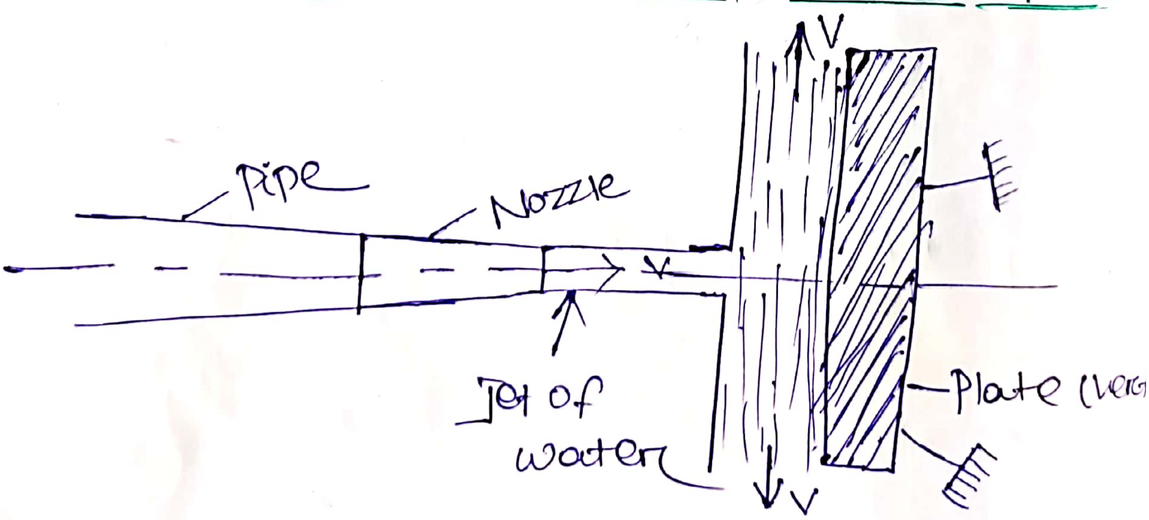


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Plate :-

ate (vertical

V = Velocity of jet
d = Diameter of jet
A = C.S. area of jet = $\pi/4\ d^2$

Force exerted by the jet on the plate in the direction of jet = Rate of change of momentum

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity}}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}}\left[\text{Initial velocity} - \text{Final velocity}\right]$$

$$= \frac{\text{Mass}}{\text{Time}}\left[\text{Initial velocity} - 0\right]$$

$$\rho A V [V - 0] = \rho A V^2$$

$$\frac{M}{V} = \rho \Rightarrow \cancel{M=}$$

$$\Rightarrow M = \rho V$$

$$\Rightarrow \frac{\text{Mass}}{\text{Time}} = \frac{\rho V}{sec} = \rho \frac{V}{sec} = \rho Q = \rho A V$$

Problem :- (1)

What is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre of the nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.

Ans :- Diameter of the nozzle = 100mm = 0.1m

$$Area = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} (0.1)^2 = 85 \times 10^{-3} m$$

$H = 100m$

$C_V = 0.95$

$$V = C_V \sqrt{2gH}$$

$$= 0.95 \sqrt{2 \times 9.81 \times 100}$$

$$= 42.07 \; m^3/sec.$$

$$F_x = \rho a V^2$$

$$= 1000 \times 7.85 \times 10^{-3} \times (42.07)^2$$

$$= 13893.59 \; N$$

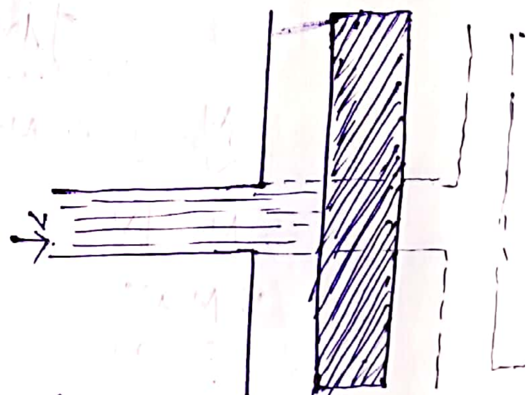
Force exerted by a jet on a flat vertical plate moving in the direction of jet :-

Let,

V = Velocity of jet

u = velocity of plate

a = Area of cross-of jet



In this case the jet does n't strike with a velocity but with a relative velocity $V-u$.

Mass of water striking the plate per sec

$$= \rho \times a \times (V-u)$$

$$F_x = \rho a (V-u) \; [(V-u) - 0]$$

$$= \rho a (V-u)^2$$

Workdone / sec by the jet on the plate

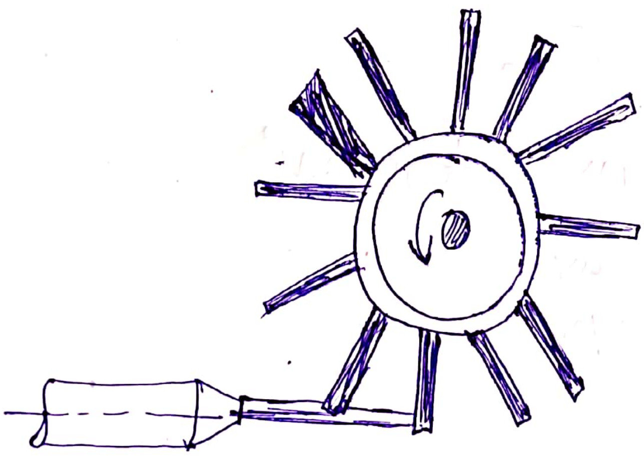
$$= \text{Force} \times \frac{\text{Distance in the direction of form}}{\text{time}}$$

$$= F_x \times u$$

$$= \rho a (V-u)^2 u$$

Force exerted by a jet of water on a series of vanes

The force exerted by a jet of water on a single moving plate is not practically feasible actually a large number of plates are moment on the circumference of a wheel of a fixed distance apart the jet strikes



Let,

V = Velocity of jet.

d = Diameter of jet.

a = Cross-Sectional area of jet.

$$= \frac{\pi}{4} d^2$$

u = Velocity of vane

In this case the mass of water comming out from the nozzle per second is always in contact with the plate when all plates are considered. mass of water per second striking = $\rho a v$.

Jet strikes the plate with a velocity = $V-u$

F_x = Mass per second [Initial velocity - final velocity]

$= \rho a v\,[(v-u) - 0]$

$= \rho a v (v-u)$

Workdone = $F_x \times u$

$= \rho a v (v-u) u$

KE of jet per second

$= \frac{1}{2} m v^2 = \frac{1}{2} \rho a v \times v^2$

$= \frac{1}{2} \rho a v^3$

$$\eta = \frac{\text{Work done/sec}}{\text{KE / sec}}$$

$$= \frac{\rho a v (v-u)\, u}{\frac{1}{2} \rho a v^3} =$$

Condition for maximum efficiency :-

Efficiency will be maximum when

$$\frac{d\eta}{du} = 0 \Rightarrow \frac{d}{du}\left[\frac{2u(v-u)}{v^2}\right] = 0$$

$$\Rightarrow \frac{d}{du}\left[2uv - 2u^2\right] = 0$$

$$\Rightarrow 2v - 4u = 0 \Rightarrow 2v = 4u$$

$$\Rightarrow \boxed{v = \frac{4u}{2} = 2u}$$

Maximum efficiency:-

Substitution, $V=2u$

$$\eta_{max} = \frac{2u(2u-u)}{4u^2}$$

$$= \frac{2u^2}{4u^2} = \frac{1}{2} = 50\%$$

Problem :- 2

A jet of water of diameter 10cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6m/sec in the direction of jet and away from the jet. Find

(i) The force exerted by the jet, on the plate

(ii) Work done by the jet on the plate /sec.

(iii) Efficiency of the jet?

S

Diameter of jet = 10cm = 0.1 m

velocity = 15 m/s

moving velocity = 6m/sec

Area = $\frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.1)^2 = 7.85 \times 10^{-3} \ m^2$

$$F_x = \rho a (V-u)^2$$

$$= 1000 \times 7.85 \times 10^{-3} (15-6)^2$$

$$= 635.85 \ N$$

(ii) Workdone = $F_x \times u$

$= 635.85 \times 6$

$= 3815.1$ J/sec.

(iii) Efficiency = $\dfrac{F_x \times u}{\frac{1}{2}\rho a v^3}$

$$= \frac{1000 \times 7.85 \times 10^{-3} (15-6)^2 \times 6}{\frac{1}{2} \times 1000 \times 7.85 \times 10^{-3} \times (15)^3}$$

$= 0.288$

Problem:- (3)

A nozzle of 50 mm diameter delivers a steam of water at 20 m/sec perpendicular to a plate that moves away from the jet at 5 m/sec. Find

(i) The force exerted on the plate.
(ii) The workdone per sec.
(iii) The efficiency of the jet.

Ans:-

Diameter of nozzle = 50 mm
= 0.005 m

velocity = 20 m/sec

Moving velocity = 5 m/sec

Area = $\frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.005)^2 = 1.96$
10^-

$F_x = \rho a (V-u)^2$

$= 1000 \times 1.96 \times 10^{-3} (20-5)^2$

$= 441 N$

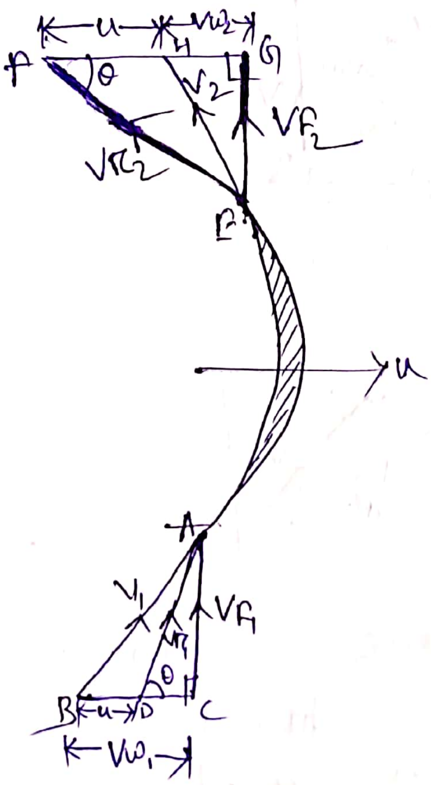
Workdone $= F_x \times u$

$= 441 \times 5 = 2205 \ J/sec$

Efficiency $= \dfrac{F_x \times u}{\frac{1}{2} \rho a V^3}$

$= \dfrac{2205}{\frac{1}{2} \times 1000 \times 1.96 \times 10^{-3} \times 20^3} = 0.28$

Force exerted by a jet of water on ~~an~~ moving curved unsymmetrically Plate when jet strikes tangentially at one of the tip:-



Where,

V_1 = Velocity of jet at inlet.

u_1 = Velocity of Plate at inlet.

V_{r1} = Relative Velocity of jet and Plate at inlet.

α = Guide blade angle.

θ = Vane angle at inlet.

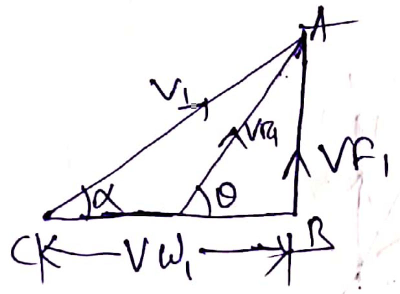
Similarly, V_2, u_2, V_{r2}, V_{f2} and V_{w2} are the Correct values at out let,

ϕ = vane angle at wheel.

β = Guide blade angle at vessel

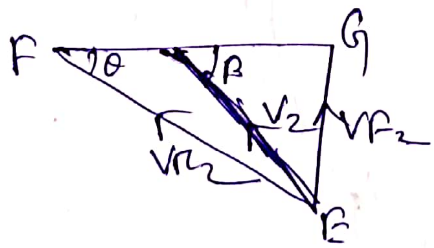
Inlet velocity traingle:-

ΔABC



Outlet velocity traingle:-

ΔEFG



If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal, Then,

$$u_1 + u_2 = u$$

$$V_{r_1} = V_{r_2}$$

$\rho A V_{r_1}$ = Mass of water striking vane/sec

$F_x = M/s$ [Initial velocity − Final velocity]

$$= \rho A V_{r_1} [V_{r_1} \cos\theta - (-V_{r_2} \cos\phi)]$$

$$= \rho A V_{r_1} [V_{w_1} - u_1 + (V_{w_2} + u_2)]$$

$$= \rho A V_{r_1} [V_{w_1} + V_{w_2}]$$

If,

$$\beta = 90^\circ$$

$$V_{w_2} = 0$$

$$F_x = \rho A V_{r_1} [V_{w_1}]$$

If,

$$\beta => 90^\circ$$

$$F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]$$

In general,

$$F_x = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]$$

Workdone per second:-

$$F_x \times u$$

$$= \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u$$